Universal enveloping algebras of free Jordan algebras and their associated graded algebras

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I plan to speak on the structure of universal enveloping algebras of free Jordan algebras and of their associated graded algebras.

With any Jordan algebra J, the two universal associative algebras can be associated: the universal associative enveloping algebra $S(J)$ with a homomorphism $i\colon J \to S(J)^{(+)}$, which is the universal object for specializations of J, that is, for homomorphisms of J into special Jordan algebras, and the universal multiplicative enveloping algebra $U(J)$, which is the universal object for representations of J. For a unital J, the algebra $S(J)$ is an ideal in $U(J)$, and any specialization of J may be considered as a representation.

The universal algebras $U(J)$ and $S(J)$ play important role for the structure theory and representations of Jordan algebras. Many important results in Jordan Theory have been proved by using relations between properties of an algebra J and of its universal enveloping algebras.

Contrary to the Lie algebra case, there is no "canonical base" for the algebras $S(J)$ and $U(J)$. On the other hand, these algebras are more closely related to the algebra J; for instance, if J is finite dimensional then so are $S(J)$ and $U(J)$. The both algebras $S(J)$ and $U(J)$ have natural ascending filtrations, and one can consider the associated graded algebras $gr\,S(J)$ and $gr\,U(J)$. These algebras have more simple structure; for instance, the algebra $gr\,S(J)$ is a homomorphic image of the exterior algebra $\Lambda(J)$.

The main objective of the talk is the structure of the algebras $gr\,U(J)$ and $gr\,S(J)$ when J is the free Jordan algebra $Jord[X]$ or the free special Jordan algebra $SJord[X]$. Observe that $S(SJord[X]) = As[X]$, so the first algebra is just the associated graded algebra of the free associative algebra $As[X]$ with respect to the Jordan filtration. More exactly, for $J = SJord[X]$ we have

$$J_0 = F \subset J_1 = J \subset J_2 \subset \cdots \subset J_k \subset \cdots As[X],$$

where J_k is the subspace of $As[X]$ generated by all products of at most k Jordan elements. It is known that if X is finite then the above filtration is finite. We find estimates for the length of this filtration and investigate the structure of terms J_k for $k > 1$. The similar questions we consider for the graded algebra $gr\,U(Jord[X])$.

It is a join research with S. Sverchkov (SO RAN, Novosibirsk).