Orthogonal arrays and their distance distributions

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Orthogonal arrays were introduced by Rao (1946) [4] and since then have been studied by many researchers from various fields. Let H_q be an alphabet of q letters and H_q^n be the Hamming space over H_q. An orthogonal array (OA) of strength τ and index λ in H_q^n consists of the rows of an $M \times n$ matrix C with the property that every $M \times t$ submatrix of C contains all ordered τ-tuples of H_q^n, each one exactly $\lambda = M/q^\tau$ times as rows. We denote C by $OA(M, n, q, \tau)$.

Some of important parameters of the (M, n, q, τ) orthogonal array C are its distance distributions, minimum distance and covering radius. There are known several methods for computing all possible distance distributions, all of them variations of a classical result of Delsarte [1, 2] (see also [3, 5]). Furthermore, the set of all possible distance distributions can be reduced by investigations of the relations between C and its related orthogonal arrays. The information about distance distributions of a given orthogonal array implies bounds for their minimum distance and covering radius. If all possible distance distributions are known, then the minimum distance and covering radius can be found. We describe various methods dealing with the sets of feasible distance distributions of orthogonal arrays. This leads to good (in some cases sharp) bounds on the minimum distance and covering radius.

References

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