On forms of G-graded simple algebras

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Let G be finite group and D a finite dimensional G-graded division algebra e-central over k (k consists of the central e-homogeneous elements of D). Restricting scalars to the algebraic closure $F = \bar{k}$, we obtain a finite dimensional G-graded simple algebra. In this lecture we consider the problem in the opposite direction, namely if A is a finite dimensional G-graded simple algebra over F (with $char(F) = 0$), then we ask under which conditions it admits a G-graded division algebra form (in the sense of descent theory)? (i.e. nonzero homogeneous elements are invertible). More restrictive, we ask under which conditions A admits a division algebra G-graded form? (i.e. nonzero elements are invertible). We provide a complete answer for the first question and only a partial one for the second. The main tools come from PI-theory. These allow us to construct the corresponding generic objects. Joint works with (1) Haile and Karasik, (2) Karasik.

Quasi-polynomial growth of minimal free resolutions

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Every module over an arbitrary ring can be "resolved" by sequences of homomorphisms of free modules. Finitely generated modules over noetherian local rings (or graded connected algebras) admit minimal resolutions that are unique up to isomorphism; the ranks of the free modules in such resolutions provide important intensely studied invariants of the module. It a long-open question to describe the rings over which those ranks are eventually given by (quasi-)polynomials. Recent results will be presented for commutative rings; their proofs hinge on the structure of super Lie-algebras canonically associated to such rings. The talk is partly based on joint work with N. Packauskas and M. Walker, and with A. Seceleanu and Z. Yang.

Delta sets and polynomial identities in pointed Hopf algebras

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We survey a vast array of known results and techniques in the area of polynomial identities in pointed Hopf algebras. Some new results are proven in the setting of Hopf algebras that appeared in the papers of D. Radford and N. Andruskiewitsch - H.-J. Schneider.

This is joint work with Sarah Whitherspoon from Texas A& M University.

Specht's theorem, commutativity theorems, and decision procedures

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We consider the following question: Given a finite set of multivariate polynomial identities $P_1 = P_2 = \cdots = P_d = 0$, is it the case that a ring which satisfies these identities is necessarily commutative? As it turns out, one can use work related to Specht's theorem on affine representability to give a decision procedure which takes as input the set of identities and terminates after finitely many steps and gives an answer to this question. We then revisit old commutativity theorems of Jacobson and Herstein in light of this algorithm and obtain general results in this vein. This is joint work with Peter Danchev.

On the asymptotics for $*$-graded Capelli identities

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The finite dimensional simple $*$-superalgebras over an algebraically closed field of characteristic zero have been classified in [3]. The main goal of this talk is to show a characterization of the $T^*_{\mathbb{Z}_2}$-ideal of $*$-graded polynomial identities of any such algebra by considering the growth of the corresponding variety.

We prove that the $*$-graded codimensions of the finite dimensional simple $*$-superalgebras are asymptotically equal to the $*$-graded codimensions of the $T^*_{\mathbb{Z}_2}$-ideal generated by a set of $*$-graded Capelli polynomials.

Similar results have been found for simple finite dimensional algebras in [4], for simple finite dimensional superalgebras in [1] and for simple finite dimensional algebra with involution in [2].

This talk is based on a joint work with A. Valenti.

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Identities and Trace Identities of Verbally Prime Algebras

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In Kemer's classification of varieties of p. i. algebras the verbally prime algebras are represented by (1) matrices over the field, (2) $M_{k,l}$ and (3) matrices over the Grassmann algebra. Inspired by this, I will show that many of the known theorems about identities and trace identities of matrices can be generalized to the other two.

The $\mathbb{Z}$-gradings on the Grassmann algebra and Arithmetic tools

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The Grassmann algebra E of an infinite-dimensional vector space L is one of the most important algebras satisfying a polynomial identity. The celebrated papers of Kemer have it as a key ingredient. In this talk, we will introduce the structures of gradings on E whose support coincides with a subgroup of the group $\mathbb{Z}$. We present in more details the so-called 2 and 3-induced $\mathbb{Z}$-gradings on E. In these cases, we provide a better criterion for their supports, and we describe the graded identities in all of them. As a consequence of this fact we give examples of $\mathbb{Z}$-gradings on E which are PI-equivalent but not $\mathbb{Z}$-isomorphic. This is the first example of graded algebras with infinite support that are PI-equivalent and not isomorphic as graded algebras. We strongly use Elementary Number Theory as a tool, providing an interesting connection between this area and PI-Theory. Our results are new and this is a joint work with A. Guimarães (UFRN), A. Brandão (UFCG) and P. Koshlukov (UNICAMP).

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Symmetric polynomials in three noncommuting variables

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In 1936 Margarete Caroline Wolf published a paper where she proved that the symmetric polynomials in the free associative algebra form a free subalgebra and described the system of free generators. The aim of the talk is to present these results from modern point of view. Their relations with other results in the frames of commutative and noncommutative invariant theory are considered.

If symmetric group of degree $n > 2$ acts as permuting the variables on the free algebra of the same number of variables, then the algebra of symmetric polynomials in noncommuting variables is not finite generating ([2, 3, 5]). It has turned out that the analogue of the theorem of Emmy Noether for the finite generation of $K[X_d]^G$ for finite groups G holds for $K\langle X_d\rangle^G$ in very special case only. Koryukin ([4]) proved that if there is an extra action of symmetric group of degree n, then algebra of invariants of every reductive group (in particular finite) is finite generating.

We present our results in the case of symmetric polynomials in three noncommuting variables.

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Decomposing Linear Codes over Finite Fields using Permutation Groups[1]

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A linear $[n, k]$ code C is a k-dimensional subspace of the vector space $\mathbb{F}_q^n$, where $\mathbb{F}_q$ is the finite field of q elements. We say that two codes C_1 and C_2 of the same length over $\mathbb{F}_q$ are equivalent provided there is a monomial matrix M and an automorphism γ of the field such that $C_2 = C_1 M\gamma$. The set of coordinate permutations that map the code C to itself forms a group, called the permutation automorphism group of C and denoted by $PAut(C) < \mathrm{S}_n$. Two more groups can be considered - the monomial automorphism group $MAut(C)$, and the group $\Gamma Aut(C)$ consisting of the maps of the form $M\gamma$, that map C to itself.

Let $(u, v) : \mathbb{F}_q^n \times \mathbb{F}_q^n \to \mathbb{F}_q$ be an inner product in the linear space $\mathbb{F}_q^n$. If C is an $[n, k]$ linear code, then its orthogonal complement $C^\perp = \{u \in \mathbb{F}_q^n : (u, v) = 0\ \forall v \in C\}$ is a linear $[n, n-k]$ code. If $C = C^\perp$, C is termed self-dual.

The purpose of this talk is to present the structure of the linear codes over a finite field with q elements that have permutation automorphisms of prime order. Methods to construct and classify self-dual codes under the assumption that they have an automorphism of a given prime order are given in [1, 2]. These methods are extended to linear codes with permutation and monomial automorphisms over larger fields

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Zero product determined algebras

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A not necessarily associative algebra A over a field F is said to be zero product determined if every bilinear functional $\varphi : A \times A \to F$ with the property that $xy = 0$ implies $\varphi(x, y) = 0$ is of the form $\varphi(x, y) = \tau(xy)$ for some linear functional τ on A. These algebras have been studied in pure algebra as well as in functional analysis where one additionally assumes that φ and τ are continuous.

The talk will survey the general theory and applications of zero product determined algebras.

On the classification of multiplicity-free products of Schur functions

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Schur functions are symmetric polynomials introduced by Schur as characters of irreducible representations of the general linear group of invertible matrices. Schur functions can be generated combinatorially using semistandard Young tableaux and form a basis for the ring of symmetric functions. Basically there are three fundamental products on the ring of symmetric functions, namely the ordinary product, the Kronecker product and the plethysm of Schur functions. All these three products of Schur functions are again symmetric functions and can be expressed in terms of Schur functions basis.

We will present the most recent results on various products of Schur functions which are multiplicity-free in the sense that the coefficients which arise in their expansion as a sum of Schur functions are $0, 1$.

Nowicki's conjecture and about

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We draw up an historical timeline of results towards Nowicki's conjecture and its related themes.

Deformations of vector bundle-valued Higgs bundles and L_∞-algebras

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In this talk we shall discuss some work in progress on obstruction spaces to deformations of Higgs bundles (over a smooth projective variety/$\mathbb{C}$) with coefficients in a vector bundle. For that we shall use some parts of the theory of deformations via differential graded Lie algebras and L-infinity algebras.

Algebraic structures in deep learning

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Exploiting the structure of data is a key component for successful generalization in deep learning that has surpassed limitations predicted by conventional machine learning and statistical learning theory. Properties of symmetries and scale separability of data are captured by algebraic structures and their representations guide the construction of invariant and equivariant models, useful to deep learning. In the first part of this talk we will survey some of the modelling trends in this direction. Then, we will explore the learning algorithms used for training the models and present preliminary results on how they might benefit from imposing algebraic structures on data. The first part of the talk will be based on arXiv:2104.13478.

Biangular lines with maximal sum of the squared inner products

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Line systems, passing through the origin of an n-dimensional Euclidean sphere generate corresponding antipodal spherical codes. When these line systems admit exactly two angles between their elements, they are called biangular and the corresponding spherical codes have five inner products. Recently Ganzhinov and Szöllösi [1] explored various connections between biangular lines and other objects such as binary codes, few-distance sets and association schemes. They also posed several open problems, connected to the existence of biangular lines with some parameters. Here we give an answer to one of them.

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On superalgebras with superinvolution and their *-graded polynomial identities

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In this talk we consider block-triangular matrix algebras with superinvolution related to a finite sequence of finite dimensional simple *-superalgebras. These superalgebras and their *-graded polynomial identities appear in the positive solution of Specht's problem for algebras with involution, given in [1] by Aljadeff, Giambruno and Karasik. The finite dimensional simple *-superalgebras are the basic buildings in order to determine the exact value of the *exponent* for finitely generated superalgebras with superinvolution (see [2]). In this talk we discuss some problem related to it.

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On the Number of Gradings on Matrix Algebras

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In this talk we determine the number of isomorphism classes of elementary gradings by a finite group on an algebra of upper block-triangular matrices. As a consequence we prove that, for a finite abelian group G, the sequence of the numbers $E(G,m)$ of isomorphism classes of elementary G-gradings on the algebra $M_m(\mathbb{F})$ of $m \times m$ matrices with entries in a field $\mathbb{F}$ characterizes G. A formula for the number of isomorphism classes of gradings by a finite abelian group on an algebra of upper block-triangular matrices over an algebraically closed field, with mild restrictions on its characteristic, is also provided. Finally, if G is a finite abelian group, $\mathbb{F}$ is an algebraically closed field and $N(G,m)$ is the number of isomorphism classes of G-gradings on $M_m(\mathbb{F})$ we prove that $N(G,m) \sim \frac{1}{|G|!} m^{|G|-1} \sim E(G,m)$. These results were obtained in [1] in collaboration with Daniel Pellegrino.

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Invariant theory in varieties of associative algebras

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Invariant theory deals with subalgebras of invariants of groups acting via algebra automorphisms on finitely generated algebras in the variety of commutative associative algebras. A natural context to develop *noncommutative* invariant theory is obtained by replacing the variety of commutative algebras by other varieties of associative algebras. The role of the commutative polynomial ring is then taken up by the relatively free algebras in the variety considered, and we are led to the study of the subalgebra of invariants of a group acting via linear substitutions of the generators of a relatively free algebra. Although some key results of commutative invariant theory about finite generation do not fully extend to this context, notable statements hold for certain varieties of associative algebras. In the talk we shall survey works in this direction, done in recent years in collaboration with Vesselin Drensky.

Locally nilpotent derivations of polynomial and other free algebras

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We shall survey some results on locally nilpotent derivations of polynomial algebras over fields of characteristic zero and their analogues for free associative algebras and for relatively free algebras of varieties of algebras. A special attention will be paid on the Weitzenböck derivations and on the Nowicki conjecture, their commutative and noncommutative generalizations, and their relations with classical and noncommutative invariant theory.

Associative-admissible operad

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An algebra is called associative-admissible, if this algebra under anti-commutator $\{a, b\} = ab + ba$ becomes associative. For example, Zinbiel algebra, i.e., algebra with identity $a(bc) - (ab + ba)c = 0$ is associative-admissible. Associative algebra is associative-admissible iff it satisfes the identity $[[a, b], c] = 0$. Let $\mathcal{A}sAdm$ and $\mathcal{L}eib$ be operads generated by associative-admissible algebras and two-sided Leibniz algebras respectively.

Associative-admissible operad has the following properties.

- Operads $\mathcal{A}sAdm$ and $\mathcal{L}eib$ are Koszul
- $\mathcal{A}sAdm^! = \mathcal{L}eib$
- $\mathcal{A}sAdm = \mathcal{A}sCom \star \mathcal{A}com$
- Dimensions of multi-linear parts $d_n = \dim \mathcal{A}sAdm(n)$ satisfy the following recurrence relations
$$d_n = \sum_{k=1}^{n-1} k! F_{k+2} B_{n-1,k}(d_1, d_2, \ldots, d_{n-k}), \quad n > 1,$$
$$d_1 = 1,$$
where F_n are Fibonacci numbers and $B_{n,k}(x_1, \ldots, x_{n-k+1})$ are Bell polynomials.
- If p is prime, then
$$d_{p-1} \equiv \begin{cases} 1(mod\, p), & \text{if } p \neq 3, \\ -1 & \text{if } p = 3, \end{cases}$$
$$d_p \equiv \begin{cases} 1(mod\, p), & \text{if } p \neq 2, \\ 0 & \text{if } p = 2, \end{cases}$$
$$d_{p+1} \equiv 2(mod\, p),$$
$$d_{p+2} \equiv 10(mod\, p).$$

Non-group gradings on simple Lie algebras

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A set grading on the split simple Lie algebra of type D_{13}, that cannot be realized as a group-grading, is constructed by splitting the set of positive roots into a disjoint union of pairs of orthogonal roots, following a pattern provided by the lines of the projective plane over $GF(3)$. This answers in the negative a question posed in 2013.

Similar non-group gradings are obtained for types D_n with $n \equiv 1 \pmod{12}$, by substituting the lines in the projective plane by blocks of suitable Steiner systems.

On the Lvov-Kaplansky conjecture

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In this talk we will discuss a conjecture attribuited to Lvov and Kaplansky which says that the image of a multilinear polynomial over the matrix algebra is a vector space. We will be particularly interested in variations of such conjecture and recent positive results, for instance the images of multilinear polynomials in some matrix subalgebras.

Symmetric Polynomials in nonassociative algebras

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Let F_n be the free metabelian Lie, Leibniz, or Poisson algebra of rank n generated by $x_1, \ldots, x_n$ over a field of characteristic zero. A polynomial $p \in F_n$ is called symmetric if $p(x_1, \ldots, x_n) = p(x_{\pi 1}, \ldots, x_{\pi n})$, for every permutation $\pi \in S_n$. The set $F_n^{S_n}$ of symmetric polynomials is the algebra of invariants of the group S_n. We review the description of the algebra $F_n^{S_n}$ in the light of recent results [1, 2, 3].

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Algebras defined by Lyndon words and Artin-Schelter regularity

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Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite alphabet, and let K be a field. We study classes $\mathfrak{C}(X,W)$ of graded K-algebras $A = K\langle X\rangle/I$, generated by X and with *a fixed set of obstructions* W. Initially we do not impose restrictions on W and investigate the case when the algebras in $\mathfrak{C}(X,W)$ have polynomial growth and finite global dimension d. Next we consider classes $\mathfrak{C}(X,W)$ of algebras whose sets of obstructions W are antichains of Lyndon words. The central question is "*when a class* $\mathfrak{C}(X,W)$ *contains Artin-Schelter regular algebras?*" We show that each class $\mathfrak{C}(X,W)$ defines a Lyndon pair (N,W) which, if N is finite, determines uniquely the Gelfand-Kirillov dimension, $GK \dim A$ and the global dimension, $gl \dim A$, for every $A \in \mathfrak{C}(X,W)$. More precisely, we prove that A has polynomial growth of degree d if and only if its sets of Lyndon atoms N has order d. In this case A has global dimension d and is standard finitely presented, with $d - 1 \leq |W| \leq d(d-1)/2$. We find a combinatorial condition in terms of (N,W), so that the class $\mathfrak{C}(X,W)$ contains the enveloping algebra $U\mathfrak{g}$, of a Lie algebra $\mathfrak{g}$. We introduce *monomial Lie algebras defined by Lyndon words*, and prove results on Gröbner-Shirshov bases of Lie ideals generated by Lyndon-Lie monomials. Finally we classify all two-generated Artin-Schelter regular algebras of global dimension 6 and 7 occurring as enveloping $U = U\mathfrak{g}$ of *standard monomial Lie algebras*. The classification is made in terms of their Lyndon pairs (N,W), each of which determines also the explicit relations of U.

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Functions holomorphic over finite-dimensional commutative associative unital $\mathbb{C}$-algebras

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Let $\mathcal{A}$ be a finite-dimensional commutative associative unital $\mathbb{R}$-algebra and let $U \subseteq \mathcal{A}$ be an open subset. A function $f : U \to \mathcal{A}$ is called $\mathcal{A}$-differentiable at the point $Z_0 \in U$ iff

$$f'(Z_0) := \lim_{\substack{H \to 0 \\ H \in \mathcal{A}^\times}} \frac{f(Z_0 + H) - f(Z_0)}{H}$$

exists. When $\mathcal{A}$ carries a complex structure, $\mathcal{A}$-*holomorphic* functions exhibit a theory very similar to the classical theory of a single complex variable despite being functions of several complex variables. In particular, such functions admit:

- generalized Cauchy-Riemann PDEs with *complex* coeffcients;
- one-variable Cauchy's and Morera's Integral Theorems;
- one-variable Homological Cauchy's Integral Formula;
- analyticity over $\mathcal{A}$.

We then discuss $\mathcal{A}$-meromorphic functions, their singularities, and finally go on to introduce Riemann "Surfaces" over $\mathcal{A}$.

Recursion operators and hierarchies of NLEE equations related to Kac-Moody algebras

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We start with a brief review of our recent results [1, 2, 3] on the nonlinear evolution equations (NLEE) of mKDV type related to Kac–Moody algebras. In [1] we constructed the three nonequivalent gradings in the algebra $D_4 \simeq so(8)$. The first one is the standard one obtained with the Coxeter automorphism $C_1 = S_{\alpha_2} S_{\alpha_1} S_{\alpha_3} S_{\alpha_4}$ using its dihedral realization; here S_{α_j} is the Weyl reflection related to the simple root α_j of D_4. In the second one we use $C_2 = S_{\alpha_1} S_{\alpha_3} R$ where R is the mirror automorphism. The third one is $C_3 = S_{\alpha_2} S_{\alpha_1} T$ where T is the external automorphism of order 3. For each of these gradings we constructed the basis in the corresponding linear subspaces $\mathfrak{g}^{(k)}$, the orbits of the Coxeter automorphisms and the related Lax pairs generating the corresponding mKdV hierarchies. We found compact expressions for each of the hierarchies in terms of the recursion operators. At the end we wrote explicitly the first nontrivial mKdV equations and their Hamiltonians.

In [2] the same approach has been applied to the Kac–Moody algebras $A_5^{(1)}$ and $A_5^{(2)}$. Again we construct explicitly the gradings of these algebras and derive the corresponding systems of mKdV equations. In [3] we analyze the spectral theory of the relevant Lax operators related to these algebras.

In addition we formulate also the well known 2-dimensional Toda field theories which must be considered as negative flows of the corresponding mKdV hierarchies. We will also briefly discuss the construction of the soliton solutions of these equations.

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Polynomial identities and trace codimensions

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An aspect of PI-theory is the study of growth functions associated to the polynomial identities of a given algebra. This is expressed by the asymptotics of the sequence of codimensions of the algebra and several results have been obtained in characteristic zero.

When the algebra is endowed with a trace function one studies the corresponding sequence of trace codimensions. I will survey on some of the results obtained in this setting comparing them with the classical case.

The general rank one matrix bispectral operators

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We prove a very general theorem establishing the bispectrality of noncommutative Darboux transformations. It has a wide range of applications that establish bispectrality of such transformations for differential operators with values in all noncommutative algebras. All known bispectral Darboux transformations are special cases of the theorem. Using the methods of the spectral theory of matrix polynomials, we explicitly classify the set of bispectral Darboux transformations from rank one differential operators with values in matrix algebras. New examples of bispectral operators are presented. In particular they include bispectral superdifferential operators.

Hilbert series and invariant theory of symplectic and orthogonal groups

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Let $A = \bigoplus_{i\geq 0} A^i$ be a finitely generated graded algebra over $\mathbb{C}$ such that each homogeneous component is a polynomial $\mathrm{GL}(n,\mathbb{C})$-module. Let G be one of the complex groups $\mathrm{O}(n,\mathbb{C})$, $\mathrm{SO}(n,\mathbb{C})$, and $\mathrm{Sp}(2d,\mathbb{C})$ (the last in the case $n = 2d$). In this talk, we present a method for computing the Hilbert series of the algebra of invariants A^G. Then, we take explicit choices of A and apply our method to compute a lot of examples. The main examples we consider for A are the symmetric algebra $S(W)$ and the exterior algebra $\Lambda(W)$ of a polynomial $\mathrm{GL}(n,\mathbb{C})$-module W and certain relatively free algebras in varieties of associative algebras. In some of the examples, we use the computed Hilbert series to determine a set of generators for the respective algebra of invariants. As a further application, we consider the question of regularity of the algebra $S(W)^{\mathrm{O}(n)}$. For $n = 2$ and $n = 3$ we give a complete list of modules W, so that if $S(W)^{\mathrm{O}(n)}$ is regular then W is in this list. The talk is based on a joint work with Vesselin Drensky.

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Regular and superabundant components of the Hilbert scheme of smooth projective curves

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Denote by $\mathcal{I}_{d,g,r}$ the union of irreducible components of the Hilbert scheme whose general points correspond to smooth integral non-degenerate complex curves of genus g and degree d in $\mathbb{P}^r$. Assume that $\rho(d,g,r) := g-(r+1)(g-d+r) \geq 0$. In 1921 Severi conjectured that if $d \geq g+r$ then $\mathcal{I}_{d,g,r}$ is irreducible. However, in the 1980's Ein and Keem gave counterexamples to the claim when $r \geq 6$. In fact, it turned out that the geometry of $\mathcal{I}_{d,g,r}$ is more diverse. I will show the existence of additional generically smooth superabundant components and the existence of at least two regular components.

The talk is based on joints works with Y.Choi and S.Kim.

Some PI-results on superalgebras with pseudoinvolution

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Let F be an algebraically closed field of characteristic zero and let us consider a superalgebra A, i.e., an algebra graded by $\mathbb{Z}_2$, the cyclic group of order 2. A pseudoinvolution on $A = A_0 \oplus A_1$ is a graded linear map $* : A \to A$ such that $a^{**} = (-1)^{|a|}a$ and $(ab)^* = (-1)^{|a||b|}b^*a^*$, for any homogeneous elements $a, b \in A_0 \cup A_1$.

The existence of pseudoinvolutions of the first kind, i.e., pseudoinvolutions fixing the ground field F, was proven in [5] by Jaber. Later on, Martinez and Zelmanov in [6] classified the irreducible bimodules over finite dimensional simple Jordan superalgebras via pseudoinvolutions.

The goal of this talk is to present several results of the theory of polynomial identities in the setting of superalgebras with pseudoinvolution ([1, 2, 3, 4]).

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Images of non-commutative polynomials evaluated on algebras

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Let p be a polynomial in several non-commuting variables with coefficients in a field K of arbitrary characteristic. It has been conjectured that for any n, for p multilinear, the image of p evaluated on the set $M_n(K)$ of n by n matrices is either zero, or the set of scalar matrices, or the set $\mathrm{sl}_n(K)$ of matrices of trace 0, or all of $M_n(K)$. This expository paper describes research on this problem and related areas. We discuss the solution of this conjecture for $n = 2$ in Section 2, some decisive results for $n = 3$ in Section 3, and partial information for $n \geq 3$ in Section 4, also for non-multilinear polynomials. In addition we consider the case of K not algebraically closed, and polynomials evaluated on other finite dimensional simple algebras (in particular the algebra of the quaternions). This review recollects results and technical material of our previous papers, as well as new results of other researches, and applies them in a new context. This article also explains the role of the Deligne trick, which is related to some nonassociative cases in new situations, underlying our earlier, more straightforward approach. We pose some problems for future generalizations and point out possible generalizations in the present state of art, and in the other hand providing counterexamples showing the boundaries of generalizations.

Finite Galois quotients of bi-elliptic surfaces

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Let Y be a bi-elliptic surface and $\mathfrak{G}$ be a finite group of holomorphic automorphisms of Y. In [1] Yoshihara classifies the smooth quotient $Y/\mathfrak{G}$.

We study the singular quotients $Y/\mathfrak{G}$ by computing their fundamental group and by obtaining the Enriques-Kodaira type of their resolution of singularities X. It turns out that X can be a $K3$ surface, an Enriques surface or a rational surface.

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Property T for some groups of tame automorphisms of K[x, y, z]

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I will describe a construction of some subgroups of automorphisms of polynomial rings like $K[x, y, z]$ consisting of tame automorphisms which have Kazdan property T. The groups can be used to provide new examples of expander graphs. This is joint work with Pierre-Emmanuel Caprace.

Accurate Eigenvectors of Symmetric Tridiagonal Matrices (based on a joint research with Frederick Vincent)

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We will present new algorithms for computing eigenvector components of symmetric tridiagonal matrices to high relative accuracy. This is in contrast with the current state-of-the-art where the eigenvectors are computed to high norm accuracy and thus only the largest components are computed accurately with the tiny ones lost to roundoff. Our new algorithms are based on a recently (re-)discovered result which expresses the components of an eigenvector in terms of the eigenvalues of the matrix and the eigenvalues of certain submatrices. These eigenvalues can be computed relative accuracy using known methods for totally nonnegative matrices. The eigenvectors are always computed with known and guaranteed relative accuracy. When certain expected and typical relative gap conditions are met, these components are computed to high relative accuracy.

Presentations of Semigroups of Order-Preserving Partial Injections on a Finite Set (joint talk with Apatsara Sareeto)

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Presentations of semigroups of transformations is a classical problem in Semigroup Theory. In 1887, Moore has given a presentation for the symmetric group S_n. In 1958, Aĭzenštat has provided a presentation for the symmetric semigroup T_n. Presentations for some monoids of transformations on a finite chain where given by Fernandes et.al. in 2005. The monoid POI_n of all injective order-preserving transformations on an n-element chain has been a object of study (for example by Fernandes). We consider a submonoid $POFI_n^{par}$ consisting of all transformations in POI_n preserving both the parity and the zig-zag order $1 < 2 > 3 < \cdots > n < 1 < n$ (for even natural number n). We provide a presentation for $POFI_n^{par}$ in $3n - 4$ generators and several classes of relations. In particular, we will point out the idea of the proof.

Stability of solitary waves for the generalized Klein-Gordon-Hartree equation (based on a joint research with Mirko Tarulli and George Venkov, Technical University of Sofia)

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We explore the generalized Klein-Gordon-Hartree equation where we are interested in finding the set of the so called "ground states". These are radially symmetric, positive and decreasing solutions to the corresponding stationary equation (also known as the Euler-Lagrange equation).

"Ground states" solutions are minimizers of the action functional associated to the generalized Klein-Gordon Hartree equation and to the stationary equation, in particular. By applying a classical result by Shatah, we provide sufficient conditions for the stability of the set of "ground states" solutions.

Fine gradings on classical simple real Lie algebras

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If G is a group and A is an algebra with any number of multilinear operations over a field $\mathbb{F}$, then a *G-grading* on A is a family of subspaces $\{A_g\}_{g\in G}$ such that $A = \bigoplus_{g\in G} A_g$ and, for any operation φ defined on A, we have $\varphi(A_{g_1}, \ldots, A_{g_n}) \subset A_{g_1\cdots g_n}$ for all $g_1, \ldots, g_n \in G$, where n is the number of variables taken by φ.

In the past two decades there has been considerable interest in classifying group gradings on algebras of different varieties including associative, associative with involution, Lie and Jordan. Of particular importance are the so-called *fine gradings* (that is, those that do not admit a proper refinement), because any grading on a finite-dimensional algebra A can be obtained from them via a group homomorphism, although not in a unique way. If the ground field $\mathbb{F}$ is algebraically closed and of characteristic 0, then the classification of fine abelian group gradings on A up to equivalence is the same as the classification of maximal quasitori in the algebraic group $\mathrm{Aut}(A)$ up to conjugation. In particular, it is known for all finite-dimensional simple complex Lie algebras.

In this talk I will present a recent joint work with A. Elduque and A. Rodrigo-Escudero in which we classify fine gradings on simple associative algebras with involution over the field of real numbers and, as a consequence, on classical simple real Lie algebras.

Codimensions of Algebras with additional structures

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Let A be an associative algebra over a field F of characteristic zero and let $\mathrm{Id}(A)$ be the T-ideal of polynomial identities of A.

One associates to A, in a natural way, a numerical sequence $c_n(A)$, $n = 1, 2, \ldots$, called the sequence of codimensions of A which is the main tool for the quantitative investigation of the polynomial identities of the algebra A. Such a sequence, in case A satisfies a non-trivial identity, is exponentially bounded.

The purpose of this talk is to survey some recent results on the growth of codimensions of algebras with additional structures.

Around Tokuyama's formula

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Tokuyama's formula offers a link between combinatorics and representation theory. Namely, it interprets an expression involving the characters of general linear groups as being a sum over one of three combinatorial objects: Gelfand-Tsetlin patterns, shifted tableaux, or gamma ice models. We first review existing literature concerning Tokuyama's formula and then present two novel proofs which avoid complicated machinery required by previous proofs. We have described progress in extending our results toward an analogous combinatorial identity for the characters of symplectic groups.

Ciphers and Ordinary Difference Equations

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Many stream or block ciphers of application interest such as Trivium, Bluetooth's E0, Keeloq, etc can be modeled as systems of explicit ordinary difference equations over finite fields. Such systems indeed determine the evolution over discrete time of the internal state of these ciphers which is simply a vector with entries in a finite field. The use of the formal theory of algebraic difference equations, so-called Difference Algebra, allows the study of some fundamental properties of difference ciphers, such as their invertibility and periodicity. This study implies the precise definition of algebraic attacks for the purpose of assessing cipher's security. Such modeling and the corresponding cryptanalysis allows hence the development of new cryptosystems.

Identities for a parametric Weyl algebra over a ring

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This is a joint work with Carlos Arturo Rodriguez Palma. In 2015 Benkart, Lopes and Ondrus introduced and studied in a series of papers the infinite-dimensional unital associative algebra A_h generated by elements x, y, which satisfy the relation $yx - xy = h$ for some $0 \neq h \in \mathbb{F}[x]$. We generalize this construction to $\mathsf{A}_h(\mathsf{B})$ by working over the fixed $\mathbb{F}$-algebra B instead of $\mathbb{F}$. Namely, for $h \in Z(\mathsf{B})[x]$, the *parametric Weyl algebra* $\mathsf{A}_h(\mathsf{B})$ *over the ring* B is the unital associative algebra over $\mathbb{F}$ generated by B and letters x, y commuting with B subject to the defining relation $yx = xy + h$ (equivalently, $[y, x] = h$, where $[y, x] = yx - xy$), i.e.,

$$\mathsf{A}_h(\mathsf{B}) = \mathsf{B}\langle x, y\rangle/\mathrm{id}\{yx - xy - h\}.$$

We describe the polynomial identities for $\mathsf{A}_h(\mathsf{B})$ over the infinite field $\mathbb{F}$ in case $h \in \mathsf{B}[x]$ satisfies certain restrictions.

Centralizers of rank one in the first Weyl algebra

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One of the most popular noncommutative algebras is the first Weyl algebra. Do not worry if you do not remember what it is, I'll remind you the definition. Since it is not commutative, centralizers of non-central elements i.e. elements which commute with a given element, are proper subalgebras. These subalgebras are the subject of the talk.

Central polynomials and exponential growth of the codimensions

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Let F be a field of characteristic zero, A an associative F-algebra and $F\langle X\rangle$ the free associative algebra, freely generated over F by the set X of variables. A non-zero polynomial $f(x_1,\dots,x_n)\in F\langle X\rangle$ is a central polynomial for A if for all $a_1,\dots,a_n\in A$, $f(a_1,\dots,a_n)\in Z(A)$, the center of A. Clearly, the polynomial identities are in particular central polynomials, thus if a polynomial is central but it is not an identity, then we call it a proper central polynomial.

If we consider for any $n\geq 1$ the space P_n of multilinear polynomials of degree n, then we attach to it three numerical sequences: $c_n(A)$, the dimension of P_n modulo the polynomial identities of A; $c_n^z(A)$, the dimension of P_n modulo the central polynomials of A; $\delta_n(A)$, the dimension of the space of multilinear central polynomials of degree n modulo the identities of A. They are called the ordinary, central and proper central codimension sequence, respectively, and for all $n\geq 1$,

$$c_n(A)=c_n^z(A)+\delta_n(A).$$

If the algebra A satisfies a non-trivial polynomial identity, then it is well-known that $c_n(A)$ is exponentially bounded. Therefore a similar result holds also for the central and the proper central codimension sequences.

In this talk we shall present some recent results about the exponential growth of such sequences in the setting of ordinary algebras, algebras with involution and algebras graded by a finite abelian group. In particular, we give a positive answer to the Amitsur conjecture concerning the existence and the integrability of the so-called PI-central and PI-proper central exponent. Finally, we compare the PI-exponent with the central one.

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The *commutator-degree* of a polynomial and images of multilinear polynomials

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Let K be a field, $X = \{x_1, x_2, \dots\}$ be an infinite set and $K\langle X\rangle$ be the free associative algebra freely generated by X.

We have a strictly descending chain of T-ideals of $K\langle X\rangle$,

$$K\langle X\rangle \supsetneqq \langle [x_1, x_2]\rangle^T \supsetneqq \langle [x_1, x_2][x_3, x_4]\rangle^T \supsetneqq \langle [x_1, x_2][x_3, x_4][x_5, x_6]\rangle^T \supsetneqq \cdots$$

We say that a polynomial $p \in K\langle X\rangle$ has *commutator-degree* r if $f \in \langle [x_1, x_2][x_3, x_4]\cdots[x_{2r-1}, x_{2r}]\rangle^T$ and $f \notin \langle [x_1, x_2][x_3, x_4]\cdots[x_{2r+1}, x_{2r+2}]\rangle^T$.

In this talk we show how multilinear polynomials of a given commutator-degree r can be characterized in terms of its coefficients.

We apply the above characterization to show that the image of a multilinear polynomial evaluated on the algebra of $n\times n$ upper triangular matrices is a vector space (a solution to the analogous of the Lvov-Kaplansky conjecture for the algebra of $n\times n$ upper triangular matrices).

This is a joint work with Ivan Gonzales Gargate.

Differential polynomial identities of $UT_3(F)$

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If A is an assigned algebra on a field F, the set $Der(A)$ of all its F-derivatives is a Lie algebra enveloped by $End_F(A)$. Then the polynomial relations on A taking into account the derivation action of $Der(A)$ on A generalize the notion of polynomial identity, and constitute a larger set concretely including the ordinary polynomial identities. In this talk, the differential polynomial identities of $A = UT_3(F)$ under the action of the full $Der(UT_3(F))$ will be described. So far, indeed, just the differential polynomial identities of $UT_2(F)$ have been computed, and for $n \geqslant 3$, the differential polynomial identities of $UT_n(F)$ under the derivation action of the subalgebra L_2 (the 2-dimensional nonabelian Lie algebra) of $Der(UT_n(F))$ have been described. Hopefully, the description for $UT_3(F)$ may give some insight to the general problem.

Decomposition, anomalies, and quantum symmetries

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Decomposition is a phenomenon in quantum physics which converts quantum field theories with non-effectively acting gauge symmetries into equivalent more tractable theories in which the fields live on a disconnected space. I will explain the mathematical content of decomposition which turns out to be a higher categorical version of Pontryagin duality. I will show how this duality interacts with quantum anomalies and secondary quantum symmetries and will show how the anomalies can be canceled by homotopy coherent actions of diagrams of groups. I will discuss in detail the case of 2-groupoids which plays a central role in anomaly cancellation, and will describe a new duality operation that yields decomposition in the presence of anomalies. This is a joint work with Eric Sharpe.

Quotient groups of IA-automorphisms of free metabelian groups

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For a group G and a positive integer c, we write $\gamma_c(G)$ for the c-th term of the lower central series of G. For a positive integer n, with $n \geq 2$, let M_n be a free metabelian group of rank n. For $c \geq 2$, let $\mathrm{I}_c\mathrm{A}(M_n)$ be the subgroup of $\mathrm{Aut}(M_n)$ consisting of all automorphisms which induce the identity mapping on $M_n/\gamma_c(M_n)$. Our aim in this talk is to study the quotient groups $\mathcal{L}^c(\mathrm{IA}(M_n)) = \mathrm{I}_c\mathrm{A}(M_n)/\mathrm{I}_{c+1}\mathrm{A}(M_n)$ for all n and c. For $c \geq 2$, we show $\mathrm{I}_c\mathrm{A}(M_2)) = \gamma_{c-1}(\mathrm{IA}(M_2))$. For $n = 3$, we show $\gamma_3(\mathrm{IA}(M_3)) \neq \mathrm{I}_4\mathrm{A}(M_3)$ and so, the Andreadakis' conjecture (for a free metabelian group of rank 3) is not valid for $n = 3$ and $c = 3$. For $n, c \geq 4$, we prove that $\mathcal{L}^c(\mathrm{IA}(M_n)) = \gamma_{c-1}(\mathrm{IA}(M_n))\mathrm{I}_{c+1}\mathrm{A}(M_n)/\mathrm{I}_{c+1}\mathrm{A}(M_n)$.

Phoenix restricted Lie algebras

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Different versions of BURNSIDE PROBLEM ask what one can say about finitely generated periodic groups under additional assumptions. For associative algebras, KUROSH type problems ask similar questions about properties of finitely generated nil (more generally, algebraic) algebras. Similarly, one considers finitely generated restricted Lie algebras with a nil p-mapping. Now we study an oscillating intermediate growth in the class of NIL restricted Lie algebras.

Namely, for any field of positive characteristic, we construct a family of 3-generated restricted Lie algebras of intermediate oscillating growth. We call them *Phoenix algebras*, because of the following. a) For infinitely many periods of time the algebra is "almost dying" by having a *quasi-linear* growth, namely the lower Gelfand-Kirillov dimension is one, more precisely, the growth is of type $n\big(\underbrace{\ln\cdots\ln}_{q\text{ times}} n\big)^{\kappa}$, where $q \in \mathbb{N}$, $\kappa > 0$ are constants. b) On the other hand, for infinitely many n the growth function has a rather fast intermediate behaviour of type $\exp(n/(\ln n)^{\lambda})$, λ being a constant determined by characteristic, for such periods the algebra is "resuscitating". c) Moreover, the growth function is bounded and oscillating between these two types of behaviour. d) These restricted Lie algebras have a nil p-mapping.

A note on the Formanek Weingarten function

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The aim of this note is to compare work of Formanek [2] on a certain construction of central polynomials with that of Collins [1] on integration on unitary groups.

These two quite disjoint topics share the construction of the same function on the symmetric group, which the second author calls *Weingarten function*.

By joining these two approaches we succeed in giving a simplified and *very natural* presentation of both Formanek and Collins's Theory.

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Derivation Algebras of Albert Algebras over a Ring

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Over a field, Albert algebras are the only exceptional simple Jordan algebras. We will recall the known results on derivation algebras of Albert algebras over a field and present recent results on derivation algebras of Albert algebras over a ring.

Identities generated by standard polynomials for some matrix algebras with Grassmann entries

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The paper gives a short survey on the standard identities for matrices over the Grassmann algebra E. It exposes author's investigations on the degree minimality of the considered identities in some special matrix algebras. In the case of a finitely generated Grassmann algebra $E^{(m)}$ the best known estimation of the degree function $k(m,n)$ of the standard identity $\mathrm{St}_k = 0$ for $M_n(E^{(m)})$ is found. The minimal identities generated by standard polynomials in skew-symmetric variables for $(M_2(E), t)$, and in symmetric ones for $(M_2(E), s)$ are investigated as well for "t" the transpose and "s" the symplectic involution. For a subalgebra of $M_{2,2}(E)$ with involution φ, the minimal identities of the considered type in symmetric and in skew-symmetric variables are given.

Finitely generated axial algebras

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We investigate generation of (perhaps nonassociative) algebras by idempotents, the main question being, "Under what conditions must an algebra generated by idempotents be finite dimensional?" The monoid algebra generated by two "free" idempotents is infinite dimensional; also Krupnik showed that 3 idempotents can generate arbitrarily large dimensional algebras (and thus infinite dimensional algebras via an ultraproduct argument), so some restriction is needed.

Motivated by group theory and associated schemes, considerable interest has arisen in studying *axes.*

Let $a \in A$ be a semisimple idempotent and $F \ni \lambda, \delta \notin \{0, 1\}$.

Notation 1. *1. For $y \in A$, write L_y for the left multiplication map $z \mapsto yz$ and R_y for the left multiplication map $z \mapsto zy$. We write T when it can be L or R. Thus $T_y(z) \in \{yz, zy\}$.*

2. Write A_η for the left η-eigenspace of A with respect to a. We assume throughout that a is absolutely primitive, in the sense that $A_1 = \mathbb{F}a$. We say that a is a left axis of type (gl) *if $(L_a - \lambda)(L_a - 1)L_a = 0$ and*

$$A = \overbrace{A_0(a) \oplus A_1(a)}^{0\text{-}part} \oplus \overbrace{A_\lambda(a)}^{1\text{-}part},$$

where this is a noncommutative (i.e. two sided) $\mathbb{Z}_2$-grading of A.

A right axis of type (δ) is defined analogously, where R_a satisfies $(R_a - \delta)(R_a - 1)R_a = 0$.

An (2-sided) axis of type (λ, δ) is a left axis of type (λ) which is also a right axis of type (δ), satisfying $L_aR_a = R_aL_a$, i.e., $a(ya) = (ay)a$ for all $y \in A$.

3. An axis of Jordan type (λ, δ) is an axis of type (λ, δ) for which $A_{\lambda,0} = A_{0,\delta} = 0$.

Our overriding goal is to develop a computable theory, and it turns out that the axioms for noncommutative axes are enough to develop this theory. The grading of A with respect to a is otherwise known as its *fusion rules*, a special case of general fusion rules. A is a *(noncommutative) axial algebra (resp. of Jordan type)* if it is generated by a set X of axes (resp. of Jordan type), but not necessarily of the same type. If all the axes of X have the same type (λ, δ), we say that A itself has *type* (λ, δ). The *classical case*, is when $A_0^2 = A_0$. The case treated in the literature often is the commutative classical case. It turns out that the axioms for noncommutative axes are enough to develop our theory. We say that A is *finitely generated* if it is generated (as an algebra) by a finite set of axes.

Axes in flexible algebras are always of Jordan type, and we shall see that although they often are commutative, there are counterexamples.

Definition 1. Assume A contains a set X of axes. A *monomial* of *length* n is a product of n axes. We denote by B_X the subspace of A spanned by **all possible monomials** in the elements of X such that in each monomial each element of X appears **at most once**.

Hall, Rehren, and Shpectorov classified all commutative axial algebras $A = A(X)$ for $|X| = 2$, and in each case $A = B(X)$. We recall the very nice main theorem of Gorshkov and Staroletov, which we use as a launching pad:

Suppose that $A = A(X)$ is a commutative axial algebra for $|X| = 3$. Then $A = B_X$.

This leads to the general case:

Main Conjecture .

1. *Suppose that $A = A(X)$ is a finitely generated axial algebra. Then A is finite dimensional.*
2. *When A is commutative, then $A = B_X$.*

Hall, Rehren, and Shpectorov solved (1) for commutative axial algebras of Jordan type $\lambda \neq \frac{1}{2}$, although the proof relies on the classification of simple groups and the given bound of the dimension is rather high. Our objective in this project is to prove that all finitely generated axial algebras are finite dimensional, with a direct proof providing a sharper bound on the dimension.

Our first main result is that Main Conjecture (1) holds even in the noncommutative setting, for axial algebras generated by three axes of Jordan type. Our method is to build an associative algebra from the adjoint algebra of A, which has a strictly larger dimension which nevertheless also is finite dimensional. This also enables us to handle the case when X does not generate A. When A is commutative we recover the Gorshkov-Staroletov Theorem.

For any axes not of Jordan type, there are counterexamples to Main Conjecture (2) even for axial algebras generated by two axes, since $(ab)a$need not be in $B_{\{a,b\}}$. Furthermore, for axial algebras generated by four axes of Jordan type, even in the commutative case, it appears likely that $(ab)((ac)d) \notin B_{\{a,b,c,d\}}$.

In proving these results and more generally attacking Main Conjecture (1), a major tool is the following noncommutative version of a result of Hall, Rehren, and Shpectorov.

Suppose A is generated by a set of axes X. Let $V \subseteq A$ be a subspace containing X such that $xV \subseteq V$ and $Vx \subseteq V$, for all $x \in X$. Then $V = A$.

Universal enveloping algebras of free Jordan algebras and their associated graded algebras

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I plan to speak on the structure of universal enveloping algebras of free Jordan algebras and of their associated graded algebras.

With any Jordan algebra J, the two universal associative algebras can be associated: the universal associative enveloping algebra $S(J)$ with a homomorphism $i\colon J \to S(J)^{(+)}$, which is the universal object for specializations of J, that is, for homomorphisms of J into special Jordan algebras, and the universal multiplicative enveloping algebra $U(J)$, which is the universal object for representations of J. For a unital J, the algebra $S(J)$ is an ideal in $U(J)$, and any specialization of J may be considered as a representation.

The universal algebras $U(J)$ and $S(J)$ play important role for the structure theory and representations of Jordan algebras. Many important results in Jordan Theory have been proved by using relations between properties of an algebra J and of its universal enveloping algebras.

Contrary to the Lie algebra case, there is no "canonical base" for the algebras $S(J)$ and $U(J)$. On the other hand, these algebras are more closely related to the algebra J; for instance, if J is finite dimensional then so are $S(J)$ and $U(J)$. The both algebras $S(J)$ and $U(J)$ have natural ascending filtrations, and one can consider the associated graded algebras $gr\,S(J)$ and $gr\,U(J)$. These algebras have more simple structure; for instance, the algebra $gr\,S(J)$ is a homomorphic image of the exterior algebra $\Lambda(J)$.

The main objective of the talk is the structure of the algebras $gr\,U(J)$ and $gr\,S(J)$ when J is the free Jordan algebra $Jord[X]$ or the free special Jordan algebra $SJord[X]$. Observe that $S(SJord[X]) = As[X]$, so the first algebra is just the associated graded algebra of the free associative algebra $As[X]$ with respect to the Jordan filtration. More exactly, for $J = SJord[X]$ we have

$$J_0 = F \subset J_1 = J \subset J_2 \subset \cdots \subset J_k \subset \cdots As[X],$$

where J_k is the subspace of $As[X]$ generated by all products of at most k Jordan elements. It is known that if X is finite then the above filtration is finite. We find estimates for the length of this filtration and investigate the structure of terms J_k for $k > 1$. The similar questions we consider for the graded algebra $gr\,U(Jord[X])$.

It is a join research with S. Sverchkov (SO RAN, Novosibirsk).

Orthogonal arrays and their distance distributions

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Orthogonal arrays were introduced by Rao (1946) [4] and since then have been studied by many researchers from various fields. Let H_q be an alphabet of q letters and H_q^n be the Hamming space over H_q. An orthogonal array (OA) of strength τ and index λ in H_q^n consists of the rows of an $M \times n$ matrix C with the property that every $M \times t$ submatrix of C contains all ordered τ-tuples of H_q^n, each one exactly $\lambda = M/q^\tau$ times as rows. We denote C by $OA(M, n, q, \tau)$.

Some of important parameters of the (M, n, q, τ) orthogonal array C are its distance distributions, minimum distance and covering radius. There are known several methods for computing all possible distance distributions, all of them variations of a classical result of Delsarte [1, 2] (see also [3, 5]). Furthermore, the set of all possible distance distributions can be reduced by investigations of the relations between C and its related orthogonal arrays. The information about distance distributions of a given orthogonal array implies bounds for their minimum distance and covering radius. If all possible distance distributions are known, then the minimum distance and covering radius can be found. We describe various methods dealing with the sets of feasible distance distributions of orthogonal arrays. This leads to good (in some cases sharp) bounds on the minimum distance and covering radius.

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Hook theorem for various types of identities

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Hook theorem is one of the key result of the classical PI-theory (the theory of polynomial identities of algebras) in the case of varieties of algebras over a field of characteristic zero. This well known result is fundamental for applications of technique of the classic representation theory of the symmetric group to study identities, and has essential connections with many important facts of PI-theory.

The theorem describes the form of Young diagrams associated with the multilinear part of the relatively free algebra of varieties. It is also well known that these parameters are also connected with structure characteristics of carriers of varieties, and define a growth of varieties of associative PI-algebras over a field of characteristic zero.

We will discuss the versions of the hook theorem for various type of identities. Particularly, we will represent some versions of the hook theorem for identities, which involve some additional structures (such as gradings, involutions, etc.). We also will discuss some possible consequences and applications of this theorem.

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Solving equations in the symmetric group

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We investigate the solutions of the general "cubic" equation

$$\alpha_1 \circ x^{r_1} \circ \alpha_2 \circ x^{r_2} \circ \alpha_3 \circ x^{r_3} = 1$$

(with $r_1, r_2, r_3 \in \{1, -1\}$) in the symmetric group S_n. In certain cases this equation can be rewritten as a conjugate equation of the form $\alpha \circ y \circ \alpha^{-1} = y^2$, where $\alpha = \alpha_1^{-1}$ and $y = x \circ \alpha_2$. It turns out that the existence of a non-trivial solution $y \neq 1$ of the above conjugate equation heavily depends on the structure of the cycles in α.

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Metabelian varieties and left nilpotent varieties (Joint with S.P. Mishchenko)

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Thirty years ago Drensky in [1] proved that for unitary classical algebras (associative algebras, Lie algebras and Jordan algebras) there are no varieties of fractional polynomial growth. More precisely the growth function $f(n)$ of a variety of polynomial growth is of the form $f(n) = rn^q + O(n^{q-1})$, for some real number r and for some natural number q. In the class of left nilpotent algebras of index two it was proved in [2] that there are no varieties of fractional polynomial growth $\approx n^\alpha$ with $1 < \alpha < 2$ and $2 < \alpha < 3$ instead in [4] it was established the existence of a variety of fractional polynomial growth with $\alpha = \frac{7}{2}$. The aim of this talk is to investigate similar problems for varieties of commutative or anticommutative metabelian algebras. Recently in [3] we constructed a correspondence between left nilpotent algebras of index two and commutative metabelian algebras or anticommutative metabelian algebras and we proved that the codimensions sequences of the corresponding algebras coincide up to a constant. This allows us to transfer the above results concerning varieties of left nilpotent algebras of index two to varieties of commutative or anticommutative metabelian algebras.

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Derivations and automorphisms of the endomorphism semiring of an infinite chain

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We introduce the endomorphism semiring of an infinite chain and deals with the increasing endomorphisms which are endomorphisms with a right inverse. Two derivations such that any left ideal of the considered semiring is closed under the derivation δ_ℓ and any right ideal is closed under the derivation δ_r are constructed. We prove that the product of these two derivations is an automorphism and consider all positive and negative integer degrees of all maps. Furthermore we construct new derivations using the positive degrees of δ_ℓ. The main result is that the set of these new derivations is a commutative additively idempotent semiring. Analogous results we propose for the derivations constructed from the positive degrees of δ_r and for the mixed derivations constructed from the positive degrees of δ_ℓ and δ_r. At the last we obtain the similar result for automorphisms constructed from the degrees of $\delta_\ell.\delta_r$.

Determining graded-simple algebras by their graded polynomial identities

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An interesting question is if the polynomial identities of an algebra can uniquely determine it, up to an isomorphism. This topic, and its graded version, was addressed by several authors. In general, the answer is no, but under some restrictions we obtain positive answers. In this talk, we shall prove that finite-dimensional graded-simple algebras over an algebraically closed field are uniquely determined by their graded polynomial identities. The proof relies on Razmyslov's theory on the same problem in the context of prime Ω-algebras (1989). This is a joint work with Yuri Bahturin (Memorial University of Newfoundland, Canada).

On existence of PI-exponent of codimension growth

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Given an algebra A, one can associate the sequence $c_n(A)$ of non-negative integers called *codimension sequence* that measures the number of polynomial identities of A. For many classes of algebras the sequence $\{c_n(A)\}$ is exponentially bounded. For example, if A is a finite dimensional algebra, $\dim A = d$, then $c_n(A) \le d^{n+1}$. Codimension sequence is also exponentially bounded for any associative PI-algebra, for any infinite dimensional simple Lie algebra of Cartan type, for any Novikov algebra, etc.

At the end of 1980's S. Amitsur conjectured that so-called PI-exponent

$$exp(A) = \lim_{n\to\infty} \sqrt[n]{c_n(A)}$$

exists for any associative PI-algebra A. Amitsur's conjecture was confirmed not only for associative algebras but also for all finite dimensional Lie algebras, for all finite dimensional nonassociative simple algebras and many others. Unlike finite dimensional case, there are counterexamples to Amitsur's conjecture in the class of infinite dimensional nonassociative algebras.

If an algebra A is equipped with involution $* : A \to A$ then the same problem concerning $*$-identities and $*$-codimensions $c_n^*(A)$ can be considered. It is known that for an associative PI-algebra A the corresponding limit

$$exp^*(A) = \lim_{n\to\infty} \sqrt[n]{c_n^*(A)}$$

always exists. Nevertheless, in general nonassociative case we have the following result.

THEOREM. *For any real $\alpha > 1$ there exists an algebra C_α with involution $* : C_\alpha \to C_\alpha$ such that*

$$\liminf_{n\to\infty} \sqrt[n]{c_n^*(A)} = 1, \qquad \text{whereas} \qquad \limsup_{n\to\infty} \sqrt[n]{c_n^*(A)} = \alpha.$$

Poisson brackets and related (super) algebras.

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We will discuss Lie and Jordan (super) algebras related to Poisson and contact brackets.