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The Evaluation of the Lebesque Function of Fourier-Jacobi Double Series

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Let $P_n^{(\alpha,\beta)}(x)$ $(n=0, 1, \ldots)$ -be the orthonormal system of Jacobi polynomials ([1], p. 70), with weight

$$h^{(\alpha,\beta)}(x)=(1-x)^{\alpha}(1+x)^{\beta} \quad (\alpha>-1,\ \beta>-1)$$

on the segment $[-1,\ 1]$ and

$$(1) \qquad \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} c_{ij}(f)\, P_i^{(\alpha,\beta)}(x)\, P_j^{(\alpha,\beta)}(y)$$

$$\left(c_{ij}(f)=\int_{-1}^{1}\int_{-1}^{1} h^{(\alpha,\beta)}(u)\, h^{(\alpha,\beta)}(v)\, f(u,v)\, P_i^{(\alpha,\beta)}(u)\, P_j^{(\alpha,\beta)}(v)\, du dv\right),$$

— Fourier-Jacobi double series of continuous function $f(x, y)$ in square $[-1, 1]\times[-1,1]$,

$$S_{n,m}^{(\alpha,\beta)}(f;x,y)=\sum_{i=0}^{n}\sum_{j=0}^{m} c_{ij}(f)\, P_i^{(\alpha,\beta)}(x)\, P_j^{(\alpha,\beta)}(y),$$

— rectangular partial sums of series (1) and

$$L_{n,m}^{(\alpha,\beta)}(x,y)=\int_{-1}^{1}\int_{-1}^{1} h^{(\alpha,\beta)}(u)\, h^{(\alpha,\beta)}(v) \left|\sum_{i=0}^{n}\sum_{j=0}^{m} P_i^{(\alpha,\beta)}(x)\, P_j^{(\alpha,\beta)}(y)\, P_i^{(\alpha,\beta)}(u)\, P_j^{(\alpha,\beta)}(v)\right| du dv,$$

— the Lebesque function of order $(n,\ m)$.

Let us signify by $E_{n,m}(f)$ the best uniform approximation of function $f(x, y)$ continuous in square $[-1, 1]\times[-1, 1]$ by algebraic polynomial of two variables of degree no higher than n on x-variable and no higher than m on y-variable.

It is well known, that

$$(2)\qquad |f(x,y)-S_{n,m}(f;\ x,y)|\leqq[1+L_{n,m}(x,y)]\,E_{n,m}(f).$$

A similar evaluation proves to be important with regard to the convergence of Fourier-Jacobi series for the function from one variable. It is worth while to note that for a two-dimensional case no evaluation gives meaningful propositions if to change n and m irrespective of each other.

In light of this a study of the partial sums of series (1) and the kind of

$$\mathfrak{S}_{n,m}^{(\alpha,\beta)}(f;x,y)=\sum_{0\leqq\frac{i}{n}+\frac{j}{m}\leqq 1} c_{ij}(f)\,P_i^{(\alpha,\beta)}(x)\,P_j^{(\alpha,\beta)}(y)$$

and the Lebesque function

$$\mathfrak{L}_{n,m}^{(\alpha,\beta)}(x,y)=\int_{-1}^{1}\int_{-1}^{1} h^{(\alpha,\beta)}(u)\,h^{(\alpha,\beta)}(v)\,\Big|\sum_{0\leqq\frac{i}{n}+\frac{j}{m}\leqq 1} P_i^{(\alpha,\beta)}(u)\,P_i^{(\alpha,\beta)}(x)\,P_j^{(\alpha,\beta)}(v)\,P_j^{(\alpha,\beta)}(y)\Big|\,du\,dv.$$

Here $n,m,p=\frac{n}{m}\in\mathrm{N}$.

A study has been given to the case when $|\alpha|=|\beta|=\frac{1}{2}$. For example, with $\alpha=\beta=-\frac{1}{2}$ equal to n,m,p the asymptotic equation is valid.

$$\mathfrak{L}_{n,m}^{(-\frac{1}{2},-\frac{1}{2})}=\max_{-1\leqq x,y\leqq 1}|\mathfrak{L}_{n,m}^{(-\frac{1}{2},-\frac{1}{2})}(x,y)|=\frac{32}{\pi^4}\ln n\ln m-\frac{16}{\pi^4}\ln^2 m+O(\ln n),$$

where the constant contained in 0 (1) is an absolute one.

References

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