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## The Evaluation of the Lebesgue Function of Fourier-Jacobi Double Series

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Presented by Bl. Sendov

Let  $P_n^{(\alpha, \beta)}(x)$  ( $n=0, 1, \dots$ ) -be the orthonormal system of Jacobi polynomials ([1], p. 70), with weight

$$h^{(\alpha, \beta)}(x) = (1-x)^\alpha (1+x)^\beta \quad (\alpha > -1, \beta > -1)$$

on the segment  $[-1, 1]$  and

$$(1) \quad \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{ij}(f) P_i^{(\alpha, \beta)}(x) P_j^{(\alpha, \beta)}(y)$$

$$(c_{ij}(f) = \int_{-1}^1 \int_{-1}^1 h^{(\alpha, \beta)}(u) h^{(\alpha, \beta)}(v) f(u, v) P_i^{(\alpha, \beta)}(u) P_j^{(\alpha, \beta)}(v) dudv),$$

– Fourier-Jacobi double series of continuous function  $f(x, y)$  in square  $[-1, 1] \times [-1, 1]$ ,

$$S_{n,m}^{(\alpha, \beta)}(f; x, y) = \sum_{i=0}^n \sum_{j=0}^m c_{ij}(f) P_i^{(\alpha, \beta)}(x) P_j^{(\alpha, \beta)}(y),$$

– rectangular partial sums of series (1) and

$$L_{n,m}^{(\alpha, \beta)}(x, y) = \int_{-1}^1 \int_{-1}^1 h^{(\alpha, \beta)}(u) h^{(\alpha, \beta)}(v) \left| \sum_{i=0}^n \sum_{j=0}^m P_i^{(\alpha, \beta)}(x) P_j^{(\alpha, \beta)}(y) P_i^{(\alpha, \beta)}(u) P_j^{(\alpha, \beta)}(v) \right| dudv,$$

– the Lebesgue function of order  $(n, m)$ .

Let us signify by  $E_{n,m}(f)$  the best uniform approximation of function  $f(x, y)$  continuous in square  $[-1, 1] \times [-1, 1]$  by algebraic polynomial of two variables of degree no higher than  $n$  on  $x$ -variable and no higher than  $m$  on  $y$ -variable.

It is well known, that

$$(2) \quad |f(x, y) - S_{n,m}(f; x, y)| \leq [1 + L_{n,m}(x, y)] E_{n,m}(f).$$

A similar evaluation proves to be important with regard to the convergence of Fourier-Jacobi series for the function from one variable. It is worth while to note that for a two-dimensional case no evaluation gives meaningful propositions if to change  $n$  and  $m$  irrespective of each other.

In light of this a study of the partial sums of series (1) and the kind of

$$\mathfrak{S}_{n,m}^{(\alpha,\beta)}(f; x, y) = \sum_{\substack{i,j \\ 0 \leq \frac{i}{n} + \frac{j}{m} \leq 1}} c_{ij}(f) P_i^{(\alpha,\beta)}(x) P_j^{(\alpha,\beta)}(y)$$

and the Lebesgue function

$$\mathcal{L}_{n,m}^{(\alpha,\beta)}(x, y) = \int_{-1}^1 \int_{-1}^1 h^{(\alpha,\beta)}(u) h^{(\alpha,\beta)}(v) \left| \sum_{\substack{i,j \\ 0 \leq \frac{i}{n} + \frac{j}{m} \leq 1}} P_i^{(\alpha,\beta)}(u) P_i^{(\alpha,\beta)}(x) P_j^{(\alpha,\beta)}(v) P_j^{(\alpha,\beta)}(y) \right| du dv.$$

Here  $n, m, p = \frac{n}{m} \in \mathbb{N}$ .

A study has been given to the case when  $|\alpha| = |\beta| = \frac{1}{2}$ . For example, with  $\alpha = \beta = -\frac{1}{2}$  equal to  $n, m, p$  the asymptotic equation is valid.

$$\mathcal{L}_{n,m}^{(-\frac{1}{2}, -\frac{1}{2})} = \max_{-1 \leq x, y \leq 1} |\mathcal{L}_{n,m}^{(-\frac{1}{2}, -\frac{1}{2})}(x, y)| = \frac{32}{\pi^4} \ln n \ln m - \frac{16}{\pi^4} \ln^2 m + O(\ln n),$$

where the constant contained in  $O(1)$  is an absolute one.

## References

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