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Separation Properties at p for the Topological Categories of Reflexive Relation Spaces and Preordered Spaces

Mehmet Baran

Presented by P. Kenderov

In [1] and [2] various generalizations of the separation properties are defined for arbitrary topological category over sets. In this paper, an explicit characterization of each of the separation properties T_0, T_1, Pre T_2, and T_2 at a point p is given in the topological categories of Reflexive Relation Spaces and Preordered Spaces. Moreover, specific relationships that arise among the various T_0, Pre T_2, and T_2 structures at p are examined in these categories.

1. Introduction

The notion of topological space has been generalized to include convergence spaces, limit spaces, bornological spaces, and preordered spaces, by H. Herrlich [3], D. C. Kent [4], O. Wyler [9], L. D. Nel, Bentley, F. Schwarz [8], among others, to the notion of a topological category. There are a number of equivalent ways to describe topological categories, for example, in terms of the existence of initial lifts of certain sources, H. Herrlich [3] or in terms of functors to the category of complete lattices, O. Wyler [9]. If one wishes to study the extent to which theorems in general topology can be formulated and proved in the more general setting of a topological category it is necessary to reformulate first certain basic concepts in terms of concepts which make sense in any topological category e. g. in terms of initial lifts, final lifts, and discreteness.

Some basic concepts in general topology are the notions of separation (T_0, T_1, T_2, T_3, T_4) which appear in many important theorems such as the Urysohn Metrization theorem, the Urysohn Lemma, the Tietze extention theorem, among others. In view of this, it is useful to be able not only to extend these various notions to arbitrary topological categories but also to have a convenient characterization of them in certain topological categories of interest.

Let E be a category and *Sets* be the category of sets.

1.1. Definition. A Functor $U: E \to Sets$ is said to be topological or E is a topological category over *Sets* iff the following conditions hold:

1. U is concrete i. e. faithful (U is mono on hom sets) and amnestic (if $U(f) = \text{id}$ and f is an isomorphism, then $f = \text{id}$).

2. U has small fibers i. e. $U^{-1}(b)$ is a set for all b in *Sets*.
3. For every U-source, i. e. family $g_i : b \to U(X_i)$ of maps in *Sets*, there exists a family $f_i : X \to X_i$ in E such that $U(f_i) = g_i$ and if $U(h_i : Y \to X_i) = kg_i : UY \to b \to U(X_i)$, then there exists a lift $\bar{k} : Y \to X$ of $k : UY \to UX$ i. e. $U(\bar{k}) = k$. This latter condition means that every U-source has an i n i t i a l l i f t. It is well known, see [3], p. 125 or [6], p. 278, that the existence of initial lifts of arbitrary U-source is equivalent to the existence of f i n a l l i f t s (the dual of the initial lifts) for arbitrary U-sink.

1.2. Definition. The C a t e g o r y o f R e f l e x i v e R e l a t i o n S p a c e s, *R Rel* has as objects the pairs (A, R) where R is a reflexive relation on the set A and has as morphisms $(A, R) \to (A_1, R_1)$ those functions $f : A \to A_1$ such that if aRb, then $f(a)R_1 f(b)$ for all a, b in A. *R Rel* is a topological category over *Sets*. See [1] p. 9.

1.3. Definition. The C a t e g o r y o f P r e o r d e r e d s p a c e s, *Prord* is the full subcategory of *R Rel* determined by those spaces (A, R) where R is a transitive relation. *Prord* is a topological category over *Sets*. See [5], p. 531 or [7], p. 1374.

1.4. The discrete structure (A, R) on A in *R Rel* and *Prord* is given by aRb iff $a = b$, for all a, b in A. [1], p. 12.

1.5. A source $\{f_i : (A, R) \to (A_i, R_i), i \in I\}$ is initial in *R Rel* and *Prord* iff for all a, b in A, aRb iff $f_i a R_i f_i b$ for all i. See [7], p. 1373 and [1], p. 13.

1.6. An epi morphism $f : (A_1, R_1) \to (A, R)$ in *Prord* is final iff for all a, b in A, aRb iff there exists a sequence a_i, $i = 1, 2, \ldots n$ of points in A, with $a = a_1 R a_2 R \ldots R a_n = b$ such that for each $k = 1, \ldots n-1$, there is a pair c_k, c_{k+1} in A_1 such that $f(c_k) = a_k$, $f(c_{k+1}) = a_{k+1}$ and $c_k R_1 c_{k+1}$. [7], p. 1373.

1.7. An epi morphism $f : (A_1, R_1) \to (A, R)$ in *R Rel* is final iff for each pair a, b in A, aRb holds in A precisely when there exists c, d in A_1 such that $cR_1 d$ and $f(c) = a$ and $f(d) = b$.

An epi sink $\{i_1, i_2 : (A, R) \to (A_1, R_1)\}$ is final in *R Rel* iff for each pair a, b in A_1, $aR_1 b$ iff there exists a pair c, d in A such that cRd and $i_k(c) = a$ and $i_k(d) = b$ for some $k = 1, 2$ [1], p. 15.

Let X be a set and p a point in X. Let $X \vee_p X$ be the wedge product of X with itself, i. e. two distinct copies of X identified at the point p. A point x in $X \vee_p X$ will be denoted by x_1 (x_2) if x is in the first (resp. second) component of $X \vee_p X$. Let $X^2 = X \times X$ be the Cartesian product of X with itself.

1.8. Definition. The p r i n c i p a l a x i s m a p, $A_p : X \vee_p X \to X^2$ is defined by $A_p(x_1) = (x_1, p)$ and $A_p(x_2) = (p, x_2)$.

1.9. Definition. The s k e w e d p a x i s m a p, $S_p : X \vee_p X \to X^2$ is defined by $S_p(x_1) = (x_1, x_1)$ and $S_p(x_2) = (p, x_2)$.

1.10. Definition. The f o l d m a p a t p, $\nabla_p : X \vee_p X \to X$ is given by $\nabla_p(x_i) = x$ for $i = 1, 2$.

Let $U : E \to Sets$ be a topological functor, X an object in E, and p a point in $UX = B$.

1.11. Definitions

1. X is $\overline{T}_0$ at p iff the initial lift of the U-source $\{A_p : B\vee_p B \to U(X^2)=B^2$ and $\nabla_p : B\vee_p B \to UDB=B\}$ is discrete, where DB is the discrete structure on B.
2. X is T'_0 at p iff the initial lift of the U-source $\{id : B\vee_p B \to U(X\vee_p X)=B\vee_p B$ and $\nabla_p : B\vee_p B \to UDB=B\}$ is discrete, where $X\vee_p X$ is the wedge in E i. e. the final lift of the U-sink $\{i_1, i_2 : UX=B \to B\vee_p B\}$ where i_1, i_2 denote the canonical injections.
3. X is $Pre\ \overline{T}_2$ at p iff the initial lift of the U-source $\{S_p : B\vee_p B \to U(X^2)=B^2\}$ and the initial lift of the U-source $\{A_p : B\vee_p B \to U(X^2)=B^2\}$ agree.
4. X is T_1 at p iff the initial lift of the U-source $\{S_p : B\vee_p B \to U(X^2)=B^2$ and $\nabla_p : B\vee_p B \to UDB=B\}$ is discrete.
5. X is $Pre\ T'_2$ at p iff the initial lift of the U-source $\{S_p : B\vee_p B \to U(X^2)=B^2$ and the final lift of the U-sink $\{i_1, i_2 : UX=B \to B\vee_p B\}$ agree.
6. X is $\overline{T}_2$ at p iff X is $\overline{T}_0$ at p and $Pre\ \overline{T}_2$ at p.
7. X is T'_2 at p iff X is T'_0 at p and $Pre\ T'_2$ at p. See [1], p. 19 or [2].

1.12. Remark. We define p_1, p_2, ∇_p, by $1+p$, $p+1$, $1+1 : B\vee_p B \to B$, respectively where $1 : B \to B$ is the identity map and $p : B \to B$ is constant map at p. Note that $\pi_1 A_p = p_1 = \pi_1 S_p$, $\pi_2 A_p = p_2$, $\pi_2 S_p = \nabla_p$. Furthermore, when showing A_p and S_p are initial, it is sufficient to show that (p_1 and p_2) and (p_1 and ∇_p) are initial lifts, respectively. See [1] p. 22.

2. Separation Properties at p

In this section, we give explicit characterizations of the generalized separation properties for the topological categories of Reflexive Relation Spaces, *R Rel* and Preordered spaces, *Prord*.

2.1. Definitions. Let R be a relation on a set B and $p\in B$. R is said to be antisymmetric at p iff for $x\in B$, if xRp and pRx, then $x=p$. R is said to be symmetric at p iff for all $x\in B$, if xRp, then pRx and pRx, then xRp. R is said to be transitive at p iff for all $x, y\in B$ if xRp and pRy, then xRy, if pRx and xRy, then pRy and if xRy and yRp, then xRp.

2.2. Theorem. *$X=(B,R)$ in R Rel or Prord is $\overline{T}_0$ at p iff R is antisymmetric at p.*

Proof. Suppose X is $\overline{T}_0$ at p i. e. by 1.4, 1.5, 1.12 for any pair u, v in the wedge, $p_1 uRp_1 v$, $p_2 uRp_2 v$, and $\nabla u = \nabla v$ iff $u=v$. We will show that R is antisymmetric at p i. e. iff xRp and pRx, then $x=p$. Note that $p_1(x, p)Rp_1(p, x)=xRp$, $p_2(x, p)Rp_2(p, x)=pRx$, and $\nabla(x, p)=x=\nabla(p, x)$. Since X is $\overline{T}_0$ at p, $(x, p)=(p, x)$ i. e. $x=p$. Hence R is antisymmetric at p.

Conversely, suppose R is antisymmetric at p. If $u=v$, then clearly $p_1 uRp_1 v$, $p_2 uRp_2 v$ and $\nabla u=\nabla v$. It remains to show that if $p_1 uRp_1 v$, $p_2 uRp_2 v$, and $\nabla u=\nabla v$, then $u=v$. Since $\nabla u=\nabla v$, u and v have the form (x, p) or (p, x) for some x. If $u=(x, p)$ and $v=(p, x)$, then $p_1 uRp_1 v=xRp$, $p_2 uRp_2 v=pRx$, and thus, by the assumption, $x=p$ i. e. $u=v$. If $u=(x, p)$ and $v=(x, p)$, then $u=v$. Therefore X is $\overline{T}_0$ at p.

2.3. Theorem. *$X=(B,R)$ in R Rel or Pror d is T_1 at p iff R is discrete at p i. e. if xRp or pRx, then $x=p$.*

Proof. Suppose X is T_1 at p i. e. by 1.4, 1.5, and 1.12 for all pairs u and v in the wedge $p_1 uRp_1 v$, $\nabla uR\nabla$, and $\nabla u=\nabla v$ iff $u=v$. If xRp, then $p_1(x, p)Rp_1(p, x)=xRp$, $\nabla(x, p)R\nabla(p, x)=xRx$ and $\nabla(x, p)=x=\nabla(p, x)$. Since X is T_1 at p, it follows that $(x, p)=(p, x)$ i. e. $x=p$. Similarly, if pRx, then $x=p$. This shows that R is discrete at p.

On the other hand, suppose R is discrete at p. We must show that X is T_1 at p. If $u=v$, then $p_1 uRp_1 v$, $\nabla uR\nabla v$, and $\nabla u=\nabla v$ since *R Rel* and *Prord* are reflexive. If $p_1 uRp_1 v$, $\nabla uR\nabla v$, and $\nabla u=\nabla v$, then $u, v=(x, p)$ or (p, x) for some x. If $u=(x, p)$ and $v=(p, x)$, then $p_1 uRp_1 v=xRp$ and $\nabla uR\nabla v=xRx$ and $\nabla u=x=\nabla v$. Since R is discrete at p, $x=p$ i. e. $u=v$. Similarly, if $u=(p, x)$ and $v=(x, p)$, then $u=v$. If $u=(x, p)$ and $v=(x, p)$, then $u=v$. Therefore X is T_1 at p.

2.4. Theorem. *All $X=(B, R)$ in R Rel are T'_0 at p.*

Proof. X is T'_0 at p means, by 1.4, 1.5, 1.7, and definition 1.11, for each pair u and v in the wedge, $\nabla u=\nabla v$ and there exist x and y in B such that xRy and $i_k x=u$ and $i_k y=v$ for some $k=1$ or 2 iff $u=v$. If $\nabla u=\nabla v$, then $u, v=(x, p)$ or (p, x) for some x. If $u=(x, p)$ and $v=(p, x)$, then clearly $x=p$ since $i_k x=u$ and $i_k y=v$ for some $k=1$ or 2. Similarly, if $u=(p, x)$ and $v=(x, p)$, then $x=p$ i. e. $u=v$. Hence we must have $u=(x, p)=v$ or $u=(p, x)=v$. If $u=v$ then $\nabla u=\nabla v$ and $i_k x=u=v$ for some $k=1$ or 2 and for some x in B (since xRx).

2.5. Theorem. *$X=(B, R)$ in Prord is T'_0 at p iff R is antisymmetric at p* (2.1)

Proof. Supose X is T'_0 at p i. e. by 1.4, 1.6, 1.5, and definition 1.11 for each pair u and v in the wedge, (a) $\nabla u=\nabla v$ and if u and v are in different component of the wedge, then there exist x and y in B such that xRp and pRy with $i_k x=u$ and $i_n y=v$ for some $k, n=1$ or 2 and $k\neq n$, and if u and v are in the same component of the wedge, then there exist x and y in B such that xRy and $i_k x=u$ and $i_k y=v$ for some $k=1$ or 2 iff (b) $u=v$. We must show that R is antisymmetric at p. If xRp and pRx, then $i_1 x=(x, p)$, $i_2 x=(p, x)$ and $\nabla(x, p)=x=\nabla(p, x)$. Since X is T'_0 at p, $(x, p)=(p, x)$ i. e. $x=p$. Conversely, suppose R is antisymmetric at p and condition (a) holds. We must show that (b) holds. If $\nabla u=\nabla v$, then u and v have the form (x, p) or (p, x) for some x in B. If $u=(x, p)$ and $v=(p, x)$, then it follows from (a) that xRp, pRx and $i_1 x=u$, $i_2 x=v$ since u and v are in different component of the wedge. By the assumption that R is antisymmetric at p, we get $x=p$. Similarly if $u=(p, x)$ and $v=(x, p)$, then $x=p$ and consequently $u=v$. Clearly (b) implies (a) since *Prord* is reflexive. Therefore X is T'_0 at p.

2.6. Theorem. *$X=(B, R)$ is Pre $\overline{T}_2$ at p iff R is symmetric at p* (2.1) *for X in Prord, and both symmetric and transitive at p* (2.1) *for X in R Rel.*

Proof. Suppose X is *Pre* $\overline{T}_2$ at p i. e. by 1.5, and 1.12 for any pair u and v in the wedge, if $p_1 uRp_1 v$, then $p_2 uRp_2 v$ iff $\nabla uR\nabla v$. We must show that R is symmetric at p if X is in *Prord* and R is symmetric and transitive at p if X is in *R Rel*. If xRp, then $p_1(x, p)Rp_1(p, x)=xRp$ and $\nabla(x, p)R\nabla(p, x)=xRx$. Since X is *Pre* $\overline{T}_2$ at p, it follows that $p_2(x, p)Rp_2(p, x)=pRx$. Similarly if pRx then xRp. Hence R is symmetric at p.

Further, if X is in $R\,Rel$ and xRp and pRy, then $p_1(x, p)Rp_1(p, y)=xRp$ and $p_2(x, p)Rp_2(p, y)=pRy$. Since X is *Pre* T_2 at p, $\nabla(x, p)R\nabla(p, y)=xRy$. If pRx and xRy for some x, y in B, then by the result of the first part, xRp (since we have already proved that R is symmetric at p). Note that $p_1(x, p)Rp_1(p, y)=xRp$ and $\nabla(x, p)R\nabla(p, y)=xRy$. Since X is *Pre* T_2 at p, $p_2(x, p)Rp_2(p, y)=pRy$.

On the other hand, suppose R is symmetric at p for X in *Prord* and both symmetric and transitive at p for X in $R\,Rel$. We must show that X is *Pre* T_2 at p. We consider various possibilities for u and v, namely, $u=(x, p)$, (p, x) or (p, p) and $v=(y, p)$, (p, y) or (p, p). If $u=(x, p)$ and $v=(y, p)$, then if $p_1uRp_1v=xRy$, then $p_2uRp_2v=pRp$ iff $\nabla uR\nabla v=xRy$. If $u=(x, p)$ and $v=(p, y)$ then if $p_1uRp_1v=xRp$, then $p_2uRp_2v=pRy$ iff $\nabla uR\nabla v=xRy$ (since R is symmetric and transitive at p). If $u=(x, p)$ and $v=(p, p)$ then if $p_1uRp_1v=xRp$, then $p_2uRp_2v=pRp$ iff $\nabla uR\nabla v=xRp$. If $u=(p, x)$ and $v=(y, p)$, then $p_1uRp_1v=pRy$, then $p_2uRp_2v=xRp$ iff $\nabla uR\nabla v=xRy$ since R is symmetric and transitive at p. Similarly, if ($u=(p, x)$ and $v=(p, y)$ or (p, p)) or ($u=(p, p)$ and $v=(y, p)$, (p, y) or (p, p)) then the *Pre* T_2 condition holds. This completes the proof.

2.7. Theorem. *$X=(B, R)$ in $R\,Rel$ is Pre T'_2 at p iff R is discrete at p.*

Proof. Suppose X is *Pre* T'_2 at p i. e. by 1.7, 1.5, and 1.12 for each pair u and v in the wedge (a) p_1uRp_1v and $\nabla uR\nabla v$ iff (b) there exist x and y in B such that xRy and $i_kx=u$, $i_ky=v$ for some $k=1$ or 2. We will show that if xRp or pRx, then $x=p$. If xRp, then $p_1(x, p)Rp_1(p, x)=xRp$ and $\nabla(x, p)R\nabla(p, x)=xRx$. Since X is *Pre* T'_2 at p, it follows that $x=p$. Similarly if pRx, then $x=p$ i. e. R is discrete at p.

Conversely, we will show that if R is discrete at p, then X is *Pre* T'_2 at p i. e. and (b) above are equivalent. By [1], (b) implies (a) since $R\,Rel$ is normalized (a). It remains to show that (a) implies (b). To this end we consider following cases for u and v: $u=(x, p)$, (p, x) or (p, p) and $v=(y, p)$, (p, y) or (p, p). If $u=(x, p)$ and $v=(y, p)$, then $p_1uRp_1v=xRy=\nabla uR\nabla v$. Clearly $i_1x=u$ and $i_1y=v$. If $u=(x, p)$ and $v=(p, y)$, then $p_1uRp_1v=xRp$ and $\nabla uR\nabla v=xRy$. Since R is discrete at p, $x=p$ and consequently, $i_1p=(p, p)=u$ and $i_1y=(y, p)=v$. If $u=(x, p)$ and $v=(p, p)$, then $p_1uRp_1v=xRp=\nabla uR\nabla v$. Clearly $i_1x=u$ and $i_1p=(p, p)=v$. If $u=(p, x)$ and $v=(y, p)$, then $p_1uRp_1v=pRy$ and $\nabla uR\nabla v=xRy$. Since R is discrete at p, $y=p$ and consequently, $i_2x=u$ and $i_2p=v$. Similarly, if $u=(p, x)$ or (p, p) and $v=(p, y)$ or (p, p), then clearly the *Pre* T'_2 condition holds. This completes the proof.

2.8. Theorem. *$X=(B, R)$ in Prord is Pre T'_2 at p iff R is symmetric at p.*

Proof. Suppose X is *Pre* T'_2 at p i. e. by 1.6, 1.5, and 1.12 for each pair u and v in the wedge (a) if u and v are in the different component of the wedge, then there exist x and y in v such that xRp and pRy, $i_kx=u$, and $i_ny=v$ for some k, $n=1$ or 2 and $k\neq n$ and if u and v are in the same component of the wedge, then there exist x and y in B such that xRy, $i_kx=u$, and $i_ky=v$ for some $k=1$ or 2 iff p_1uRp_1v and $\nabla uR\nabla v$. We must show that R is symmetric at p i. e. if xRp or pRx, then pRx or xRp. If xRp, then $p_1(x, p)Rp_1(p, x)=xRp$ and $\nabla(x, p)R\nabla(p, x)=xRx$. Since X is *Pre* T'_2 at p, it follows that pRx (since $i_1x=(x, p)$, $i_2x=(p, x)$, and xRp). Similarly, if pRx, then xRp. Hence R is symmetric at p.

On the other hand, suppose R is symmetric at p. We will show that X is *Pre* T'_2 at p i. e. (a) and (b) above are equivalent. By [1] (a) implies (b) since *Prord* is normalized. It remains to show that (b) implies (a). To this end we consider the following cases for u and v : $u=(x, p)$, (p, x), and (p, p) and $v=(y, p)$, (p, y), and (p, p). If $u=(x, p)$ and $v=(y, p)$, then $p_1 uRp_1 v=xRy=\nabla uR\nabla v$. Clearly, $i_1 x=u$ and $i_1 y=v$. If $u=(x, p)$ and $v=(p, y)$, then $p_1 uRp_1 v=xRp$ and $\nabla uR\nabla v=xRy$. Since R is symmetric at p, pRx. R is transitive implies pRy since pRx and xRy. Hence we have xRp and pRy with $i_1 x=u$ and $i_2 y=v$. If $u=(x, p)$ and $v=(p, p)$, then $p_1 uRp_1 v =(x, p)=\nabla uR\nabla v$. Clearly, $i_1 x=u$ and $i_1 p=v$. If $u=(p, x)$ and $v=(y, p)$, then $p_1 uRp_1 v=pRy$ and $\nabla uR\nabla v=xRy$. Since R is symmetric at p, yRp. Hence by transitivity of R, xRp and consequently, $i_2 x=u$ and $i_1 y=v$. If $u=(p, x)$ and $v=(p, y)$, then $p_1 uRp_1 v=pRp$ and $\nabla uR\nabla v=xRy$. Clearly, $i_2 x=u$ and $i_2 y=v$. If $u=(p, x)$ and $v=(p, p)$, then $p_1 uRp_1 v=pRp$ and $\nabla uR\nabla v=xRp$ and clearly $i_2 x=u$ and $i_2 p=v$. Similarly, if $u=(p, p)$ and $v=(y, p)$, (p, y) or (p, p), then $i_k p=u$ and $i_1 y=(y, p)$, $i_2 y=(p, y)$ or $i_k p=(p, p)$ for some $k=1$ or 2. This shows that X is *Pre* T'_2 at p.

2.9. Theorem. *$X=(B, R)$ in $R\,Rel$ or Pror d is T_2 at p iff R is discrete at p.*

P r o o f. Combine 2.2, 2.6, and definition 1.11. Note that if R is antisymmetric at p and symmetric at p, then it is discrete since if xRp, then by symmetry at p, pRx and consequently, by antisymmetry at p, $x=p$. Similarly, if pRx, then $x=p$.

2.10. Theorem. *$X=(B, R)$ in $R\,Rel$ is T'_2 at p iff R is discrete at p.*

P r o o f. Combine 2.4, 2.7, and definition 1.11.

2.11. Theorem. *$X=(B, R)$ in Pror d is T'_2 at p iff R is discrete at p.*

P r o o f. Combine 2.5, 2.8, and definition 1.11 and note that if R is antisymmetric and symmetric at p, then it is discrete at p.

2.12. R e m a r k. In *Prord*, both T_0 and T'_0 at p, *Pre* T_2 and *Pre* T'_2 at p, and both T_2 and T'_2 at p are equivalent. T_0 at p and *Pre* T'_2 at p imply T'_0 at p and *Pre* T_2 at p, respectively in $R\,Rel$ and both T_2 and T'_2 at p are identical in $R\,Rel$.

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Erciyes University
Faculty of Arts and Sciences
Department of Mathematics
38039 Kayseri
TURKEY

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