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Generalized Stability of Integral Manifolds and Lyapunov Vector Function

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We use in this paper generalized space which permit to introduce definition of generalized stability of integral manifold. Theorems are proved referring to various characteristics of integral manifolds using generalized norms

The second method of Lyapunov for stability of integral manifold of differential equations using Lyapunov vector functions and comparision method is well known I. Russinov [1], I. Muhametzianov [2]. For Dynamical systems with large dimension F. N. Bailey [3], R. W. Mitchel, D. A. Pace [4] the problem for construction of Lyapunov vector function and development of quality theory of ordinary differential equations is small effectiveness. One of the reasons for this situaiton that definitions of stability given by means of ordinary norm are insufficient flexible.

Definition 1. [4] A generalised norm from $\mathbb{R}^n$ to $\mathbb{R}^k$ is the operator $\|\cdot\|_G : \mathbb{R}^n \to \mathbb{R}^k$ denoted with $\|x\|_G = (\theta_1(x), \dots, \theta_k(x))$ such that

[a)] $\|x\|_G \geq 0$ (i.e. $\theta_i(x) \geq 0$ for $i = \overline{1,k}$)

[b)] $\|x\|_G = 0$ if and only if $x = 0$

[c)] $\|\lambda x\|_G = |\lambda| \|x\|_G$ (i.e. $\theta_i(\lambda x) = |\lambda| \theta_i(x)$ for $i = \overline{1,k}$)

[d)] $\|x + y\|_G \leq \|x\|_G + \|y\|_G$ (i.e. $\theta_i(x+y) \leq \theta_i(x) + \theta_i(y)$)

We consider the space $E^n = (\mathbb{R}^n, \|\cdot\|_G)$ with generalized norm $\|x\|_G = (\|y\|_G^2 + \|z\|_G^2)^{1/2}$ as $\|y\|_G = (\theta_1(y), \dots, \theta_r(y))$, $\|z\|_G = (\theta_{r+1}(z), \dots, \theta_k(z))$ are generalized norms and

$$\|y\|_G : \mathbb{R}^m \to \mathbb{R}^r, \ \|z\|_G : \mathbb{R}^p \to \mathbb{R}^{k-r}; \ x = (y^1, \dots, y^m, z^1, \dots, z^p)^T$$

We shall consider the diferential system

$$\dot{x} = X(t,x),\ x(t_0) = x_0 \tag{1}$$

where $X \in C[\mathbb{R}_+ \times S(H) \times \mathbb{R}^p, \mathbb{R}^n]$, $X = (Y^1,\dots,Y^m,Z^1,\dots,Z^p)^T$ as

$$\begin{aligned} &\mathbb{R}_+ = [0,+\infty],\quad S(H) = \{y \in E^m/\|y\|_G < H\}, \\ &H \in \mathbb{R}^r_+,\quad 0 < H_\tau < +\infty,\ \text{for } \tau = \overline{1,r} \end{aligned} \tag{2}$$

Here $E^m = (\mathbb{R}^m, \|y\|_G)$ and $E^p = (\mathbb{R}^p, \|z\|_G)$ are generalized sub-space of E^n.

We suppose the solutions of the system (1) are z - prolongable (i.e. every solution $x(t)$ is defined for each $t \in \mathbb{R}_+$ for which $\|y\|_G \le H$. This condition means that none of the coordinates $z^j (j = \overline{1,p})$ on finite time interval tends infinity.

We denote by $x = x(t;t_0,x_0)$ the solution of the system (1) with initial values $x(t;t_0,x_0) = x_0$.

We write the system (1) in the form

$$\dot{y} = Y(t,y,z) \tag{3}$$

$$\dot{z} = Z(t,y,z) \tag{4}$$

as $Y(t,0,z) \equiv 0$. Then the system (3)-(4) and consequently the system (1) possess p dimensional integral manifold $y = 0$ [2]. We investigate the stability of this manifold. For that purpose we use the following definition

Definition 2. The integral manifold is said to be with respect to the system (3)–(4) :

[M1)] stable if for each vector $\mathcal{E} \in \mathbb{R}^r_+$ and for each $t_0 \in \mathbb{R}_+$ there exists vector function $\delta(t_0,\mathcal{E}) \in \mathbb{R}^r_+$ which is continious in t_0 for each vector $\mathcal{E}$ such that $\|y_0\|_G < \delta$, $0 \le (\|z\|_G)_j < +\infty$ $(j = \overline{1,p})$, $x_0 = (y_0, z_0)$ implies $\|y(t;t_0,x_0)\|_G < \mathcal{E}$ for each $t \ge t_0$.

[M2)] equi-asymptotically stable if it is stable and for each vector $\mathcal{E} \in \mathbb{R}^r_+$ and for each $t_0 \in \mathbb{R}_+$ exist $\delta_0 = \delta_0(t_0)$, $\delta_0 \in \mathbb{R}^r_+$ and $T = T(t_0,\mathcal{E})$ such that $\|y_0\|_G < \delta_0$, $0 \le (\|z\|_G)_j$ $(j = \overline{1,p})$, implies $\|y(t;t_0,x_0)\|_G < \mathcal{E}$.

We consider vector functions $V(t,y) = (V^1(t,y),\dots,V^s(t,y))$ defined and continious in the region

$$\overline{\Gamma}'_0 = \{(t,y)\ :\ t \in \mathbb{R}_+, \|y\|_G \le H^1_0\},\ 0 < H^1_0 < H$$

together with their derivatives with respect to t.

$\dot{V} = (\dot{V}^1, \ldots, \dot{V}^s)$ as $V(t, 0) \equiv 0$. For the set of these functions we deduce the norm

$$\|V\| = |V^1| + \ldots |V^s|, \quad s \leq n.$$

We also consider the functions $f^1(t, V, y), \ldots, f^s(t, V, y)$ which are defined and continuous in the region $\mathbb{R}_+ \times \Omega_1$. The region is determined in following way

$$\Omega_1 = \{V, y \ : |V^1| + \ldots + |V^l| < K_1, \ |V^{l+1}| + \ldots + |V^s| < +\infty, \ \|y\|_G \leq H_0^1\}$$

besider

$$K = \sup[|V^1| + \ldots + |V^l| \ : \ (t, y) \in \overline{\Gamma}_0'] < K_1 < +\infty \text{ or } K_1 = +\infty$$

We use the comparison system

$$\dot{u} = f(t, u, \eta(t)), \ u \in \mathbb{R}_+^s, \ \eta \in \mathbb{R}^m \tag{5}$$

when $u = (v^1, \ldots, v^l, w^1, \ldots, w^{s-l})^T$, $f = (F^1, \ldots, F^l, G^1, \ldots, G^{s-l})^T$, $\|v\| = |v^1| + \ldots |v^l|$, $\|w\| = |w^1| + \ldots |w^{s-l}|$, $\|u\| = |u^1| + \ldots |u^s| = (\|v\|^2 + \|w\|^2)^{1/2}$

We suppose that $f(t, 0, 0) \equiv 0$ and the solutions of the system (5) are w-prolongable.

We write the system (5) in the form

$$\dot{v} = F(t, v, w) \tag{6}$$

$$\dot{w} = G(t, v, w) \tag{7}$$

and let the condition $F(t, 0, w, \eta(t)) \equiv 0$ holds as $t \geq 0$ which shows that system (6)-(7) possesses $(s + 1 - l)$-dimensional right integral $v = 0$, $(t \geq 0)$ in the extended phase space. If in $\mathbb{R}^s$ we deduce partial consequence setting for $u_1, u_2 \in \mathbb{R}^s$: $u_1 \leq u_2 \iff u_1^1 \leq u_2^1, \ldots, u_1^s \leq u_2^s$. Then one can use the vectoral inequalities

$$\dot{V} \ \leq \ f(t, V(t), y(t)), \ V \in \mathbb{R}^s, \ y(t) \in \mathbb{R}^m \tag{8}$$

of Chapligin's type.

We use the definitions for quasimonotone nondecreasing function $f(t, V, y)$ and for maximal (minimal) solution $\overline{u}(t; t_0, x_0)$ $(\underline{u}(t; t_0, x_0))$ of system (5) from [1].

Let us the function $\varphi(A)$ $(A \in \mathbb{R}^r_+)$ is given which is defined , continuous and nondecreasing on the interval $[0, H_0^1]$, $(H_0^1 < H)$ in addition $\varphi(0) = 0$, $\varphi(H_0^1) \leq K$. Obviousle for the positive definite function $W(\theta_1(y), \dots, \theta_r(y))$, $(W(0, \dots, 0) = 0,\ W(\theta_1(y), \dots, \theta_r(y)) \leq K$ as $\|y\|_G \leq H_0^1)$ the functions

$$\varphi_1(A) = \inf[W(\theta_1(y), \dots, \theta_r(y)) \ : \ A \leq \|y\|_G \leq H_0^1]$$
$$\varphi_2(A) = \sup[W(\theta_1(y), \dots, \theta_r(y)) \ : \ \|y\|_G \leq A]$$

possesses the same properties (since $W(\theta_1(y), \dots, \theta_r(y))$ is uniformly continuous).

We introduce

Definition 3. The right integral manifold $v = 0$ $(t \geq 0)$ is said to be with respect to the system (6)-(7) :

[$M1^*$)] quasi-stable at function $\varphi(A)$ if for each vector $A \in \mathbb{R}^r_+$ $(A < H_1^0)$ and $t_0 \in \mathbb{R}_+$ there exist functions $\alpha(t_0, A) \in \mathbb{R}^r_+$ and $\hat{\alpha}(t_0, A) \in \mathbb{R}_+$ $(\hat{\alpha} \leq \min \alpha_\tau$, $\tau = \overline{1,r}$, $\alpha < A$, $\hat{\alpha} < \varphi(A))$ such that for each continously differentiable function $\eta(t)$ in $\mathbb{R}_+$ for which $\|\eta(t_0)\|_G \leq \alpha$ each solution $(v(t), w(t))$ of system (6)-(7) with initial conditions $\|v(t_0)\| \leq \hat{\alpha}$, $\|w(t_0)\| < +\infty$ is defined and satisfy the inequality $\|v(t)\| < \varphi(A)$ on arbitrary time interval $[t_0, t^*]$ of which $\|\eta(t)\|_G \leq A$.

[$M2^*$)] equi-asymptotically quasi-stable at function $\varphi(A)$ if it is quasi-stable in this function $\varphi(A)$ and each vector $A \in \mathbb{R}^r_+$, for each $B \in \mathbb{R}_+$ $(A < H,\ B < \min A_\tau,\ \tau = \overline{1,r},\ B < \varphi(A))$ and for each $t_0 \in \mathbb{R}_+$ there exist vectors $\alpha(t_0, A) \in \mathbb{R}^r_+$ and $\hat{\alpha}(t_0, A) \in \mathbb{R}_+$ $(\alpha < A,\ \hat{\alpha} \leq \min \alpha_\tau,\ \tau = \overline{1,r}\ \hat{\alpha} < \varphi(A))$ and as well as for each continously differentiable function $\eta(t)$ for which $\|\eta(t_0)\|_G \leq \alpha$, $\|\eta(t_0)\|_G \leq A$ as $t \in \mathbb{R}_+$ and for each solution $(V(t), W(t))$ of system (6)-(7) with initial conditions $\|v(t_0)\| \leq \hat{\alpha}$, $\|w(t_0)\| < +\infty$ exists $T > 0$ such that $(V(t), W(t))$ is defined for $t \in [t_0, +\infty]$ and the inequalities $\|v(t)\| < \varphi(A)$ for $t \geq t_0$ $\|v(t)\| < B$ for $t \geq t_0 + T$ hold.

Theorem 1. *Assume that there exists vector function $V(t,y)$ satisfying in the region $\overline{\Gamma}'_0$ the following conditions:*

1) $\sum_{i=1}^{l} |V^i(t,y)| \geq W(\theta_1(y), \dots, \theta_r(y))$, *where W is positive definite function;*

2) $\dot{V} \leq f(t, V, y)$

3) The function $f(t, V, y)$ is quasimonotone nondecreasing

4) $\|y\|_G = (\theta_1(y), \ldots, \theta_r(y))$ is generalized norm and $\|y\|_G : \mathbb{R}^m \to \mathbb{R}^r$

5) The right integral manifold $v = 0$, $(t \geq 0)$ of the system (6)-(7) is quasi-stable (respectively equi-asymptotically quasi stable) at function $\varphi(A) = \inf[W(\theta_1(y), \ldots, \theta_r(y)) \ : \ A \leq \|y\|_G \leq H_0^1]$ with the initial values $u_0 = V(t_0, y_0)$, $(t_0, y_0) \in \overline{\Gamma}_0'$

Then the manifold $y = 0$ of the system (3)-(4) is stable (respectively equi-asymptotically stable).

Proof. Let the conditions 1)-5) of the theorem are hold and given vector $A \in \mathbb{R}_+^r$ $(A < H_0^1)$. From 1) and 5) it follows

$$0 < \varphi(A) = \inf[W(\theta_1(y), \ldots, \theta_r(y)) \ : \ A \leq \|y\|_G \leq H_0^1] \leq K$$

therefore $\|y\|_G < a$ at $W(\theta_1(y), \ldots, \theta_r(y)) < \varphi(A)$

For each vector $A \in \mathbb{R}_+^r$ and for each $t_0 \in \mathbb{R}_+$ there exist functions $\alpha(t_0, A) \in \mathbb{R}_+^r$ and $\hat{\alpha}(t_0, A) \in \mathbb{R}_+(\hat{\alpha} \leq \min \alpha_\tau,\ \tau = \overline{1, r},\ \alpha < A,\ \hat{\alpha} < \varphi(A))$ such that for each continously differentiable function $\eta(t)$ in $\mathbb{R}_+$ for which $\|\eta(t_0)\|_G \leq \alpha$ each solution $u(t; t_0, u_0)$ of the system (5) with initial conditions $\|v_0\| \leq \hat{\alpha}$, $\|w_0\| < +\infty$ is defined and satisfy the inequality $\|v(t; t_0, u_0)\| < \varphi(A)$ on arbitrary time interval $[t_0, t^*]$ on which $\|\eta(t)\|_G \leq A$.

The function $|V^1(t_0, y)| + \ldots + |V^l(t_0, y)|$ possesses infinitesimal superior limit so that for this $\hat{\alpha}$ and for t_0 there exists function $0 < \beta(\hat{\alpha}, t_0) = \beta(A, t_0) \leq \alpha$, $\beta \in \mathbb{R}_+^r$ which is continuous in t_0 for each A such that

$$|V^1(t_0, y_0)| + \ldots + |V^l(t_0; y_0)| \leq \hat{\alpha}, \text{ as } \|y_0\|_G \leq \beta,\ 0 \leq (\|z\|_G)_j < +\infty,\ j = \overline{1, p}$$

We show that for each solution $x(t; t_0, x_0)$ with initial conditions $t_0 \geq 0$, $\|y_0\|_G \leq \beta$, $(\|z\|_G)_j < +\infty$, $j = \overline{1, p}$ $\|y(t; t_0, x_0)\|_* < A$ holds as $t \geq t_0$. If this is not true, then there exist t^*, y_0^*, z_0^* $(\|y_0^*\|_G \leq \beta,\ 0 \leq (\|z_0^*\|_G)_j < +\infty,\ j = \overline{1, p},\ t^* > t_0)$ such that $\|y(t^*; t_0, y_0^*, z_0^*)\|_G = A$, $\|y(t; t_0, y_0^*, z_0^*)\|_G < A$ as $t \in [t_0, t^*)$

Setting $\eta(t) = y(t; t_0, y_0^*, z_0^*)$ then

$$\|\eta(t_0)\|_G = \|y_0^*\|_G \leq \beta \leq \alpha$$

and

$$\|\eta(t)\|_G \leq A \text{ as } t \in [t_0, t^*).$$

If we assume $u_0^* = u_0^*(t_0; t_0, u_0^*) = V(t_0, y_0^*)$ then for this function β $\|v_0^*\| \leq \hat{\alpha}$, $\|w_0^*\| < +\infty, u_0^* = (v_0^*, w_0^*) \in \overline{\Gamma}_0'$ therefore each solution $u(t; t_0, u_0^*)$ of the system (6)-(7) is defined and satisfy the inequality $\|v(t; t_0, v_0^*, w_0^*)\| < \varphi(A)$ as $t \in [t_0, t^*]$. This implies that the maximal solution $\overline{u}(t; t_0, u_0^*)$ in time interval $[t_0, t^* + \Delta t)$ ($\Delta t > 0$ and sufficiently small) satisfy the condition $\|\overline{v}(t; t_0, u_0^*)\| < \varphi(A)$.

We consider the function which is continuously differentiable as $V(t, y(t; t_0, y_0^*, z_0^*))$. We have from the condition 2) of the theorem

$$\text{(9)} \qquad \frac{dV(t, y(t; t_0, y_0^*, z_0^*))}{dt} \leq f(t, V(t, y(t; t_0, y_0^*, z_0^*)), y(t, y(t; t_0, y_0^*, z_0^*))$$

as $t \in [t_0, t^* + \Delta t)$

Inequaliti (9) and condition 3) of the theorem permit us to apply the Wazewski's theorem. Thus

$$V(t, y(t; t_0, y_0^*, z_0^*)) \leq \overline{u}(t; t_0, v_0^*, w_0^*) \text{ as } t \in [t_0, t^* + \Delta t).$$

We have in special case

$$\begin{aligned} &W((\theta_1(y(t; t_0, y_0^*, z_0^*)), \ldots, \theta_1(y(t; t_0, y_0^*, z_0^*)))) \\ &\leq \sum_{i=1}^{l} |V^i(t, y(t; t_0, y_0^*, z_0^*))| \\ &\leq \|\overline{v}(t; t_0, v_0^*, w_0^*)\| < \varphi(A) \end{aligned}$$

as $t \in [t_0, t^*]$. Then $\|y(t; t_0, y_0^*, z_0^*)\|_G < A$ as $t \in [t_0, t^*]$ which is a contradiction. Thus we prove the stability of the integral manifold $y = 0$ of the system (3) (4).

If the quasi-stability in condition 5) of the theorem is equi-asymptotically then for each B ($B < \min A_\tau$, $\tau = \overline{1, r}$, $B < \varphi(A)$) and for each solution $x(t; t_0, x_0)$ with initial values $\|y_0\|_G \leq \beta \leq \alpha$, $0 < (\|z_0\|_G)_j < +\infty$ (i.e. the condition $\|y(t; t_0, y_0^*, z_0^*)\| < A$ holds as $t \in [t_0, +\infty)$ and for its corresponding function $V(t, y(t; t_0, y_0, z_0))$ which satisfy the condition $|V^1(t_0, y_0) + \ldots + V^l(t_0, y_0)| \leq \hat{\alpha}$ (i.e. for t_0, $u_0 = (v_0, w_0)$, $\|v_0\| \leq \hat{\alpha}$, $\|w_0\| < +\infty$, $u_0 \in V(\overline{\Gamma}_0')$) there exist $T(B, A, , t_0, v_0, w_0)$ such that $\|v(t; t_0, v_0, w_0)\| < B$ as $t \geq t_0 + T$. Then

$$\sum_{i=1}^{l} |V^i(t, y(t; t_0, x_0))| < B \text{ as } t \geq t_0 + T$$

Consequently

$$\lim_{t\to\infty} \sum_{i=1}^{l} |V^i(t, y(t; t_0, x_0))| = 0$$

From here and from condition 1) of the theorem we have

$$\lim_{t\to\infty} (\|y(t; t_0, x_0)\|_G)_j = 0, \quad j = \overline{1, m}$$

which shows that the integral manifold $y = 0$ of the system (3)-(4) is equi-asymptotically stable. This completes the proof.

Let the system (3)-(4) possesses $(n - m_1)$-dimensional manifold $y_1 = 0$ and $(n - m_2)$-dimensional manifold $y_2 = 0$, $m_1 + m_2 = n - p$. We state a result which demonstrates with higher accuracy the idea of the generalized norm. The proof is direct cotrollary from Theorem 1.

Theorem 2. *Assume that there exists vector-function*

$$V(t, y) = (V^1(t, y_1), \ldots, V^{l_1}(t, y_1), V^{l_1+1}(t, y_2), \ldots, V^{l_1+l_2}(t, y_2))$$

which satisfy the folloing properties in the region $\overline{\Gamma}'_0$:

1) $\|x\|_G = (\theta_1(y_1), \ldots, \theta_{r_1}(y_1), \ \theta_{r_1+1}(y_2), \ldots, \theta_{r_1+r_2}(y_2), \ \theta_{r_1+r_2+1}(z), \ldots, \theta_k(z))$

$y_1 = (y^1, \ldots, y^{m_1})$, $y_2 = (y^{m_1+1}, \ldots, y^{m_1+m_2})$, $z = (z^1, \ldots, z^p)$,
$x = (y_1, y_2, z)$, $\|x\|_G : \mathbb{R}^n \to \mathbb{R}^k$, $\|y_1\|_G : \mathbb{R}^{m_1} \to \mathbb{R}^{r_1}$,
$\|y_2\|_G : \mathbb{R}^{m_2} \to \mathbb{R}^{r_2}$, $\|z\|_G : \mathbb{R}^p \to \mathbb{R}^{k-(r_1+r_2)}$;

2) $\sum_{i=1}^{l_1} |V^i(t, y_1)| \ \geq \ W_2(\theta_1(y_1), \ldots, \theta_{r_1}(y_1))$,

$\sum_{i=1}^{l_2} |V^{l_1+i}(t, y_2)| \ \geq \ W_2(\theta_{r_1+1}(y_1), \ldots, \theta_{r_1+r_2}(y_2))$,

where W_1 *and* W_2 *are positive definite functions;*

3) $\dot{V} \ \leq \ f(t, V, y)$

4) The function $f(t, V, y)$ *is quasimonotone nondecreasing*

5) The right integral manifolds $v_1 = 0$ $(t \geq 0)$ *and* $v_2 = 0$ $(t \geq 0)$ *of the system (6)-(7) are quasi-stable and equi-asymptotically quasi-stable respectively* *at functions*

$$\varphi_1(A_1) = \inf[W_1(\theta_1(y_1), \dots, \theta_{r_1}(y_1)) \ : \ A_1 \le \|y_1\|_G \le H^1_{0_r}, \ \tau = \overline{1, m_1}\ ]$$

$$\varphi_2(A_2) = \inf[W_2(\theta_{r_1+1}(y_2), \dots, \theta_{r_1+r_2}(y_2)) \ : \ A_2 \le \|y_2\|_G \le H^1_{0_{m_1+r}}, \ \tau = \overline{1, m_2}\ ]$$

Then the integral manifolds $y_1 = 0$ and $y_2 = 0$ of the system (3)-(4) are quasi-stable and equi-asymptotically quasi-stable respectively.

Example.

$$\begin{aligned}
\dot y^{(1)} &= (e^{-t} + \sin t) y^{(1)} - y^{(1)} {y^{(3)}}^2 \\
\dot y^{(2)} &= (e^{-t} - \sin t) y^{(2)} - y^{(2)} {y^{(3)}}^2 \\
\dot y^{(3)} &= (e^{-t} - 2) y^{(3)} - \frac{1}{4} y^{(3)} (y^{(1)} + y^{(2)})^2 \\
\dot y^{(4)} &= -3 y^{(4)} - \frac{1}{4} y^{(4)} (y^{(1)} + y^{(2)})^2 \\
\dot z^{(1)} &= e^{-t} - \frac{1}{4} z^{(1)} (y^{(1)} - y^{(2)})^2
\end{aligned} \tag{10}$$

The system (10) possesses the integral manifolds $y_1 = (y^{(1)}, y^{(2)}) = 0$ and $y_2 = (y^{(3)}, y^{(4)}) = 0$.

We deduce the generalized norm

$$\|y\|_G = \begin{pmatrix} |y^{(1)}| \\ |y^{(2)}| \\ \sqrt{{y^{(3)}}^2 + {y^{(4)}}^2} \end{pmatrix}$$

We choose The Lyapunov vector function in the form

$$V(y_1, y_2) = (V^1(y^{(1)}), V^2(y^{(2)}), V^3(y^{(3)}, y^{(4)}) \ .$$

where

$$V^1(y^{(1)}) = {y^{(1)}}^2, \ V^2(y^{(2)}) = {y^{(2)}}^2, \ V^3(y^{(3)}, y^{(4)}) = {y^{(3)}}^2 + {y^{(4)}}^2 .$$

$$W_1 = \sqrt{{y^{(1)}}^2, + {y^{(2)}}^2}, W_2 \equiv V^3$$

The comparision system is

$$\dot{v}^{(1)} = 2\dot{v}^{(1)}\,(e^{-t} + \sin t)$$
$$\dot{v}^{(2)} = 2\dot{v}^{(2)}\,(e^{-t} - \sin t)$$
$$\dot{v}^{(3)} = -\,2v^{(3)}$$

which possesses right integral manifolds $v_1 = (v^{(1)}, v^{(2)}) = 0,\ (t \geq 0)$ and $v_2 = v^{(3)} = 0\ (t \geq 0)$. At this $v_1 = 0,\ (t \geq 0)$ is quasi-stable and $v_2 = 0,\ (t \geq 0)$ is equi-asymptotically quasi-stable. Theorem 2. implies the integral manifolds $y_1 = 0$ and $y_2 = 0$ of the system (10) possess stability of the same kinds respectively.

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