Mathematica Balkanica

Mathematical Society of South-Eastern Europe
A quarterly published by
the Bulgarian Academy of Sciences – National Committee for Mathematics

---

Mathematica Balkanica

New Series Vol. 9, 1995, Fasc. 2-3

Subgroups of the Isometry Group in a Galilean Space I: The Cases of Two and Three-Parametric Subgroups [1]

Adrijan V. Borisov

Presented by P. Kenderov

In this paper the two and three-parametric subgroups of the isometry group in three-dimensional Galilean space are determined.

1. Introduction

With respect to nonhomogeneous coordinates an isometry of the six-parametric isometry group B_6 in the three-dimensional Galilean space G_3 has the form

$$\overline{x} = a + x,$$

$$\overline{y} = b + cx + cos\varphi.y + \sin\varphi.z,$$

$$\overline{z} = d + ex - \sin\varphi.y + cos\varphi.z,$$

where a, b, c, d, e and φ are real numbers [3]. The infinitesimal operators of B_6 are

$$X_1 = \frac{\partial}{\partial x}, \quad X_2 = \frac{\partial}{\partial y}, \quad X_3 = x\frac{\partial}{\partial y},$$

$$X_4 = \frac{\partial}{\partial z}, \quad X_5 = x\frac{\partial}{\partial z}, \quad X_6 = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}$$

[1] This work was in part supported by MES grant MM-18/91.

and satisfy the system

$$\begin{array}{lll} [X_1,X_2]=0, & [X_1,X_3]=X_2, & [X_1,X_4]=0, \\ [X_1,X_5]=X_4, & [X_1,X_6]=0, & [X_2,X_3]=0, \\ [X_2,X_4]=0, & [X_2,X_5]=0, & [X_2,X_6]=-X_4, \\ [X_3,X_4]=0, & [X_3,X_5]=0, & [X_3,X_6]=-X_5, \\ [X_4,X_5]=0, & [X_4,X_6]=X_2, & [X_5,X_6]=X_3, \end{array}$$

where [...] is the bracket of Poisson.

For the necessities of some applications the natural problem which arises is to classify the subgroups of B_6. That is the aim of this paper and we give the two and the three-parametric subgroups of B_6 which are different up to a Galilean isometry. The results have been announced without proofs [1], which are given here.

2. Two-parametric subgroups of B_6

A two-parametric subgroup of B_6 can be defined by two infinitesimal operators (see [2], p.163)

$$Y_h = \sum_{k=1}^{6} a_{hk} X_k, \quad h = 1, 2, \tag{1}$$

satisfying the conditions

$$[Y_i, Y_j] = \sum_{k=1}^{2} c_{ij}^k Y_k, \quad i, j = 1, 2; \quad i \neq j \tag{2}$$

where a_{hk} and c_{ij}^k are real numbers. Now we shall consider the possible cases.

1. $a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = 0$. Then $a_{11} \neq 0$ and by a suitable change of the variables we can obtain $a_{11} = 1, \quad a_{21} = 0$.

1.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Then $a_{22} \neq 0$ and choosing $a_{22} = 1$ we obtain the subgroup

$$B_{21} = \{X_1, X_2\}.$$

1.2. $a_{23} \neq 0, \quad a_{24} = a_{25} = a_{26} = 0$. We put $a_{23} = 1$ and therefore $Y_1 = X_1$, $Y_2 = a_{22}X_2 + X_3$. From $[Y_1, Y_2] = X_2$ we deduce that the operators Y_1 and Y_2 do not define a group.

1.3. $a_{24} \neq 0, \quad a_{25} = a_{26} = 0$. We take $a_{24} = 1$ and after the substitution

$$\overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z), \quad \overline{z} = \frac{1}{\sqrt{1 + a_{22}^2}}(-y + a_{22}z) \tag{3}$$

we get the subgroup B_{21}.

1.4. $a_{25} \neq 0, \quad a_{26} = 0$. Choosing $a_{25} = 1$ we have that $Y_h, \quad h = 1, 2$, do not define a group.

1.5. $a_{26} \neq 0$. We assume $a_{26} = 1$. The operators define a group iff $a_{23} = a_{25} = 0$. Now we make the change

$$\overline{x} = x, \quad \overline{y} = -a_{24} + y, \quad \overline{z} = a_{22} + z$$

and obtain the subgroup

$$B_{22} = \{X_1, X_6\}.$$

2. $a_{12} \neq 0, \quad a_{13} = a_{14} = a_{15} = a_{16} = 0$. We suppose $a_{12} = 1, \quad a_{22} = 0$.

2.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Consequently $a_{21} \neq 0$ and taking $a_{21} = 1, \quad a_{11} = 0$ we get B_{21}.

2.2. $a_{23} \neq 0, \quad a_{24} = a_{25} = a_{26} = 0$. We put $a_{23} = 1$. The operators define a group iff $a_{11} = 0$ and we find the subgroup

$$B_{23} = \{X_2, \alpha X_1 + X_3 | \alpha \in \mathbb{R}\}.$$

2.3. $a_{24} \neq 0, \quad a_{25} = a_{26} = 0$. We assume $a_{24} = 1$. Then the operators define a group if and only if $a_{11}a_{23} = 0$.

2.3.1. $a_{11} = 0$.

2.3.1.1. $a_{21} = 0$. In this case we obtain the subgroup

$$B_{24} = \{X_2, \alpha X_3 + X_4 | \alpha \in \mathbb{R}\}.$$

2.3.1.2. $a_{21} \neq 0$. We make the substitution

$$\overline{x} = x, \quad \overline{y} = y, \quad \overline{z} = -\frac{1}{a_{21}}x + z \tag{4}$$

and we have B_{21} or B_{23} if $a_{23} = 0$ or $a_{23} \neq 0$, respectively.

2.3.2. $a_{23} = 0$.

2.3.2.1. $a_{11} = a_{21} = 0$. Now we get B_{24} by $\alpha = 0$.

2.3.2.2. $a_{11} = 0, \quad a_{21} \neq 0$. Applying (4) we obtain B_{21}.

2.3.2.3. $a_{11} \neq 0, \quad a_{21} = 0$. By means of the change

$$\overline{x} = x, \quad \overline{y} = y, \quad \overline{z} = \frac{1}{a_{11}}x - y,$$

we get again B_{21}.

2.3.2.4. $a_{11} \neq 0, \quad a_{21} \neq 0$. Now replacing

$$\overline{x} = x,$$
$$\overline{y} = \frac{1}{\sqrt{a_{11}^2 + a_{21}^2}}\left(-\frac{a_{21}}{a_{11}}x + a_{21}y - a_{11}z\right),$$
$$\overline{z} = \frac{1}{\sqrt{a_{11}^2 + a_{21}^2}}(-x + a_{11}y + a_{21}z),$$

we have B_{21}.

2.4. $a_{25} \neq 0$, $a_{26} = 0$. Choosing $a_{25} = 1$ we obtain that Y_1 and Y_2 define a group iff $a_{11} = 0$. By the change

$$\overline{x} = a_{24} + x, \quad \overline{y} = y, \quad \overline{z} = z$$

we get the subgroup

$$B_{25} = \{X_2, \alpha X_1 + \beta X_3 + X_5 | \alpha\beta \in \mathbb{R}\}.$$

2.5. $a_{26} \neq 0$. We put $a_{26} = 1$. Then the operators define a group if and only if $a_{11} \neq 0$, $a_{23} = a_{11}a_{25} - 1 = 0$. Making the substitution

$$\overline{x} = x, \quad \overline{y} = -a_{24} - \frac{1}{a_{11}}x + y, \quad \overline{z} = -\frac{a_{21}}{a_{11}} + z,$$

we find B_{22}.

3. $a_{13} \neq 0$, $a_{14} = a_{15} = a_{16} = 0$. We suppose $a_{13} = 1$, $a_{23} = 0$.

3.1. $a_{22} = a_{24} = a_{25} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and taking $a_{21} = 1$, $a_{11} = 0$ we obtain 1.2.

3.2. $a_{22} \neq 0$, $a_{24} = a_{25} = a_{26} = 0$. Now we choose $a_{22} = 1$, $a_{12} = 0$ and we get 2.2.

3.3. $a_{24} \neq 0$, $a_{25} = a_{26} = 0$. We put $a_{24} = 1$. The operators define a group iff $a_{21} = 0$. Replacing

$$\overline{x} = a_{12} + x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z), \quad \overline{z} = \frac{1}{\sqrt{1 + a_{22}^2}}(-y + a_{22}z),$$

we get B_{25}.

3.4. $a_{25} \neq 0$, $a_{26} = 0$. We take $a_{25} = 1$. Y_1 and Y_2 define a group if and only if $a_{11} = a_{21} = 0$. After the change

$$\overline{x} = a_{12} + x, \quad \overline{y} = y, \quad \overline{z} = z$$

we obtain the subgroup

$$B_{26} = \{X_3, \alpha X_2 + \beta X_4 + X_5 | \alpha, \beta \in \mathbb{R}\}.$$

3.5. $a_{26} \neq 0$. Choosing $a_{26} = 1$ we get that the operators do not determine a group.

4. $a_{14} \neq 0$, $a_{15} = a_{16} = 0$. We assume $a_{14} = 1$, $a_{24} = 0$.

4.1. $a_{22} = a_{23} = a_{25} = a_{26} = 0$. Consequently $a_{21} \neq 0$ and taking $a_{21} = 1$, $a_{11} = 0$ we have 1.3.

4.2. $a_{22} \neq 0$, $a_{23} = a_{25} = a_{26} = 0$. Now we put $a_{22} = 1$, $a_{12} = 0$ and obtain 2.3.

4.3. $a_{23} \neq 0$, $a_{25} = a_{26} = 0$. Choosing $a_{23} = 1$, $a_{13} = 0$ we get 3.3.

4.4. $a_{25} \neq 0$, $a_{26} = 0$. We take $a_{25} = 1$. The operators define a group iff $a_{11} = a_{13}\ a_{21} = 0$.

4.4.1. $a_{11} = a_{13} = 0$.

4.4.1.1. $a_{12} = a_{22} = 0$. By the change

$$\overline{x} = x, \quad \overline{y} = z, \quad \overline{z} = -y \tag{5}$$

we obtain B_{23} or B_{25} if $a_{23} = 0$ or $a_{23} \neq 0$, respectively.

4.4.1.2. $a_{12} \neq 0$, $a_{22} = 0$. In this case we make the change

$$\overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{12}^2}}(a_{12}y + z), \quad \overline{z} = \frac{1}{\sqrt{1 + a_{12}^2}}(-y + a_{12}z), \tag{6}$$

and we get B_{23} or B_{25} if $a_{23} = a_{12}$ or $a_{23} \neq a_{12}$, respectively.

4.4.1.3. $a_{12} = 0$, $a_{22} \neq 0$.

4.4.1.3.1. $a_{21} = a_{23} = 0$. Now we apply (5) and we find B_{24}.

4.4.1.3.2. $a_{21} \neq 0$, $a_{23} = 0$. Replacing

$$\overline{x} = x, \quad \overline{y} = z, \quad \overline{z} = \frac{a_{22}}{a_{21}}x - y, \tag{7}$$

we have B_{23}.

4.4.1.3.3. $a_{21} = 0,\ a_{23} \neq 0$. By the substitution

$$\overline{x} = \frac{a_{22}}{a_{23}} + x, \quad \overline{y} = z, \quad \overline{z} = -y,$$

we obtain B_{25} by $\alpha = 0$.

4.4.1.3.4. $a_{21} \neq 0 a_{23} \neq 0$. In this case we apply (7) and we get B_{25}.

4.4.1.4. $a_{12} \neq 0,\ a_{22} \neq 0$.

4.4.1.4.1. $a_{21} = a_{23} = 0$. We make the change

$$(8) \quad \overline{x} = -\frac{a_{22}}{a_{12}} + x, \quad \overline{y} = \frac{1}{\sqrt{1+a_{12}^2}}(a_{12}y + z),\ \overline{z} = \frac{1}{\sqrt{1+a_{12}^2}}(-y + a_{12}z)$$

and we find B_{25} by $\alpha = 0$.

4.4.1.4.2. $a_{21} \neq 0,\ a_{23} = 0$. Applying (8) we have B_{25}.

4.4.1.4.3. $a_{21} = 0,\ a_{23} \neq 0$. If $a_{23} \neq a_{12}$, then we make the substitution

$$\overline{x} = \frac{a_{22}}{a_{23} - a_{12}} + x, \quad \overline{y} = \frac{1}{\sqrt{1+a_{12}^2}}(a_{12}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1+a_{12}^2}}(-y + a_{12}z)$$

and we obtain B_{25} by $\alpha = 0$. If $a_{23} = a_{12}$, then we apply the change (6) and we get B_{24}.

4.4.1.4.4. $a_{21} \neq 0,\ a_{23} \neq 0$. Changing the variables in the form

$$\overline{x} = x,\ \overline{y} = \frac{1}{\sqrt{1+a_{12}^2}}(-\frac{a_{12}a_{22}}{a_{21}}x + a_{12}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1+a_{12}^2}}(\frac{a_{22}}{a_{21}}x - y + a_{12}z),$$

we find B_{23} or B_{25} if $a_{23} = a_{12}$ or $a_{23} \neq a_{12}$, respectively.

4.4.2. $a_{11} = a_{21} = 0$.

4.4.2.1. $a_{13} = 0$.

4.4.2.1.1. $a_{12} = a_{22} = 0$. In this case we have 4.4.1.1.

4.4.2.1.2. $a_{12} \neq 0,\ a_{22} = 0$. We obtain 4.4.1.2.

4.4.2.1.3. $a_{12} = 0,\ a_{22} \neq 0$. Now we get 4.4.1.3.

4.4.2.1.4. $a_{12} \neq 0,\ a_{22} \neq 0$. We have 4.4.1.4.

4.4.2.2. $a_{13} \neq 0$.

4.4.2.2.1. $a_{22} = a_{23} = 0$. Applying (5) we obtain B_{26}.

4.4.2.2.2. $a_{22} \neq 0$, $a_{23} = 0$.

4.4.2.2.2.1. $a_{12} = 0$. If $a_{13}a_{22} < 0$, then we have the subgroup

$$B_{27} = \{\alpha X_3 + X_4,\ \beta X_2 + X_5 | \alpha\beta < 0;\ \alpha, \beta \in \mathbb{R}\}.$$

and if $a_{13}a_{22} > 0$, then we use the change

$$\overline{x} = \sqrt{\frac{a_{22}}{a_{13}}} + x,$$
$$\overline{y} = \frac{1}{\sqrt{1 + a_{13}a_{22}}}(\sqrt{a_{13}a_{22}}y + z),$$
$$\overline{z} = \frac{1}{\sqrt{1 + a_{13}a_{22}}}(-y + \sqrt{a_{13}a_{22}}z)$$

and we get B_{26}.

4.4.2.2.2.2. $a_{12} \neq 0$.

i) $D = a_{12}^2 + 4a_{13}a_{22} \geq 0$. Using the substitution

$$\overline{x} = \lambda + x,\quad \overline{y} = \frac{1}{\sqrt{A}}(2a_{13}a_{22}y + (-a_{12} + \sqrt{D})z),$$
$$\overline{z} = \frac{1}{\sqrt{A}}((a_{12} - \sqrt{D})y + 2a_{13}a_{22}z),$$

where

$$\lambda = \frac{2a_{22}}{A}(a_{12} - a_{12}a_{13}a_{22} + (1 + a_{12}a_{13}a_{22})\sqrt{D}),$$
$$A = (-a_{12} + \sqrt{D})^2 + 4a_{13}^2a_{22}^2,$$

we have B_{26}.

ii) $D = a_{12}^2 + 4a_{13}a_{22} < 0$. Replacing

$$\overline{x} = \frac{a_{12}}{2a_{13}} + x,\quad \overline{y} = \frac{1}{\sqrt{A}}(a_{12}y + (1 + a_{13}a_{22} + \sqrt{D_1})z),$$
$$\overline{z} = \frac{1}{\sqrt{A}}(-(1 + a_{13}a_{22} + \sqrt{D_1})y + a_{12}z),$$

where

$$D_1 = a_{12}^2 + (1 + a_{13}a_{22})^2,\quad A = a_{12}^2 + (1 + a_{13}a_{22} + \sqrt{D_1})^2,$$

we get B_{27}.

4.4.2.2.3. $a_{22} = 0$, $a_{23} \neq 0$. If $a_{12} = 0$, then we use the substitution

$$\overline{x} = -\frac{a_{23}}{a_{13}} + x, \quad \overline{y} = z, \quad \overline{z} = -y$$

and we find B_{26}. If $a_{12} \neq 0$, then replacing

$$\overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{23}^2}}(a_{23}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1 + a_{23}^2}}(-y + a_{23}z)$$

we have again B_{26}.

4.4.2.2.4. $a_{22} \neq 0$, $a_{23} \neq 0$.

4.4.2.2.4.1. $a_{12} = 0$.

i) $D = a_{23}^2 + 4a_{13}a_{22} \geq 0$. By the change

$$\overline{x} = \lambda + x, \quad \overline{y} = \frac{1}{\sqrt{A}}(2a_{13}a_{22}y + (-a_{23} + \sqrt{D})z),$$

$$\overline{z} = \frac{1}{\sqrt{A}}(-(-a_{23} + \sqrt{D})y + 2a_{13}a_{22}z),$$

where

$$\lambda = \frac{-a_{23} + \sqrt{D}}{a_{13}A}(a_{23}(a_{23} - \sqrt{D}) + 2a_{13}a_{22}(1 + a_{13}a_{22})),$$

$$A = 4a_{13}^2a_{22}^2 + (-a_{23} + \sqrt{D})^2,$$

we get B_{26}.

ii) $D = a_{23}^2 + 4a_{13}a_{22} < 0$. Now making the change

$$\overline{x} = \lambda + x, \quad \overline{y} = \frac{1}{\sqrt{A}}(a_{23}y + (1 + a_{13}a_{22} + \sqrt{D_1})z),$$

$$\overline{z} = \frac{1}{\sqrt{A}}(-(1 + a_{13}a_{22} + \sqrt{D_1})y + a_{23}z),$$

where

$$\lambda = -\frac{a_{23}\sqrt{D_1}}{a_{13}A}(1 + a_{13}a_{22} + \sqrt{D_1}),$$

$$D_1 = a_{23}^2 + (1 + a_{13}a_{22})^2,$$

$$A = a_{23}^2 + (1 + a_{13}a_{22} + \sqrt{D_1})^2,$$

we obtain B_{27}.

4.4.2.2.4.2. $a_{12} \neq 0$.

i) $D = (a_{12} - a_{23})^2 + 4a_{13}a_{22} \geq 0$. By the change

$$\overline{x} = \lambda + x, \quad \overline{y} = \frac{1}{\sqrt{A}}(2(a_{12}a_{23} - a_{13}a_{22})y + (a_{12} + a_{23} + \sqrt{D})z),$$

$$\overline{z} = \frac{1}{\sqrt{A}}(-(a_{12} + a_{23} + \sqrt{D})y + 2(a_{12}a_{23} - a_{13}a_{22})z),$$

where

$$\lambda = \frac{1}{a_{13}A}(-a_{23}(a_{12} + a_{23} + \sqrt{D})^2 + 2(1 - a_{12}a_{23} + a_{13}a_{22})(a_{12} + a_{23} + \sqrt{D})\times$$

$$\times(a_{12}a_{23} - a_{13}a_{22}) + 4a_{12}(a_{12}a_{23} - a_{13}a_{22})^2),$$

$$A = 4(a_{12}a_{23} - a_{13}a_{22})^2 + (a_{12} + a_{23} + \sqrt{D})^2,$$

we find B_{26}.

ii) $D = (a_{12} - a_{23})^2 + 4a_{13}a_{22} < 0$. Now applying the substitution

$$\overline{x} = \lambda + x, \quad \overline{y} = \frac{1}{\sqrt{A}}((a_{12} + a_{23})y + (1 - a_{12}a_{23} + a_{13}a_{22} + \sqrt{D_1})z),$$

$$\overline{z} = \frac{1}{\sqrt{A}}(-(1 - a_{12}a_{23} + a_{13}a_{22} + \sqrt{D_1})y + (a_{12} + a_{23})z),$$

where

$$\lambda = \frac{1}{a_{13}A}(-a_{23}(1 - a_{12}a_{23} + a_{13}a_{22} + \sqrt{D_1})^2 + a_{12}(a_{12} + a_{23})^2 +$$

$$+(1 - a_{12}a_{23} + a_{13}a_{22})(1 - a_{12}a_{23} + a_{13}a_{22} + \sqrt{D_1})(a_{12} + a_{23})),$$

$$A = (a_{12} + a_{23})^2 + (1 - a_{12}a_{23} + a_{13}a_{22}\sqrt{D_1})^2,$$

$$D_1 = (a_{12} + a_{23})^2 + (1 - a_{12}a_{23} + a_{13}a_{22})^2,$$

we obtain B_{27}.

4.5. $a_{26} \neq 0$. We assume $a_{26} = 1$. The operators define a group if and only if

$$a_{11} \neq 0, \quad a_{13} = 0, \quad a_{23} = -\frac{1}{a_{11}}, \quad a_{25} = \frac{a_{12}}{a_{11}}.$$

Replasing

$$\overline{x} = x, \quad \overline{y} = \frac{a_{21}}{a_{11}} - \frac{a_{12}}{a_{11}}x + y,$$

$$\overline{z} = a_{22} - \frac{a_{12}a_{21}}{a_{11}} - \frac{1}{a_{11}}x + z,$$

we obtain B_{22}.

5. $a_{15} \neq 0$, $a_{16} = 0$. We suppose $a_{15} = 1$, $a_{25} = 0$.

5.1. $a_{22} = a_{23} = a_{24} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and choosing $a_{21} = 1$, $a_{11} = 0$ we find 1.4.

5.2. $a_{22} \neq 0$, $a_{23} = a_{24} = a_{26} = 0$. We put $a_{22} = 1$, $a_{12} = 0$ and we get 2.4.

5.3. $a_{23} \neq 0$, $a_{24} = a_{26} = 0$. Taking $a_{23} = 1$, $a_{13} = 0$ we have 3.4.

5.4. $a_{24} \neq 0$, $a_{26} = 0$. Now we choose $a_{24} = 1$, $a_{14} = 0$ and we obtain 4.4.

5.5. $a_{26} \neq 0$. We put $a_{26} = 1$. In this case the operators do not define a group.

6. $a_{16} \neq 0$. We assume $a_{16} = 1$, $a_{26} = 0$.

6.1. $a_{22} = a_{23} = a_{24} = a_{25} = 0$. Then $a_{21} \neq 0$ and putting $a_{21} = 1$, $a_{11} = 0$ we find 1.5.

6.2. $a_{22} \neq 0$, $a_{23} = a_{24} = a_{25} = 0$. We take $a_{22} = 1$, $a_{12} = 0$ and we get 2.5.

6.3. $a_{23} \neq 0$, $a_{24} = a_{25} = 0$. Choosing $a_{23} = 1$, $a_{13} = 0$ we obtain 3.5.

6.4. $a_{24} \neq 0$, $a_{25} = 0$. Now we put $a_{24} = 1$, $a_{14} = 0$ and we have 4.5.

6.5. $a_{25} \neq 0$. In this case we suppose $a_{25} = 1$, $a_{15} = 0$ and we find 5.5.
Thus the following theorem is stated:

Theorem 1. *The two-parametric subgroups of B_6 can be reduced to one of the subgroups:*

$$B_{21} = \{X_1, X_2\},$$

$$B_{22} = \{X_1, X_6\},$$

$$B_{23} = \{X_2, \alpha X_1 + X_3 | \alpha \in \mathbb{R}\}.$$

$$B_{24} = \{X_2, \alpha X_3 + X_4 | \alpha \in \mathbb{R}\}.$$

$$B_{25} = \{X_2, \alpha X_1 + \beta X_3 + X_5 | \alpha\beta \in \mathbb{R}\}.$$

$$B_{26} = \{X_3, \alpha X_2 + \beta X_4 + X_5 | \alpha, \beta \in \mathbb{R}\}.$$

$$B_{27} = \{\alpha X_3 + X_4, \beta X_2 + X_5 | \alpha\beta < 0;\ \alpha, \beta \in \mathbb{R}\}.$$

3. Three-parametric subgroups of B_6

A three-parametric subgroup of B_6 can be define by three infinitesimal operators $Y_h, h = 1, 2, 3,$ in the form (1), which satisfy (2) for $i, j = 1, 2, 3,\ \ i \neq j$. Consider the possible cases.

1. $a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = 0$. Then $a_{11} \neq 0$ and we can assume that $a_{11} = 1,\ a_{21} = a_{31} = 0$.

1.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Therefore $a_{22} \neq 0$ and we can choose $a_{22} = 1,\ a_{32} = 0$. The operators $Y_1 = X_1,\ Y_2 = X_2$ and $Y_3 = a_{33}X_3 + a_{34}X_4 + a_{35}X_5 + a_{36}X_6$ define a group if and only if $a_{35} = 0,\ \ a_{36} = 0$ and $|a_{33}| + |a_{34}| \neq 0$. Thus we find the subgroup

$$B_{31} = \{X_1, X_2, \alpha X_3 + \beta X_4 | |\alpha| + |\beta| \neq 0;\ \alpha, \beta \in \mathbb{R}\}.$$

1.2. $a_{23} \neq 0,\ \ a_{24} = a_{25} = a_{26} = 0$. Now we can assume $a_{23} = 1,\ a_{33} = 0$ and applying the condition (2) for $i, j = 1, 2, 3,\ \ i \neq j$, we obtain $a_{34} = a_{35} = a_{36} = 0$. Then we choose $a_{32} = 1$ and making the change $\overline{x} = a_{22} + x,\ \overline{y} = y,\ \overline{z} = z$ we obtain B_{31} by $\beta = 0$.

1.3. $a_{24} \neq 0,\ \ a_{25} = a_{26} = 0$. We suppose $a_{24} = 1,\ a_{34} = 0$.

1.3.1. $a_{33} = a_{35} = a_{36} = 0$. Consequently $a_{32} \neq 0$ and putting $a_{32} = 1,\ a_{22} = 0$ we get B_{31} by $\beta \neq 0$.

1.3.2. $a_{33} \neq 0,\ \ a_{35} = a_{36} = 0$. We choose $a_{33} = 1,\ a_{23} = 0$ and in this case the operators do not define a group.

1.3.3. $a_{35} \neq 0,\ \ a_{36} = 0$. We assume $a_{35} = 1$. The operators define a group iff $a_{23} = 0,\ \ a_{22} = a_{33}$. Then we make the substitution

$$\overline{x} = \frac{a_{22}a_{32}}{1 + a_{22}^2} + x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1 + a_{22}^2}}(-y + a_{22}z)$$

and we obtain B_{31} by $\alpha \neq 0$.

1.3.4. $a_{36} \neq 0$. Taking $a_{36} = 1$ we have that the operators do not define a group.

1.4. $a_{25} \neq 0, \quad a_{26} = 0$. Now we suppose $a_{25} = 1, \quad a_{35} = 0$.

1.4.1. $a_{33} = a_{34} = a_{36} = 0$. Therefore $a_{32} \neq 0$ and we put $a_{32} = 1$, $a_{22} = 0$. The operators do not define a group.

1.4.2. $a_{33} \neq 0, \quad a_{34} = a_{36} = 0$. We assume $a_{33} = 1$, $a_{23} = 0$ and we find that the operators do not define a group.

1.4.3. $a_{34} \neq 0, \quad a_{36} = 0$. Taking $a_{34} = 1$, $a_{24} = 0$, we get 1.3.3.

1.4.4. $a_{36} \neq 0$. Putting $a_{36} = 1$ we obtain the operators do not define a group.

1.5. $a_{26} \neq 0$. We suppose $a_{26} = 1$, $a_{36} = 0$.

1.5.1. $a_{33} = a_{34} = a_{35} = 0$. Then $a_{32} \neq 0$ and we can choose $a_{32} = 1, \quad a_{22} = 0$. In this case the operators do not define a group.

1.5.2. $a_{33} \neq 0, \quad a_{34} = a_{35} = 0$. We take $a_{33} = 1$, $a_{23} = 0$ and from 1.2. it follows that the operators do not define a group.

1.5.3. $a_{34} \neq 0, \quad a_{35} = 0$. Choosing $a_{34} = 1$, $a_{24} = 0$, we have 1.3.4.

1.5.4. $a_{35} \neq 0$. We put $a_{35} = 1, \quad a_{25} = 0$ and we obtain 1.4.4.

2. $a_{12} \neq 0, \quad a_{13} = a_{14} = a_{15} = a_{16} = 0$. We suppose $a_{12} = 1, \quad a_{22} = a_{32} = 0$.

2.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and putting $a_{21} = 1, \quad a_{11} = a_{31} = 0$ we have 1.1.

2.2. $a_{23} \neq 0, \quad a_{24} = a_{25} = a_{26} = 0$. We assume $a_{23} = 1$, $a_{33} = 0$.

2.2.1. $a_{34} = a_{35} = a_{36} = 0$. Consequently $a_{31} \neq 0$ and taking $a_{31} = 1, \quad a_{11} = a_{21} = 0$ we get B_{31} by $\beta = 0$.

2.2.2. $a_{34} \neq 0, \quad a_{35} = a_{36} = 0$. Now we choose $a_{34} = 1$ and obtain that in this case the operators do not define a group iff $a_{11} = 0$.

2.2.2.1. $a_{31} = 0$. We have the subgroup

$$B_{32} = \{X_2, \alpha X_1 + X_3, X_4 | \alpha \in \mathbb{R}\}.$$

2.2.2.2. $a_{31} \neq 0$. We make the change

$$\overline{x} = x, \quad \overline{y} = y, \quad \overline{z} = -\frac{1}{a_{31}}x + z$$

and we get the subgroup B_{31} by $\alpha \neq 0$.

2.2.3. $a_{35} \neq 0,\ a_{36} = 0$. We suppose $a_{35} = 1$. The operators define a group if and only if $a_{11} = a_{21} = 0$. Applying a obvious change of the variables we find the subgroup

$$B_{33} = \{X_2, X_3, \alpha X_1 + X_5 | \alpha \in \mathbb{R}\}.$$

2.2.4. $a_{36} \neq 0$. Putting $a_{36} = 1$ we obtain that the operators do not define a group.

2.3. $a_{24} \neq 0,\ \ a_{25} = a_{26} = 0$. We assume $a_{24} = 1,\ a_{34} = 0$.

2.3.1. $a_{33} = a_{35} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1,\ \ a_{11} = a_{21} = 0$ we obtain 1.3.1.

2.3.2. $a_{33} \neq 0,\ \ a_{35} = a_{36} = 0$. Choosing $a_{33} = 1,\ \ a_{23} = 0$ we have 2.2.2.

2.3.3. $a_{35} \neq 0,\ \ a_{36} = 0$. We put $a_{35} = 1$. Now the operators define a group iff (i) $a_{11} = a_{21} = 0$ or (ii) $a_{23} = a_{11}a_{23} + a_{21} = 0$.

2.3.3.1. $a_{11} = a_{21} = 0$. For the coefficients a_{23} and a_{33} we have the following possibilities:

2.3.3.1.1. $a_{23} = a_{33} = 0$. Applying (5) we get B_{32}.

2.3.3.1.2. $a_{23} \neq 0,\ \ a_{33} = 0$. Now we obtain the subgroup

$$B_{34} = \{X_2, \alpha X_3 + X_4, \beta X_1 + X_5 | \alpha \neq 0; \alpha, \beta \in \mathbb{R}\}.$$

2.3.3.1.3. $a_{23} = 0,\ \ a_{33} \neq 0$. We make the substitution

$$\overline{x} = x,\ \ \overline{y} = \frac{1}{\sqrt{1 + a_{33}^2}}(a_{33}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1 + a_{33}^2}}(-y + a_{33}z)$$

and we find the subgroup B_{32}.

2.3.3.1.4. $a_{23} \neq 0,\ \ a_{33} \neq 0$. Changing the variables in the from

$$\overline{x} = -\frac{a_{33}}{a_{23}} + x,\ \ \overline{y} = y,\ \ \overline{z} = z$$

we get the subgroup B_{34}.

2.3.3.2. $a_{23} = a_{11}a_{33} + a_{21} = 0$.

2.3.3.2.1. $a_{11} = a_{33} = 0$. By the change (5) we get B_{32}.

2.3.3.2.2. $a_{11} \neq 0,\ a_{33} = 0$. We make the substitution

$$\overline{x} = x, \quad \overline{y} = z, \quad \overline{z} = \frac{1}{a_{11}}x - y$$

and we obtain B_{31} by $\alpha \neq 0$.

2.3.3.2.3. $a_{11} = 0,\ a_{33} \neq 0$. Now we have 2.3.3.1.3.

2.3.3.2.4. $a_{11} \neq 0,\ \ a_{33} \neq 0$. Replasing

$$\overline{x} = \frac{a_{31}}{a_{11}a_{33}} + x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{33}^2}}(-\frac{a_{33}}{a_{11}}x + a_{33}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1 + a_{33}^2}}(\frac{1}{a_{11}}x - y + a_{33}z),$$

we have B_{31} by $\alpha \neq 0$.

2.3.4. $a_{36} = 0$. Then we can choose $a_{36} = 1$. The operators define a group iff $a_{11} = a_{21} = a_{23} = 0$. We make the change

$$\overline{x} = x, \quad \overline{y} = -a_{31}a_{35} - a_{35}x + y,$$

$$\overline{z} = -a_{31}a_{35} + a_{33}x + z$$

and we obtain the subgroup

$$B_{35} = \{X_2, X_4, \alpha X_1 + X_6, |\alpha \in \mathbb{R}\}.$$

2.4. $a_{25} \neq 0,\ a_{26} = 0$. We assume $a_{25} = 1,\ a_{35} = 0$.

2.4.1. $a_{33} = a_{34} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1,\ \ a_{11} = a_{21} = 0$ we have 1.4.1.

2.4.2. $a_{33} \neq 0,\ \ a_{34} = a_{36} = 0$. We put $a_{33} = 1,\ a_{23} = 0$ and we get 2.2.3.

2.4.3. $a_{34} \neq 0,\ \ a_{36} = 0$. Choosing $a_{34} = 1,\ \ a_{24} = 0$ we obtain 2.3.3.

2.4.4. $a_{36} \neq 0$. Now we take $a_{36} = 1$ and we get that the operators do not define a group.

2.5. $a_{26} \neq 0$. We suppose $a_{26} = 1,\ a_{36} = 0$.

2.5.1. $a_{33} = a_{34} = a_{35} = 0$. Consequently $a_{31} \neq 0$ and putting $a_{31} = 1,\ \ a_{11} = a_{21} = 0$ we get 1.5.1.

2.5.2. $a_{33} \neq 0,\ \ a_{34} = a_{35} = 0$. We choose $a_{33} = 1,\ a_{23} = 0$ and we obtain 2.2.4.

2.5.3. $a_{34} \neq 0,\ \ a_{35} = 0$. Now we assume $a_{34} = 1,\ a_{24} = 0$ and we have 2.3.4.

2.5.4. $a_{35} \neq 0$. Taking $a_{35} = 1,\ \ a_{25} = 0$ we get 2.4.4.

3. $a_{13} \neq 0,\ \ a_{14} = a_{15} = a_{16} = 0$. We suppose $a_{13} = 1,\ a_{23} = a_{33} = 0$.

3.1. $a_{22} = a_{24} = a_{25} = a_{26} = 0$. Then $a_{21} \neq 0$ and putting $a_{21} = 1,\ \ a_{11} = a_{31} = 0$ we get 1.2.

3.2. $a_{22} \neq 0,\ \ a_{24} = a_{25} = a_{26} = 0$. We choose $a_{22} = 1,\ \ a_{12} = a_{32} = 0$ and we obtain 2.2.

3.3. $a_{24} \neq 0,\ \ a_{25} = a_{26} = 0$. We assume $a_{24} = 1,\ a_{34} = 0$.

3.3.1. $a_{32} = a_{35} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1,\ \ a_{11} = a_{21} = 0$ we have 1.3.2.

3.3.2. $a_{32} \neq 0,\ \ a_{35} = a_{36} = 0$. We put $a_{32} = 1,\ \ a_{12} = a_{22} = 0$ and we get 2.2.2.

3.3.3. $a_{35} \neq 0,\ \ a_{36} = 0$. Now we suppose $a_{35} = 1$. The operators define a droup iff $a_{21} = a_{11}a_{22} + a_{31} = 0$.

3.3.3.1. $a_{22} = 0$. Replacing $\overline{x} = a_{12} + x,\ \ \overline{y} = z,\ \ \overline{z} = -y$, we obtain B_{33} or B_{34} if $a_{32} = 0$ or $a_{32} \neq 0$, respectively.

3.3.3.2. $a_{22} \neq 0$.

3.3.3.2.1. $a_{11} = a_{12} = a_{32} = 0$. Now after the substitution (3) we get B_{33} by $\alpha = 0$.

3.3.3.2.2. $a_{11} = a_{32} = 0,\ \ a_{12} \neq 0$. Replasing

$$\overline{x} = \frac{a_{12}}{1 + a_{22}^2} + x,\quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1 + a_{22}^2}}(-y + a_{22}z)$$

we find B_{34} by $\beta = 0$.

3.3.3.2.3. $a_{11} \neq 0,\ \ a_{12} = a_{32} = 0$. Applying (3) we get B_{33}.

3.3.3.2.4. $a_{11} \neq 0,\ a_{12} \neq 0,\ a_{32} = 0$. In this case the change

$$\overline{x} = x,\quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{a_{12}}{a_{11}\sqrt{(1 + a_{22}^2)^3}}x + \frac{1}{\sqrt{1 + a_{22}^2}}(-y + a_{22}z)$$

brings to B_{34} .

3.3.3.2.5. $a_{11} = a_{12} = 0,\ \ a_{32} \neq 0$. Replasing

$$\overline{x} = -\frac{a_{22}a_{32}}{1 + a_{22}^2} + x,\quad \overline{y} = \frac{1}{\sqrt{1 + a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1+a_{22}^2}} x(-y + a_{22}z)$$

we obtain B_{34} by $\beta = 0$.

3.3.3.2.6. $a_{11} = 0, \quad a_{12} \neq 0, \quad a_{32} \neq 0$. We make the change

$$\overline{x} = \frac{a_{12} - a_{22}a_{32}}{1 + a_{22}^2} + x, \quad \overline{y} = \frac{1}{\sqrt{1+a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{1}{\sqrt{1+a_{22}^2}}(-y + a_{22}z)$$

and we get B_{33} or B_{35} if $a_{12}a_{22} + a_{32} = 0$ or $a_{12}a_{22} + a_{32} \neq 0$, respectively.

3.3.3.2.7. $a_{11} \neq 0, \; a_{12} = 0, \; a_{32} \neq 0$. By the substitution

$$\overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1+a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = -\frac{a_{22}a_{32}}{a_{11}\sqrt{(1+a_{22}^2)^3}} x + \frac{1}{\sqrt{1+a_{22}^2}}(-y + a_{22}z)$$

we obtain B_{34}.

3.3.3.2.8. $a_{11} \neq 0, \; a_{12} \neq 0, \; a_{32} \neq 0$. Replacing

$$\overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1+a_{22}^2}}(a_{22}y + z),$$

$$\overline{z} = \frac{a_{12} - a_{22}a_{32}}{a_{11}\sqrt{(1+a_{22}^2)^3}} x + \frac{1}{\sqrt{1+a_{22}^2}}(-y + a_{22}z)$$

we find B_{33} or B_{34} if $a_{12}a_{22} + a_{32} = 0$ or $a_{12}a_{22} + a_{32} \neq 0$, respectively.

3.3.4. $a_{36} \neq 0$. Choosing $a_{36} = 1$ we get that the operators do not define a group.

3.4. $a_{25} \neq 0, \; a_{26} = 0$. We assume $a_{25} = 1, \; a_{35} = 0$.

3.4.1. $a_{32} = a_{34} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and putting $a_{31} = 1, \; a_{11} = a_{21} = 0$ we obtain 1.4.2.

3.4.2. $a_{32} \neq 0, \; a_{34} = a_{36} = 0$. We take $a_{32} = 1, \; a_{12} = a_{22} = 0$ and we have 2.2.3.

3.4.3. $a_{34} \neq 0, \; a_{36} = 0$. We choose $a_{34} = 1, \; a_{24} = 0$ and we get 3.3.3.

3.4.4. $a_{36} \neq 0$. Now we suppose $a_{36} = 1$. The operators define a group iff $a_{11} = a_{21} = a_{22} = a_{31} = a_{12} - a_{24} = 0$. Making the change

$$\overline{x} = a_{12} + x, \quad \overline{y} = -a_{34} + y, \quad \overline{z} = a_{32} + z,$$

we find the subgroup

$$B_{36} = \{X_3, X_5, X_6\}.$$

3.5. $a_{26} \neq 0$. We assume $a_{26} = 1$ and $a_{36} = 0$.

3.5.1. $a_{32} = a_{34} = a_{35} = 0$. Then $a_{31} \neq 0$ and putting $a_{31} = 1$, $a_{11} = a_{21} = 0$ we obtain 1.5.2.

3.5.2. $a_{32} \neq 0$, $a_{34} = a_{35} = 0$. We take $a_{32} = 1$, $a_{12} = a_{22} = 0$ and we have 2.2.4.

3.5.3. $a_{34} \neq 0$, $a_{35} = 0$. Choosing $a_{34} = 1$, $a_{24} = 0$ we get 3.3.4.

3.5.4. $a_{35} \neq 0$. We put $a_{35} = 1$, $a_{25} = 0$ and we obtain 3.4.4.

4. $a_{14} \neq 0$, $a_{15} = a_{16} = 0$. We suppose $a_{14} = 1$, $a_{24} = a_{34} = 0$.

4.1. $a_{22} = a_{23} = a_{25} = a_{26} = 0$. Conseguently $a_{21} \neq 0$ and putting $a_{21} = 1$, $a_{11} = a_{31} = 0$ we get 1.3.

4.2. $a_{22} \neq 0$, $a_{23} = a_{25} = a_{26} = 0$. We take $a_{22} = 1$, $a_{12} = a_{32} = 0$ and we get 2.3.

4.3. $a_{23} \neq 0$, $a_{25} = a_{26} = 0$. Choosing $a_{23} = 1$, $a_{13} = a_{33} = 0$ we obtain 3.3.

4.4. $a_{25} \neq 0$, $a_{26} = 0$. Now we assume $a_{25} = 1$, $a_{35} = 0$.

4.4.1. $a_{32} = a_{33} = a_{36} = 0$. Then $a_{31} \neq 0$ and taking $a_{31} = 1$, $a_{11} = a_{21} = 0$ we have 1.3.3.

4.4.2. $a_{32} \neq 0$, $a_{33} = a_{36} = 0$. We put $a_{32} = 1$, $a_{12} = a_{22} = 0$ and we get 2.3.3.

4.4.3. $a_{33} \neq 0$, $a_{36} = 0$. In this case we choose $a_{33} = 1$, $a_{13} = a_{23} = 0$ and we obtain 3.3.3.

4.4.4. $a_{36} \neq 0$. We take $a_{36} = 1$. The operators define a droup iff

$$a_{11} = a_{21} = a_{31} = 0, \quad a_{23} = -a_{12}, \quad a_{22} = -\frac{1 + a_{12}^2}{a_{13}}, \quad a_{13} \neq 0.$$

We make the substitution

$$\overline{x} = \frac{a_{12}}{a_{13}} + x, \quad \overline{y} = y, \quad \overline{z} = a_{32} + a_{33}x + z$$

and we get the subgroup

$$B_{37} = \{\alpha X_3 + X_4, -X_2 + \alpha X_5, X_6 | \alpha \neq 0; \alpha \in \mathbb{R}\}.$$

4.5. $a_{26} \neq 0$. We suppose $a_{26} = 1$, $a_{36} = 0$.

4.5.1. $a_{32} = a_{33} = a_{35} = 0$. Therefore $a_{31} \neq 0$ and putting $a_{31} = 1,\ a_{11} = a_{21} = 0$ we obtain 1.3.4.

4.5.2. $a_{32} \neq 0,\ \ a_{33} = a_{35} = 0$. Taking $a_{32} = 1,\ \ a_{12} = a_{22} = 0$ we have 2.3.4.

4.5.3. $a_{33} \neq 0,\ a_{35} = 0$. Choosing $a_{33} = 1,\ a_{13} = a_{23} = 0$ we get 3.3.4.

4.5.4. $a_{35} \neq 0$. We put $a_{35} = 1,\ a_{25} = 0$ and we obtain 4.4.4.

5. $a_{15} \neq 0,\ a_{16} = 0$. We suppose $a_{15} = 1,\ a_{25} = a_{35} = 0$.

5.1. $a_{22} = a_{23} = a_{24} = a_{26} = 0$. Conseguently $a_{21} \neq 0$ and choosing $a_{21} = 1,\ a_{11} = a_{31} = 0$ we have 1.4.

5.2. $a_{22} \neq 0,\ a_{23} = a_{24} = a_{26} = 0$. Putting $a_{22} = 1,\ a_{12} = a_{32} = 0$ we get 2.4.

5.3. $a_{23} \neq 0,\ a_{24} = a_{26} = 0$. Now we choose $a_{23} = 1,\ a_{13} = a_{33} = 0$ and we have 3.4.

5.4. $a_{24} \neq 0,\ a_{26} = 0$. We take $a_{24} = 1,\ a_{14} = a_{34} = 0$ and we get 4.4.

5.5. $a_{26} \neq 0$. We assume $a_{26} = 1, a_{36} = 0$.

5.5.1. $a_{32} = a_{33} = a_{34} = 0$. Then $a_{31} \neq 0$ and taking $a_{31} = 1,\ a_{11} = a_{21} = 0$ we obtain 1.4.4.

5.5.2. $a_{32} \neq 0,\ a_{33} = a_{34} = 0$. We put $a_{32} = 1,\ a_{12} = a_{22} = 0$ and we find 2.4.4.

5.5.3. $a_{33} \neq 0,\ a_{34} = 0$. Choosing $a_{33} = 1,\ a_{13} = a_{23} = 0$ we have 3.4.4.

5.5.4. $a_{34} \neq 0$. Now we take $a_{34} = 1,\ a_{14} = a_{24} = 0$ and we get 4.4.4.

6. $a_{16} \neq 0$. We suppose $a_{16} = 1,\ a_{26} = a_{36} = 0$.

6.1. $a_{22} = a_{23} = a_{24} = a_{25} = 0$. Therefore $a_{21} \neq 0$ and putting $a_{21} = 1,\ a_{11} = a_{31} = 0$ we obtain 1.5.

6.2. $a_{22} \neq 0,\ a_{23} = a_{24} = a_{25} = 0$. We choose $a_{22} = 1,\ a_{12} = a_{32} = 0$ and we have 2.5.

6.3. $a_{23} \neq 0,\ a_{24} = a_{25} = 0$. Taking $a_{23} = 1,\ a_{13} = a_{33} = 0$ we get 3.5.

6.4. $a_{24} \neq 0,\ a_{25} = 0$. Now we put $a_{24} = 1,\ a_{14} = a_{34} = 0$ and we have 4.5.

6.5. $a_{25} \neq 0$. We assume $a_{25} = 1,\ a_{15} = a_{35} = 0$ and we obtain 5.5.

Now we may summarize the foregoing rezults in the following

Theorem 2. *The three-parametric subgroups of B_6 can be reduced to one of the subgroups*

$$B_{31} = \{X_1, X_2, \alpha X_3 + \beta X_4 || \alpha| + |\beta| \neq 0; \alpha, \beta \in \mathbb{R}\}.$$

$$B_{32} = \{X_2, \alpha X_1 + X_3, X_4 | \alpha \in \mathbb{R}\}.$$

$$B_{33} = \{X_2, X_3, \alpha X_1 + X_5 | \alpha \in \mathbb{R}\}.$$

$$B_{34} = \{X_2, \alpha X_3 + X_4, \beta X_1 + X_5 | \alpha \neq 0; \alpha, \beta \in \mathbb{R}\}.$$

$$B_{35} = \{X_2, X_4, \alpha X_1 + X_6, | \alpha \in \mathbb{R}\}.$$

$$B_{36} = \{X_3, X_5, X_6\}.$$

$$B_{37} = \{\alpha X_3 + X_4, -X_2 + \alpha X_5, X_6 | \alpha \neq 0; \alpha \in \mathbb{R}\}.$$

Remark. The one-parametric subgroups of B_6 have been found by O. Röschel in [3].

References

1. A. V. Borisov. On the subgroups of the isometry group in the three-dimensional Galilean spase. *C. R. Acad.Bulg. Sci.* **44**, 1991, 31–32.
2. G. Kowalewski. Einführung in die theorie der kontinuierlichen gruppen. Leipzig, 1931.
3. O. Röschel. Die geometrie des Galileischen raumes. *Ber. Math. Statist. Sekt. Forsch. Graz*, **256**, 1985, 1–120.

Institute of Mathematics
Bulgarian Academy of Sciences
P.O.B. 373
1090 Sofia
BULGARIA

Received 15.06.1993