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Subgroups of the Isometry Group in a Galilean Space II: The Cases of Four and Five-Parametric Subgroups [1]

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In this paper the four and five-parametric subgroups of the isometry group in three-dimensional Galilean space are determined.

1. Introduction

With respect to nonhomogeneous coordinates an isometry of the six-parametric isometry group B_6 in the three-dimensional Galilean space G_3 has the form

$$x = a + x,$$

$$y = b + cx + cos\varphi.y + \sin\varphi.z,$$

$$z = d + ex - \sin\varphi.y + cos\varphi.z,$$

where a, b, c, d, e and φ are real numbers [4]. The infinitesimal operators of B_6 are

$$X_1 = \frac{\partial}{\partial x}, \quad X_2 = \frac{\partial}{\partial y}, \quad X_3 = \frac{\partial}{\partial y},$$

$$X_4 = \frac{\partial}{\partial z}, \quad X_5 = x\frac{\partial}{\partial z}, \quad X_6 = z\frac{\partial}{\partial y} - y\frac{\partial}{\partial z}$$

and satisfy the system

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$$[X_1, X_2] = 0, \qquad [X_1, X_3] = x_2, \qquad [X_1, X_4] = 0,$$

$$[X_1, X_5] = x_4, \qquad [X_1, X_6] = 0$$

$$[X_2, X_3] = 0, \qquad [X_2, X_4] = 0, \qquad [X_2, X_5] = 0,$$

$$[X_2, X_6] = -X_4, \qquad [X_3, X_4] = 0$$

$$[X_3, X_5] = 0, \qquad [X_3, X_6] = -X_5, \qquad [X_4, X_5] = 0,$$

$$[X_4, X_6] = X_2, \qquad [X_5, X_6] = X_3$$

where $[.,.]$ is the bracket of Poisson.

For the necessities of some applications the natural problem which arises is to classify the subgroups of B_6. That is the aim of this paper and we give the four and five-parametric subgroups of B_6 which are different up to a Galilean isometry. The results have been announced without proofs [1], which are given here.

We note that the one-parametric subgroups of B_6 have been found by 0. Röschel in [4] and the two and three-parametric subgroups - by the author in [2].

2. Four-parametric subgroups of B_6.

A four-parametric subgroup of B_6 can be defined by four infinitesimal operators (see [3], p. 163)

$$Y_h = \sum_{k=1}^{6} a_{hk} x_k, \quad h = 1, .., 4, \tag{1}$$

satisfying the conditions

$$[Y_i, Y_j] = \sum_{k=1}^{4} c_{ij}^k Y_k, \quad i, j = 1, .., 4; i \neq j \tag{2}$$

where a_{hk} and c_{ij}^k are real numbers. Now we shall consider the possible cases.

1. $a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = 0$. Then $a_{11} \neq 0$ and by a suitable change of the variables we can obtain $a_{11} = 1$, $a_{21} = a_{31} = a_{41} = 0$.

1.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Consequently $a_{22} \neq 0$ and we suppose $a_{22} = 1, \quad a_{32} = a_{42} = 0$.

1.1.1. $a_{34} = a_{35} = a_{36} = 0$. Now $a_{33} \neq 0$ and we choose $a_{33} = 1$, $a_{43} = 0$. The operators $Y_1 = X_1, \quad Y_2 = X_2, \quad Y_3 = X_3$, $Y_4 = a_{44}X_4 + a_{45}X_5 + a_{46}X_6$ define a group if and only if $a_{45} = a_{46} = 0$. Then $a_{44} \neq 0$ and taking $a_{44} = 1$ we get the subgroup

$$B_{41} = \{X_1, X_2, X_3, X_4\}$$

1.1.2. $a_{34} \neq 0, \quad a_{35} = a_{36} = 0$. We assume $a_{34} = 1, \quad a_{44} = 0$.

1.1.2.1. $a_{45} = a_{46} = 0$. Therefore $a_{43} \neq 0$ and choosing $a_{43} = 1, \quad a_{33} = 0$ we find B_{41}.

1.1.2.2. $a_{45} \neq 0, \quad a_{46} = 0$. We take $a_{45} = 1$ and from the condition $Y_h, \quad h = 1, 2, 3, 4$, to determine a group we get $a_{33} = 0$. Changing the variables in the form

$$(3) \qquad \overline{x} = x, \quad \overline{y} = \frac{1}{\sqrt{1 + a_{43}^2}}(a_{43}y + z), \overline{z} = \frac{1}{\sqrt{1 + a_{43}^2}}(-y + a_{43}z)$$

we find again B_{41}.

1.1.2.3. $a_{46} \neq 0$. We choose $a_{46} = 1$ and applying the condition the operators to define a group we obtain $a_{33} = 0$. By the substitution

$$\overline{x} = x, \quad \overline{y} = -a_{45} + y, \quad \overline{z} = a_{43} + z$$

we get the subgroup

$$B_{42} = \{X_1, X_2, X_4, X_6\}$$

1.1.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = 0$.

1.1.3.1. $a_{44} = a_{46} = 0$. Therefore $a_{43} \neq 0$ and we take $a_{43} = 1, a_{33} = 0$. In this case the operators do not define a group.

1.1.3.2. $a_{44} \neq 0, a_{46} = 0$. Choosing $a_{44} = 1, a_{34} = 0$ we have 1.1.2.2.

1.1.3.3. $a_{46} \neq 0$. We take $a_{46} = 1$. Now the operators do not determine a group.

1.1.4. $a_{36} \neq 0$. Let to put $a_{36} = 1, a_{46} = 0$.

1.1.4.1. $a_{44} = a_{45} = 0$. Consequently $a_{43} \neq 0$ and choosing $a_{43} = 1, a_{33} = 0$ we obtain that the operators do not define a group.

1.1.4.2. $a_{44} \neq 0, a_{45} = 0$. We assume $a_{44} = 1, a_{34} = 0$ and we get 1.1.2.3.

1.1.4.3. $a_{45} \neq 0$. Putting $a_{45} = 1, a_{35} = 0$ we obtain 1.1.3.3.

1.2. $a_{23} \neq 0, a_{24} = a_{25} = a_{26} = 0$. Now we suppose $a_{23} = 1$, $a_{33} = a_{43} = 0$.

1.2.1. $a_{34} = a_{35} = a_{36} = 0$. Then $a_{32} \neq 0$ and taking $a_{32} = 1, a_{22} = a_{42} = 0$ we have 1.1.1.

1.2.2. $a_{34} \neq 0, a_{35} = a_{36} = 0$. We assume $a_{34} = 1, a_{44} = 0$.

1.2.2.1. $a_{45} = a_{46} = 0$. Therefore $a_{42} \neq 0$ and putting $a_{42} = 1, a_{22} = a_{32} = 0$ we get B_{41}.

1.2.2.2. $a_{45} \neq 0, a_{46} = 0$. Choosing $a_{45} = 1$ we find that the operators do not define a group.

1.2.2.3. $a_{46} \neq 0$. Now we assume $a_{46} = 1$. The operators do not determine a group.

1.2.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = 0$.

1.2.3.1. $a_{44} = a_{46} = 0$. Then $a_{42} \neq 0$ and taking $a_{42} = 1$, $a_{22} = a_{32} = 0$ we obtain 1.1.3.1.

1.2.3.2. $a_{44} \neq 0, a_{46} = 0$. Putting $a_{44} = 1, a_{34} = 0$ we get 1.2.2.2.

1.2.3.3. $a_{46} \neq 0$. We choose $a_{46} = 1$ and we find that the operators do not defined a group.

1.2.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = 0$.

1.2.4.1. $a_{44} = a_{45} = 0$. Consequently $a_{42} \neq 0$ and taking $a_{42} = 1, a_{22} = a_{32} = 0$ we have 1.1.4.1.

1.2.4.2. $a_{44} \neq 0, a_{45} = 0$. We put $a_{44} = 1, a_{34} = 0$ and we obtain 1.2.2.3.

1.2.4.3. $a_{45} \neq 0$. Choosing $a_{45} = 1, a_{35} = 0$ we get 1.2.3.3.

1.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. Now we suppose $a_{24} = 1$, $a_{34} = a_{44} = 0$.

1.3.1. $a_{33} = a_{35} = a_{36} = 0$. Therefore $a_{32} \neq 0$ and putting $a_{32} = 1, a_{22} = a_{42} = 0$ we get 1.1.2.

1.3.2. $a_{33} \neq 0, a_{35} = a_{36} = 0$. We take $a_{33} = 1$, $a_{23} = a_{43} = 0$ and we have 1.2.2.

1.3.3. $a_{35} \neq 0, a_{36} = 0$. We assume $a_{35} = 1, a_{45} = 0$.

1.3.3.1. $a_{43} = a_{46} = 0$. Then $a_{42} \neq 0$ and choosing $a_{42} = 1$, $a_{22} = a_{32} = 0$ we obtain 1.1.2.2.

1.3.3.2. $a_{43} \neq 0, a_{46} = 0$. We take $a_{43} = 1, a_{23} = a_{33} = 0$ and we get 1.2.3.2.

1.3.3.3. $a_{46} \neq 0$. In this case we can assume $a_{46} = 1$. The verification indicates that the operators do not define a group.

1.3.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = 0$.

1.3.4.1. $a_{43} = a_{45} = 0$. Consequently $a_{42} \neq 0$ and replacing $a_{42} = 1, a_{22} = a_{32} = 0$ we have 1.1.2.3.

1.3.4.2. $a_{43} \neq 0, a_{45} = 0$. Taking $a_{43} = 1, a_{23} = a_{33} = 0$ we obtain 1.2.2.3.

1.3.4.3. $a_{45} \neq 0$. We put $a_{45} = 1, a_{35} = 0$ and we get 1.3.3.3.

1.4. $a_{25} \neq 0, a_{26} = 0$. We suppose $a_{25} = 1, a_{35} = a_{45} = 0$.

1.4.1. $a_{33} = a_{34} = a_{36} = 0$. Then $a_{32} \neq 0$ and choosing $a_{32} = 1, a_{22} = a_{24} = 0$ we have 1.1.3.

1.4.2. $a_{33} \neq 0, a_{34} = a_{36} = 0$. We take $a_{33} = 1$, $a_{23} = a_{43} = 0$ and we obtain 1.2.3.

1.4.3. $a_{34} \neq 0, a_{36} = 0$. Putting $a_{34} = 1, a_{24} = a_{44} = 0$ we get 1.3.3.

1.4.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = 0$.

1.4.4.1. $a_{43} = a_{44} = 0$. Therefore $a_{42} \neq 0$ and choosing $a_{42} = 1, a_{22} = a_{32} = 0$ we have 1.1.3.3.

1.4.4.2. $a_{43} \neq 0, a_{44} = 0$. Taking $a_{43} = 1, a_{23} = a_{33} = 0$ we get 1.2.3.3.

1.4.4.3. $a_{44} \neq 0$. Now we put $a_{44} = 1$, $a_{24} = a_{34} = 0$ and we obtain 1.3.3.3.

1.5. $a_{26} \neq 0$. We suppose $a_{26} = 1, a_{36} = a_{46} = 0$.

1.5.1. $a_{33} = a_{34} = a_{35} = 0$. Then $a_{32} \neq 0$ and assuming $a_{32} = 1, a_{22} = a_{42} = 0$ we obtain 1.1.4.

1.5.2. $a_{33} \neq 0, a_{34} = a_{35} = 0$. We take $a_{33} = 1$, $a_{23} = a_{43} = 0$ and we obtain 1.2.4.

1.5.3. $a_{34} \neq 0, a_{35} = 0$. Putting $a_{34} = 1, a_{24} = a_{44} = 0$ we get 1.3.4.

1.5.4. $a_{35} \neq 0$. Now we choose $a_{35} = 1, a_{25} = a_{45} = 0$ and we have 1.4.4.

2. $a_{12} \neq 0, a_{13} = a_{14} = a_{15} = a_{16} = 0$. In this case we can suppose $a_{12} = 1, a_{22} = a_{32} = a_{42} = 0$.

2.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and taking $a_{21} = 1, a_{11} = a_{31} = a_{41} = 0$ we obtain 1.1.

2.2. $a_{23} \neq 0, a_{24} = a_{25} = a_{26} = 0$. We choose $a_{23} = 1$, $a_{33} = a_{43} = 0$.

2.2.1. $a_{34} = a_{35} = a_{36} = 0$. Then $a_{31} \neq 0$ and replacing $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we get 1.1.1.

2.2.2. $a_{34} \neq 0, a_{35} = a_{36} = 0$. Now we assume $a_{34} = 1, a_{44} = 0$.

2.2.2.1. $a_{45} = a_{46} = 0$. Consequently $a_{41} \neq 0$ and taking $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we obtain B_{41}.

2.2.2.2. $a_{45} \neq 0, a_{46} = 0$. We suppose $a_{45} = 1$. Then the operators define a group iff $a_{11} = a_{31} = 0$.

2.2.2.2.1. $a_{21} = a_{41} = 0$. In this case the operators determine the subgroup

$$B'_{43} = \{X_2, X_3, X_4, X_5\}$$

2.2.2.2.2. $|a_{21}| + |a_{41}| \neq 0$ If we make the change

$$\overline{x} = x, \qquad \overline{y} = \frac{1}{\sqrt{a_{21}^2 + a_{41}^2}}(a_{41}y - a_{21}z),$$

$$\overline{z} = \frac{1}{\sqrt{a_{21}^2 + a_{41}^2}}(a_{21}y + a_{41}z)$$

then we get the subgroup

$$B_{23}'' = \{X_2, X_3, X_4, \alpha X_1 + X_5 | \alpha \neq 0; \alpha \in R\}.$$

Unifying the last two cases we have the subgroup

$$B_{43} = \{X_2, X_3, X_4, \alpha X_1 + X_5 | \alpha \in R\}.$$

2.2.2.3. $a_{46} \neq 0$. We replace $a_{46} = 1$. In this case the operators do not define a group.

2.2.3. $a_{35} \neq 0, a_{36} = 0$. Now we suppose $a_{35} = 1, a_{45} = 0$.

2.2.3.1. $a_{44} = a_{46} = 0$. Therefore $a_{41} \neq 0$ and putting $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we obtain 1.1.3.1.

2.2.3.2. $a_{44} \neq 0, a_{46} = 0$. We take $a_{44} = 1, a_{34} = 0$ and we have 2.2.2.2.

2.2.3.3. $a_{46} \neq 0$. Choosing $a_{46} = 1$ we get that the operators do not define a group.

2.2.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = 0$.

2.2.4.1. $a_{44} = a_{45} = 0$. Then $a_{41} \neq 0$ and replacing $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = 0$ we find 1.1.4.1.

2.2.4.2. $a_{44} \neq 0, a_{45} = 0$. We take $a_{44} = 1, a_{34} = 0$ and we have 2.2.2.3.

2.2.4.3. $a_{45} \neq 0$. Choosing $a_{45} = 1, a_{35} = 0$ we get 2.2.3.3.

2.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. Now we assume $a_{24} = 1$, $a_{34} = a_{44} = 0$.

2.3.1. $a_{33} = a_{35} = a_{36} = 0$. Consequently $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we obtain 1.1.2.

2.3.2. $a_{33} \neq 0, a_{35} = a_{36} = 0$.We take $a_{33} = 1, a_{23} = a_{43} = 0$ and we have 2.2.2.

2.3.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = 0$.

2.3.3.1. $a_{43} = a_{46} = 0$. Then $a_{41} \neq 0$ and choosing $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = 0$ we get 1.1.2.2.

2.3.3.2. $a_{43} \neq 0, a_{46} = 0$. Putting $a_{43} = 1, a_{23} = a_{33} = 0$ we obtain 2.2.2.2.

2.3.3.3. $a_{46} \neq 0$. Now we take $a_{46} = 1$ and in this case the operators do not define a group.

2.3.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = 0$.

2.3.4.1. $a_{43} = a_{45} = 0$. Therefore $a_{41} \neq 0$ and replacing $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we have 1.1.2.3.

2.3.4.2. $a_{43} \neq 0, a_{45} = 0$. We choose $a_{43} = 1, a_{23} = a_{33} = 0$ and we get 2.2.2.3.

2.3.4.3. $a_{45} \neq 0$. Putting $a_{45} = 1, a_{35} = 0$ we obtain 2.3.3.3.

2.4. $a_{25} \neq 0, a_{26} = 0$. We suppose $a_{25} = 1, a_{35} = a_{45} = 0$.

$a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we have 1.1.3.

2.4.2. $a_{33} \neq 0, a_{34} = a_{36} = 0$. Now we choose $a_{33} = 1$, $a_{23} = a_{43} = 0$ and we get 2.2.3.

2.4.3. $a_{34} \neq 0, a_{36} = 0$. Replacing $a_{34} = 1, a_{24} = a_{44} = 0$ we get 2.3.3.

2.4.4. $a_{36} \neq 0$. We put $a_{36} = 1, a_{46} = 0$.

2.4.4.1. $a_{43} = a_{44} = 0$. Consequently $a_{41} \neq 0$ and taking $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we have 1.1.3.3.

2.4.4.2. $a_{43} \neq 0, a_{44} = 0$. We put $a_{43} = 1, a_{23} = a_{33} = 0$ and we find 2.2.3.3.

2.4.4.3. $a_{44} \neq 0$. Choosing $a_{44} = 1, a_{24} = a_{34} = 0$ we get 2.3.3.3.

2.5. $a_{26} \neq 0$. We suppose $a_{26} = 1, a_{36} = a_{46} = 0$.

2.5.1. $a_{33} = a_{34} = a_{35} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1$, $a_{11} = a_{21} = a_{41} = 0$ we have 1.1.4.

2.5.2. $a_{33} \neq 0, a_{34} = a_{35} = 0$. We choose $a_{33} = 1$, $a_{23} = a_{43} = 0$ and we get 2.2.4.

2.5.3. $a_{34} \neq 0, a_{35} = 0$. Now we replace $a_{34} = 1$, $a_{24} = a_{44} = 0$ and we obtain 2.3.4.

2.5.4. $a_{35} \neq 0$. Taking $a_{35} = 1, a_{25} = a_{45} = 0$ we have 2.4.4.

3. $a_{13} \neq 0, a_{14} = a_{15} = a_{16} = 0$. We assume $a_{13} = 1$, $a_{23} = a_{33} = a_{43} = 0$.

3.1. $a_{22} = a_{24} = a_{25} = a_{26} = 0$. Then $a_{21} \neq 0$ and choosing $a_{21} = 1, a_{11} = a_{31} = a_{41} = 0$ we get 1.2.

3.2. $a_{22} \neq 0, a_{24} = a_{25} = a_{26} = 0$. Now we take $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = 0$ and we obtain 2.2.

3.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. We suppose $a_{24} = 1$, $a_{34} = a_{44} = 0$.

3.3.1. $a_{32} = a_{35} = a_{36} = 0$. Consequently $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we have 1.2.2.

3.3.2. $a_{32} \neq 0, a_{35} = a_{36} = 0$. We choose $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ and we get 2.2.2.

3.3.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = 0$.

3.3.3.1. $a_{42} = a_{46} = 0$. Then $a_{41} \neq 0$ and taking $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = 0$ we obtain 1.2.2.2.

3.3.3.2. $a_{42} \neq 0, a_{46} = 0$. Replacing $a_{42} = 1$, $a_{12} = a_{22} = a_{32} = 0$ we have 2.2.2.2.

3.3.3.3. $a_{46} \neq 0$. Now we put $a_{46} = 1$ and we get that the operators do not define a group.

3.3.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = 0$.

3.3.4.1. $a_{42} = a_{45} = 0$. Therefore $a_{41} \neq 0$ and putting $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we get 1.2.2.3.

3.3.4.2. $a_{42} \neq 0, a_{45} = 0$. We take $a_{42} = 1$ $a_{12} = a_{22} = a_{32} = 0$ and we obtain 2.2.2.3.

3.3.4.3. $a_{45} \neq 0$. Choosing $a_{45} = 1, a_{35} = 0$ we have 3.3.3.3.

3.4. $a_{25} \neq 0, a_{26} = 0$. We assume $a_{25} = 1, a_{35} = a_{45} = 0$.

3.4.1. $a_{32} = a_{34} = a_{36} = 0$. Consequently $a_{31} \neq 0$ and taking $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we get 1.2.3.

3.4.2. $a_{32} \neq 0, a_{34} = a_{36} = 0$. Now we put $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ and we obtain 2.2.3.

3.4.3. $a_{34} \neq 0, a_{36} = 0$.Choosing $a_{34} = 1, a_{24} = a_{44} = 0$ we have 3.3.3.

3.4.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = 0$.

3.4.4.1. $a_{42} = a_{44} = 0$. Then $a_{41} \neq 0$ and taking $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = 0$ we get 1.2.3.3.

3.4.4.2. $a_{42} \neq 0, a_{44} = 0$. Now we put $a_{42} = 1$, $a_{12} = a_{22} = a_{32} = 0$ and we obtain 2.2.3.3.

3.4.4.3. $a_{44} \neq 0$. Replacing $a_{44} = 1, a_{24} = a_{34} = 0$ we have 3.3.3.3.

3.5. $a_{26} \neq 0$. We assume $a_{26} = 1, a_{36} = a_{46} = 0$.

3.5.1. $a_{32} = a_{34} = a_{35} = 0$. Therefore $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we get 1.2.4.

3.5.2. $a_{32} \neq 0, a_{34} = a_{35} = 0$. We take $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ and we obtain 2.2.4.

3.5.3. $a_{34} \neq 0, a_{35} = 0$. Now we choose $a_{34} = 1, a_{24} = a_{44} = 0$ and we have 3.3.4.

3.5.4. $a_{35} \neq 0$. Putting $a_{35} = 1, a_{25} = a_{45} = 0$ we get 3.3.4.

4. $a_{14} \neq 0, a_{15} = a_{16} = 0$. We suppose $a_{14} = 1$, $a_{24} = a_{34} = a_{44} = 0$.

4.1. $a_{22} = a_{23} = a_{25} = a_{26} = 0$. Then $a_{21} \neq 0$ and replacing $a_{21} = 1, a_{11} = a_{31} = a_{41} = 0$ we get 1.3.

4.2. $a_{22} \neq 0, a_{23} = a_{25} = a_{26} = 0$. Now we choose $a_{22} = 1$, $a_{12} = a_{31} = a_{42} = 0$ and we obtain 2.3.

4.3. $a_{23} \neq 0, a_{25} = a_{26} = 0$. We put $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = 0$ and we have 3.3.

4.4. $a_{25} \neq 0, a_{26} = 0$. We assume $a_{25} = 1, a_{35} = a_{45} = 0$.

4.4.1. $a_{32} = a_{33} = a_{36} = 0$. Consequently $a_{31} \neq 0$ and taking $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ we get 1.3.3.

4.4.2. $a_{32} \neq 0, a_{33} = a_{36} = 0$. Choosing $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ we obtain 2.3.3.

4.4.3. $a_{33} \neq 0, a_{36} = 0$. Now we put $a_{33} = 1$, $a_{13} = a_{23} = a_{43} = 0$ and we have 3.3.3.

4.4.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = 0$.

4.4.4.1.$a_{42} = a_{43} = 0$. Therefore $a_{41} \neq 0$ and putting $a_{41} = 1, a_{11} = a_{21} = a_{31} = 0$ we get 1.3.3.3.

4.4.4.2. $a_{42} \neq 0, a_{43} = 0$.We choose $a_{42} = 1$, $a_{12} = a_{22} = a_{32} = 0$ and we obtain 2.3.3.3.

4.4.4.3. $a_{43} \neq 0$. Taking $a_{43} = 1, a_{13} = a_{23} = a_{33} = 0$ we have 3.3.3.3.

4.5. $a_{26} \neq 0$. We suppose $a_{26} = 1, a_{36} = a_{46} = 0$.

4.5.1. $a_{32} = a_{33} = a_{35} = 0$. Then $a_{31} \neq 0$ and we replace $a_{31} = 1, a_{11} = a_{21} = a_{41} = 0$ and we get 1.3.4.

4.5.2. $a_{32} \neq 0, a_{33} = a_{35} = 0$. Now we choose $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ and we obtain 2.3.4.

4.5.3. $a_{33} \neq 0, a_{35} = 0$. Taking $a_{33} = 1, a_{13} = a_{23} = a_{43} = 0$ we have 3.3.4.

4.5.4. $a_{35} \neq 0$. We put $a_{35} = 1, a_{25} = a_{45} = 0$ and we get 4.4.4.

5. $a_{15} \neq 0, a_{16} = 0$. We suppose $a_{15} = 1$, $a_{25} = a_{35} = a_{45} = 0$.

5.1. $a_{22} = a_{23} = a_{24} = a_{26} = 0$. Consequently $a_{21} \neq 0$ and choosing $a_{21} = 1, a_{11} = a_{31} = a_{41} = 0$ we obtain 1.4.

5.2. $a_{22} \neq 0, a_{23} = a_{24} = a_{26} = 0$. We take $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = 0$ and we have 2.4.

5.3. $a_{23} \neq 0, a_{24} = a_{26} = 0$. Putting $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = 0$ we get 3.4.

5.4. $a_{24} \neq 0, a_{26} = 0$. Now we choose $a_{24} = 1$, $a_{14} = a_{34} = a_{44} = 0$ and we obtain 4.4.

5.5. $a_{26} \neq 0$. We assume $a_{26} = 1, a_{36} = a_{46} = 0$.

5.5.1. $a_{32} = a_{33} = a_{34} = 0$. Then $a_{31} \neq 0$ and replacing $a_{31} = 1$, $a_{11} = a_{21} = a_{41} = 0$ we have 1.4.4.

5.5.2. $a_{32} \neq 0, a_{33} = a_{34} = 0$. Choosing $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = 0$ we get 2.4.4.

5.5.3. $a_{33} \neq 0, a_{34} = 0$. We put $a_{33} = 1, a_{13} = a_{23} = a_{43} = 0$ and we obtain 3.4.4.

5.5.4. $a_{34} \neq 0$. Now taking $a_{34} = 1, a_{14} = a_{24} = a_{44} = 0$ we have 4.4.4.

6. $a_{16} \neq 0$. We suppose $a_{16} = 1, a_{26} = a_{36} = a_{46} = 0$.

6.1. $a_{22} = a_{23} = a_{24} = a_{25} = 0$. Therefore $a_{21} \neq 0$ and choosing $a_{21} = 1, a_{11} = a_{31} = a_{41} = 0$ we get 1.5.

6.2. $a_{22} \neq 0, a_{23} = a_{24} = a_{25} = 0$. Assuming $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = 0$ we obtain 2.5.

6.3. $a_{23} \neq 0, a_{24} = a_{25} = 0$. We take $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = 0$ and we have 3.5.

6.4. $a_{24} \neq 0, a_{25} = 0$. Now we choose $a_{24} = 1$, $a_{14} = a_{34} = a_{44} = 0$ and we get 4.5.

6.5. $a_{25} \neq 0$. Putting $a_{25} = 1, a_{15} = a_{35} = a_{45} = 0$ we obtain 5.5.

Therefore we may state the following.

Theorem 1. *The four-parametric subgroups of B_6 can be reduced to one of the subgroups*

$$B_{41} = \{X_1, X_2, X_3, X_4\}, \quad B_{42} = \{X_1, X_2, X_4, X_6\},$$

$$B_{43} = \{X_2, X_3, X_4, \alpha X_1 + X_5 | \alpha \in \mathbb{R}\}.$$

3. Five-parametric subgroups of B_6.

A five-parametric subgroup of B_6 may be determined by five infinitesimal operators in the form (1), satisfying the conditions (2) for $i, j = 1, ..., 5; i \neq j$. Consider the possible cases.

1. $a_{12} = a_{13} = a_{14} = a_{15} = a_{16} = 0$. Then $a_{11} \neq 0$ and we can put $a_{11} = 1, a_{21} = a_{31} = a_{41} = a_{51} = 0$.

1.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. We have $a_{22} \neq 0$ and we assume that $a_{22} = 1, a_{32} = a_{42} = a_{52} = 0$.

1.1.1. $a_{34} = a_{35} = a_{36} = 0$. Now $a_{33} \neq 0$ and we take $a_{33} = 1, a_{43} = a_{53} = 0$.

1.1.1.1. $a_{45} = a_{46} = 0$. We get $a_{44} = 1, a_{54} = 0$. The operators

$Y_1 = X_1, \quad Y_2 = X_2, \quad Y_3 = X_3, \quad Y_4 = X_4, \quad Y_5 = a_{55}X_5 + a_{56}X_6$

determine a group if and only if $a_{56} = 0$. Then $a_{55} \neq 0$ and by a change of the variables we find the subgroups

$$B_{51} = \{X_1, X_2, X_3, X_4, X_5\}.$$

1.1.1.2. $a_{45} \neq 0, a_{46} = 0$. We suppose $a_{45} = 1, a_{55} = 0$. The operators define a group iff $a_{54} \neq 0, a_{56} = 0$. We obtain the subgroup B_{51}.

1.1.1.3. $a_{46} \neq 0$. Therefore we have $a_{46} = 1, a_{56} = 0$ and a simple calculation gives that the operators do not define a group.

1.1.2. $a_{34} \neq 0, a_{35} = a_{36} = 0$. Then $a_{34} = 1, a_{44} = a_{54} = 0$.

1.1.2.1. $a_{45} = a_{46} = 0$. Now $a_{43} \neq 0$ and assuming $a_{43} = 1$, $a_{33} = a_{53} = 0$ we get 1.1.1.1.

1.1.2.2. $a_{45} \neq 0, a_{46} = 0$. We take $a_{45} = 1, a_{55} = 0$.

1.1.2.2.1. $a_{56} = 0$. Then $a_{53} \neq 0$ and choosing $a_{53} = 1$, $a_{33} = a_{43} = 0$ we obtain B_{51}.

1.1.2.2.2. $a_{56} \neq 0$. The operators do not define a group.

1.1.2.3. $a_{46} \neq 0$. We have $a_{46} = 1, a_{56} = 0$.

1.1.2.3.1. $a_{55} = 0$. Now $a_{53} \neq 0$ and the operators do not determine a group.

1.1.2.3.2. $a_{55} \neq 0$. The operators again do not generate a group.

1.1.3. $a_{35} \neq 0, a_{36} = 0$. We choose $a_{35} = 1, a_{45} = a_{55} = 0$.

1.1.3.1. $a_{54} = a_{56} = 0$. Then $a_{53} \neq 0$ and we put $a_{53} = 1$, $a_{33} = a_{43} = 0$.

1.1.3.1.1. $a_{46} = 0$. Therefore $a_{44} \neq 0$ and replacing $a_{44} = 1$, $a_{34} = 0$ we obtain B_{51}.

1.1.3.1.2. $a_{46} \neq 0$. In this case the operators do not define a group.

1.1.3.2. $a_{54} \neq 0, a_{56} = 0$. Now $a_{54} = 1, a_{34} = a_{44} = 0$ and we have 1.1.2.2.

1.1.3.3. $a_{56} \neq 0$. We can choose $a_{56} = 1, a_{46} = 0$.

1.1.3.3.1. $a_{44} = 0$. Then $a_{43} \neq 0$ and putting $a_{43} = 1$, $a_{33} = a_{53} = 0$ we obtain 1.1.3.1.2.

1.1.3.3.2. $a_{44} \neq 0$. Consequently we can replace $a_{44} = 1$, $a_{34} = a_{54} = 0$ and we have 1.1.2.2.2.

1.1.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = a_{56} = 0$.

1.1.4.1. $a_{54} = a_{55} = 0$. Therefore $a_{53} \neq 0$ and choosing $a_{53} = 1, a_{33} = a_{43} = 0$ we get 1.1.1.3.

1.1.4.2. $a_{54} = 0, a_{55} \neq 0$. We have $a_{55} = 1, a_{35} = a_{45} = 0$.

1.1.4.2.1. $a_{44} = 0$. Now $a_{43} \neq 0$ and the operators do not define a group.

1.1.4.2.2. $a_{44} \neq 0$. Replacing $a_{44} = 1, a_{34} = 0$ we get 1.1.2.2.2.

1.1.4.3. $a_{54} \neq 0$. We put $a_{54} = 1, a_{34} = a_{44} = 0$.

1.1.4.3.1. $a_{45} = 0$. Therefore $a_{43} \neq 0$ and assuming $a_{43} = 1$, $a_{33} = a_{53} = 0$ we have 1.1.1.3.

1.1.4.3.2. $a_{45} \neq 0$. We can choose $a_{45} = 1, a_{35} = a_{55} = 0$ and we find 1.1.2.2.2.

1.2. $a_{23} \neq 0, a_{24} = a_{25} = a_{26} = 0$. We put $a_{23} = 1$, $a_{33} = a_{43} = a_{53} = 0$.

1.2.1. $a_{34} = a_{35} = a_{36} = 0$. Now $a_{32} \neq 0$ and replacing $a_{32} = 1, a_{22} = a_{42} = a_{52} = 0$ we obtain 1.1.1.

1.2.2. $a_{34} \neq 0, a_{35} = a_{36} = 0$. We assume that $a_{34} = 1$, $a_{44} = a_{54} = 0$.

1.2.2.1. $a_{55} = a_{56} = 0$. Consequently $a_{52} \neq 0$ and putting $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we obtain 1.1.1.1.

1.2.2.2. $a_{55} \neq 0, a_{56} = 0$. Then we choose $a_{55} = 1, a_{45} = 0$.

1.2.2.2.1. $a_{46} = 0$. Therefore $a_{42} \neq 0$ and taking $a_{42} = 1$, $a_{22} = a_{32} = a_{52} = 0$ we receive B_{51}.

1.2.2.2.2. $a_{46} \neq 0$. Now the operators do not define a group.

1.2.2.3. $a_{56} \neq 0$. We take $a_{56} = 1, a_{46} = 0$.

1.2.2.3.1 $a_{45} = 0$. Then $a_{42} \neq 0$ and choosing $a_{42} = 1$, $a_{22} = a_{32} = a_{52} = 0$ we find 1.1.2.3.1.

1.2.2.3.2. $a_{45} \neq 0$. We replace $a_{45} = 1, a_{55} = 0$ and we get 1.2.2.2.2.

1.2.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = a_{55} = 0$.

1.2.3.1. $a_{54} = a_{56} = 0$. Consequently $a_{52} \neq 0$ and putting $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we obtain 1.1.3.1.

1.2.3.2. $a_{54} \neq 0, a_{56} = 0$. Now we take $a_{54} = 1, a_{34} = a_{44} = 0$ and we arrive to 1.2.2.2.

1.2.3.3. $a_{56} \neq 0$. We can have $a_{56} = 1, a_{46} = 0$.

1.2.3.3.1. $a_{44} = 0$. Then $a_{42} \neq 0$ and assuming $a_{42} = 1$, $a_{22} = a_{32} = a_{52} = 0$ we obtain 1.1.3.1.2.

1.2.3.3.2. $a_{44} \neq 0$. We put $a_{44} = 1, a_{34} = a_{54} = 0$ and get 1.2.2.2.2.

1.2.4. $a_{36} \neq 0$. We choose $a_{36} = 1, a_{46} = a_{56} = 0$.

1.2.4.1. $a_{54} = a_{55} = 0$. Therefore $a_{52} \neq 0$ and taking $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we have 1.1.1.3.

1.2.4.2. $a_{54} \neq 0, a_{55} = 0$. We suppose $a_{54} = 1, a_{34} = a_{44} = 0$.

1.2.4.2.1. $a_{45} = 0$. Replacing $a_{42} = 1, a_{22} = a_{32} = a_{52} = 0$ we get that the operators do not define a group.

1.2.4.2.2. $a_{45} \neq 0$. Now taking $a_{45} = 1$ we arrive 1.2.2.2.2.

1.2.4.3. $a_{55} \neq 0$. We assume $a_{55} = 1, a_{35} = a_{45} = 0$.

1.2.4.3.1. $a_{44} = 0$. Consequently $a_{42} \neq 0$ and choosing $a_{42} = 1, a_{22} = a_{32} = a_{52} = 0$ we find 1.1.3.1.2.

1.2.4.3.2. $a_{44} \neq 0$. Now we can do $a_{44} = 1, a_{34} = a_{54} = 0$ and we obtain 1.2.2.2.2.

1.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. We suppose $a_{24} = 1$, $a_{34} = a_{44} = a_{54} = 0$.

1.3.1. $a_{33} = a_{35} = a_{36} = 0$. Then $a_{32} \neq 0$ and putting $a_{32} = 1, a_{22} = a_{42} = a_{52} = 0$ we get 1.1.2.

1.3.2. $a_{33} \neq 0, a_{35} = a_{36} = 0$. Assuming $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ we have 1.2.2.

1.3.3. $a_{35} \neq 0, a_{36} = 0$. We put $a_{35} = 1, a_{45} = a_{55} = 0$.

1.3.3.1. $a_{53} = a_{56} = 0$. Therefore $a_{52} \neq 0$ and replacing $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we obtain 1.1.2.2.

1.3.3.2. $a_{53} \neq 0, a_{56} = 0$. Choosing $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ we arrive to 1.2.3.2.

1.3.3.3. $a_{56} \neq 0$. Now we suppose $a_{56} = 1, a_{46} = 0$.

1.3.3.3.1. $a_{43} = 0$. Then $a_{42} \neq 0$ and putting $a_{42} = 1$, $a_{22} = a_{32} = a_{52} = 0$ we have 1.1.2.2.2.

1.3.3.3.2. $a_{43} \neq 0$. We replace $a_{43} = 1, a_{23} = a_{33} = a_{53} = 0$ and we get 1.2.2.2.2.

1.3.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = a_{56} = 0$.

1.3.4.1. $a_{53} = a_{55} = 0$. Consequently $a_{52} \neq 0$ and taking $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we have 1.1.2.3.

1.3.4.2. $a_{53} \neq 0, a_{55} = 0$. We put $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ and we obtain 1.2.2.3.

1.3.4.3. $a_{55} \neq 0$. Choosing $a_{55} = 1, a_{35} = a_{45} = 0$ we get 1.3.3.3.

1.4. $a_{25} \neq 0, a_{26} = 0$. We suppose $a_{25} = 1$, $a_{35} = a_{45} = a_{55} = 0$.

1.4.1. $a_{33} = a_{34} = a_{36} = 0$. Therefore $a_{32} \neq 0$ and taking $a_{32} = 1, a_{22} = a_{42} = a_{52} = 0$ we find 1.1.3.

1.4.2. $a_{33} \neq 0, a_{34} = a_{36} = 0$. Then we replace $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ and we have 1.2.3.

1.4.3. $a_{34} \neq 0, a_{36} = 0$. Now we choose $a_{34} = 1$, $a_{24} = a_{44} = a_{54} = 0$ and we get 1.3.3.

1.4.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = a_{56} = 0$.

1.4.4.1. $a_{53} = a_{54} = 0$. Consequently $a_{52} \neq 0$ and putting $a_{52} = 1, a_{22} = a_{32} = a_{42} = 0$ we have 1.1.3.3.

1.4.4.2. $a_{53} \neq 0, a_{54} = 0$. Then we replace $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ and we arrive to 1.2.4.3.

1.4.4.3. $a_{54} \neq 0$. Taking $a_{54} = 1, a_{24} = a_{34} = a_{44} = 0$ we obtain 1.3.4.3.

1.5. $a_{26} \neq 0$. In this case we can suppose $a_{26} = 1$, $a_{36} = a_{46} = a_{56} = 0$.

1.5.1 $a_{33} = a_{34} = a_{35} = 0$. Therefore $a_{32} \neq 0$ and assuming $a_{32} = 1, a_{22} = a_{42} = a_{52} = 0$ we get 1.1.4.

1.5.2. $a_{33} \neq 0, a_{34} = a_{35} = 0$. We replace $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ and we find 1.2.4.

1.5.3. $a_{34} \neq 0, a_{35} = 0$. Putting $a_{34} = 1$, $a_{24} = a_{44} = a_{54} = 0$ we obtain 1.3.4.

1.5.4. $a_{35} \neq 0$. Now we choose $a_{35} = 1, a_{25} = a_{45} = a_{55} = 0$ and we get 1.4.4.

2. $a_{12} \neq 0, a_{13} = a_{14} = a_{15} = a_{16} = 0$. Changing the variables we can obtain $a_{12} = 1, a_{22} = a_{32} = a_{42} = a_{52} = 0$.

2.1. $a_{23} = a_{24} = a_{25} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and taking $a_{21} = 1, a_{11} = a_{31} = a_{41} = a_{51} = 0$ we have 1.1.

2.2. $a_{23} \neq 0, a_{24} = a_{25} = a_{26} = 0$. We suppose $a_{23} = 1$, $a_{33} = a_{43} = a_{53} = 0$.

2.2.1. $a_{34} = a_{35} = a_{36} = 0$. Then $a_{31} \neq 0$ and choosing $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we get 1.1.1.

2.2.2. $a_{34} \neq 0, a_{35} = a_{36} = 0$. We assume $a_{34} = 1$, $a_{44} = a_{54} = 0$.

2.2.2.1. $a_{55} = a_{56} = 0$. Consequently $a_{51} \neq 0$ and putting $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we have 1.1.1.1.

2.2.2.2. $a_{55} \neq 0, a_{56} = 0$. We replace $a_{55} = 1, a_{45} = 0$.

2.2.2.2.1. $a_{46} = 0$. Now $a_{41} \neq 0$ and taking $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = a_{51} = 0$ we obtain B_{51}.

2.2.2.2.2. $a_{46} \neq 0$. We put $a_{46} = 1$. Then the operators

$$Y_1 = a_{11}X_1 + X_2, \qquad Y_2 = a_{21}X_1 + X_3,$$

$$Y_3 = a_{31}X_1 + X_4, \qquad Y_4 = a_{41}X_1 + X_6,$$

$$Y_5 = a_{51}X_1 + X_5$$

define a group iff $a_{11} = a_{21} = a_{31} = a_{51} = 0$ and we get the subgroup

$$B_{52} = \{X_2, X_3, X_4, X_5, \alpha X_1 + X_6 | \alpha \in \mathbb{R}\}.$$

2.2.2.3. $a_{56} \neq 0$. We suppose $a_{56} = 1, a_{46} = 0$.

2.2.2.3.1. $a_{45} = 0$. Then $a_{41} \neq 0$ and putting $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = a_{51} = 0$ we find that the operators do not define a group.

2.2.2.3.2. $a_{45} \neq 0$. Now we take $a_{45} = 1, a_{55} = 0$ and we obtain 2.2.2.2.2.

2.2.3. $a_{35} \neq 0, a_{36} = 0$. We assume $a_{35} = 1, a_{45} = a_{55} = 0$.

2.2.3.1. $a_{54} = a_{56} = 0$. Therefore $a_{51} \neq 0$ and choosing $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we have 1.1.1.2.

2.2.3.2. $a_{54} \neq 0, a_{56} = 0$. Replacing $a_{54} = 1, a_{34} = a_{44} = 0$ we arrive to 2.2.2.2.

2.2.3.3. $a_{56} \neq 0$. We choose $a_{56} = 1, a_{46} = 0$.

2.2.3.3.1. $a_{44} = 0$. Then $a_{41} \neq 0$ and taking $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = a_{51} = 0$ we get 1.1.3.1.2.

2.2.3.3.2. $a_{44} \neq 0$. We put $a_{44} = 1, a_{34} = a_{54} = 0$ and we have 2.2.2.2.2.

2.2.4. $a_{36} \neq 0$. Now we assume $a_{36} = 1, a_{46} = a_{56} = 0$.

2.2.4.1. $a_{54} = a_{55} = 0$. Consequently $a_{51} \neq 0$ and replacing $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we obtain 1.1.1.3.

2.2.4.2. $a_{54} \neq 0, a_{55} = 0$. We put $a_{54} = 1, a_{34} = a_{44} = 0$ and we get 2.2.2.3.

2.2.4.3. $a_{55} \neq 0$. Then we choose $a_{55} = 1, a_{35} = a_{45} = 0$ and we have 2.2.3.3.

2.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. Now we can assume that $a_{24} = 1$, $a_{34} = a_{44} = a_{54} = 0$.

2.3.1. $a_{33} = a_{35} = a_{36} = 0$. In this case certainly $a_{31} \neq 0$ and choosing $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we find 1.1.2.

2.3.2. $a_{33} \neq 0, a_{35} = a_{36} = 0$. We take $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ and we have 2.2.2.

2.3.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = a_{55} = 0$.

2.3.3.1. $a_{53} = a_{56} = 0$. Therefore $a_{51} \neq 0$ and putting $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we obtain 1.1.2.2.

2.3.3.2. $a_{53} \neq 0, a_{56} = 0$. We replace $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ and we get 2.2.2.2.

2.3.3.3. $a_{56} \neq 0$. Now we assume $a_{56} = 1, a_{46} = 0$.

2.3.3.3.1. $a_{43} = 0$. Then $a_{41} \neq 0$ and choosing $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = a_{51} = 0$ we arrive to 1.1.2.2.2.

2.3.3.3.2. $a_{43} \neq 0$. We put $a_{43} = 1, a_{23} = a_{33} = a_{53} = 0$ and we obtain 2.2.2.2.2.

2.3.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = a_{56} = 0$.

2.3.4.1. $a_{53} = a_{55} = 0$. Consequently $a_{51} \neq 0$ and taking $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we have 1.1.2.3.

2.3.4.2. $a_{53} \neq 0, a_{55} = 0$. Now we can assume that $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ and we arrive to 2.2.2.3.

2.3.4.3. $a_{55} \neq 0$. We choose $a_{55} = 1, a_{35} = a_{45} = 0$ and we get 2.3.3.3.

2.4. $a_{25} \neq 0, a_{26} = 0$. We can take $a_{25} = 1$, $a_{35} = a_{45} = a_{55} = 0$.

2.4.1. $a_{33} = a_{34} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we have 1.1.3.

2.4.2. $a_{33} \neq 0, a_{34} = a_{36} = 0$. Supposing $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ we get 2.2.3.

2.4.3. $a_{34} \neq 0, a_{36} = 0$. Now we replace $a_{34} = 1$, $a_{24} = a_{44} = a_{54} = 0$ and we obtain 2.3.3.

2.4.4. $a_{36} \neq 0$. We assume $a_{36} = 1, a_{46} = a_{56} = 0$.

2.4.4.1. $a_{53} = a_{54} = 0$. Consequently $a_{51} \neq 0$ and choosing $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we have 1.1.3.3.

2.4.4.2. $a_{53} \neq 0, a_{54} = 0$. In this case we can put $a_{53} = 1$, $a_{23} = a_{33} = a_{43} = 0$ and we obtain 2.2.3.3.

2.4.4.3. $a_{54} \neq 0$. Taking $a_{54} = 1, a_{24} = a_{34} = a_{44} = 0$ we get 2.3.3.3.

2.5. $a_{26} \neq 0$. Now we suppose $a_{26} = 1, a_{36} = a_{46} = a_{56} = 0$.

2.5.1. $a_{33} = a_{34} = a_{35} = 0$. Therefore $a_{31} \neq 0$ and replacing $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we arrive 1.1.4.

2.5.2. $a_{33} \neq 0, a_{34} = a_{35} = 0$. We choose $a_{33} = 1$, $a_{23} = a_{43} = a_{53} = 0$ and we get 2.2.4.

2.5.3. $a_{34} \neq 0, a_{35} = 0$. We put $a_{34} = 1, a_{24} = a_{44} = a_{54} = 0$ and we obtain 2.3.4.

2.5.4. $a_{35} \neq 0$. We assume $a_{35} = 1, a_{25} = a_{45} = a_{55} = 0$ and we have 2.4.4.

3. $a_{13} \neq 0, a_{14} = a_{15} = a_{16} = 0$. Now we replace $a_{13} = 1$, $a_{23} = a_{33} = a_{43} = a_{53} = 0$.

3.1. $a_{22} = a_{24} = a_{25} = a_{26} = 0$. Consequently $a_{21} \neq 0$ and taking $a_{21} = 1, a_{11} = a_{31} = a_{41} = a_{51} = 0$ we get 1.2.

3.2. $a_{22} \neq 0, a_{24} = a_{25} = a_{26} = 0$. We put $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = a_{52} = 0$ and we obtain 2.2.

3.3. $a_{24} \neq 0, a_{25} = a_{26} = 0$. Now we choose $a_{24} = 1$, $a_{34} = a_{44} = a_{54} = 0$.

3.3.1. $a_{32} = a_{35} = a_{36} = 0$. Then $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we have 1.2.2.

3.3.2. $a_{32} \neq 0, a_{35} = a_{36} = 0$. We take $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ and we get 2.2.2.

3.3.3. $a_{35} \neq 0, a_{36} = 0$. We suppose $a_{35} = 1, a_{45} = a_{55} = 0$.

3.3.3.1. $a_{52} = a_{56} = 0$. Therefore $a_{51} \neq 0$ and choosing $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we obtain 1.2.2.2.

3.3.3.2. $a_{52} \neq 0, a_{56} = 0$. Now we take $a_{52} = 1$, $a_{12} = a_{22} = a_{32} = a_{42} = 0$ and we have 2.2.2.2.

3.3.3.3. $a_{56} \neq 0$. We can assume $a_{56} = 1, a_{46} = 0$.

3.3.3.3.1. $a_{42} = 0$. Then $a_{41} \neq 0$ and replacing $a_{41} = 1$, $a_{11} = a_{21} = a_{31} = a_{51} = 0$ we get 1.2.2.2.2.

3.3.3.3.2. $a_{42} \neq 0$. We put $a_{42} = 1$, $a_{12} = a_{22} = a_{32} = a_{52} = 0$ and we obtain 2.2.2.2.2.

3.3.4. $a_{36} \neq 0$. We suppose $a_{36} = 1, a_{46} = a_{56} = 0$.

3.3.4.1. $a_{52} = a_{55} = 0$. In this case $a_{51} \neq 0$ and choosing $a_{51} = 1, a_{11} = a_{21} = a_{31} = a_{41} = 0$ we have 1.2.2.3.

3.3.4.2. $a_{52} \neq 0, a_{55} = 0$. We replace $a_{52} = 1$, $a_{12} = a_{22} = a_{32} = a_{42} = 0$ and we get 2.2.2.3.

3.3.4.3. $a_{55} \neq 0$. Then we can assume $a_{55} = 1, a_{35} = a_{45} = 0$ and we obtain 3.3.3.3.

3.4. $a_{25} \neq 0, a_{26} = 0$. Now we choose $a_{25} = 1$, $a_{35} = a_{45} = a_{55} = 0$.

3.4.1. $a_{32} = a_{34} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we have 1.2.3.

3.4.2. $a_{32} \neq 0, a_{34} = a_{36} = 0$. We suppose $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ and we get 2.2.3.

3.4.3. $a_{34} \neq 0, a_{36} = 0$. Choosing $a_{34} = 1$, $a_{24} = a_{44} = a_{54} = 0$ we have 3.3.3.

3.4.4. $a_{36} \neq 0$. Then we take $a_{36} = 1, a_{46} = a_{56} = 0$.

3.4.4.1. $a_{52} = a_{54} = 0$. Now $a_{51} \neq 0$ and putting $a_{51} = 1$, $a_{11} = a_{21} = a_{31} = a_{41} = 0$ we obtain 1.2.3.3.

3.4.4.2. $a_{52} \neq 0, a_{54} = 0$. We assume $a_{52} = 1$, $a_{12} = a_{22} = a_{32} = a_{42} = 0$ and we arrive 2.2.3.3.

3.4.4.3. $a_{54} \neq 0$. Taking $a_{54} = 1, a_{24} = a_{34} = a_{44} = 0$ we get 3.3.3.3.

3.5. $a_{26} \neq 0$. We suppose $a_{26} = 1, a_{36} = a_{46} = a_{56} = 0$.

3.5.1. $a_{32} = a_{34} = a_{35} = 0$. Consequently $a_{31} \neq 0$ and replacing $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we find 1.2.4.

3.5.2. $a_{32} \neq 0, a_{34} = a_{35} = 0$. Now we put $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ and we get 2.2.4.

3.5.3. $a_{34} \neq 0, a_{35} = 0$. Assuming $a_{34} = 1$, $a_{24} = a_{44} = a_{54} = 0$ we have 3.3.4.

3.5.4. $a_{35} \neq 0$. We choose $a_{35} = 1, a_{25} = a_{45} = a_{55} = 0$ and we obtain 3.4.4.

4. $a_{14} \neq 0, a_{15} = a_{16} = 0$. Now we suppose $a_{14} = 1$, $a_{24} = a_{34} = a_{44} = a_{54} = 0$.

4.1. $a_{22} = a_{23} = a_{25} = a_{26} = 0$. Then $a_{21} \neq 0$ and taking $a_{21} = 1, a_{11} = a_{31} = a_{41} = a_{51} = 0$ we get 1.3.

4.2. $a_{22} \neq 0, a_{23} = a_{25} = a_{26} = 0$. We replace $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = a_{52} = 0$ and we have 2.3.

4.3. $a_{23} \neq 0, a_{25} = a_{26} = 0$. Putting $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = a_{53} = 0$ we arrive to 3.3.

4.4. $a_{25} \neq 0, a_{26} = 0$. Now we can assume $a_{25} = 1$, $a_{35} = a_{45} = a_{55} = 0$.

4.4.1. $a_{32} = a_{33} = a_{36} = 0$. Therefore $a_{31} \neq 0$ and taking $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we get 1.3.3.

4.4.2. $a_{32} \neq 0, a_{33} = a_{36} = 0$. Choosing $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ we have 2.3.3.

4.4.3. $a_{33} \neq 0, a_{36} = 0$. We put $a_{33} = 1$, $a_{13} = a_{23} = a_{43} = a_{53} = 0$ and we obtain 3.3.3.

4.4.4. $a_{36} \neq 0$. In this case we suppose $a_{36} = 1, a_{46} = a_{56} = 0$.

4.4.4.1. $a_{52} = a_{53} = 0$. Then $a_{51} \neq 0$ and replacing $a_{51} = 1$, $a_{11} = a_{21} = a_{31} = a_{41} = 0$ we get 1.3.3.3.

4.4.4.2. $a_{52} \neq 0, a_{53} = 0$. We put $a_{52} = 1$, $a_{12} = a_{22} = a_{32} = a_{42} = 0$ and we have 2.3.3.3.

4.4.4.3. $a_{53} \neq 0$. We choose $a_{53} = 1$, $a_{13} = a_{23} = a_{33} = a_{43} = 0$ and we obtain 3.3.3.3.

4.5. $a_{26} \neq 0$. Now we can assume that $a_{26} = 1$, $a_{36} = a_{46} = a_{56} = 0$.

4.5.1. $a_{32} = a_{33} = a_{35} = 0$. Consequently $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we get 1.3.4.

4.5.2. $a_{32} \neq 0, a_{33} = a_{35} = 0$. We choose $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ and we have 2.3.4.

4.5.3. $a_{33} \neq 0, a_{35} = 0$. Taking $a_{33} = 1$, $a_{13} = a_{23} = a_{43} = a_{53} = 0$ we get 3.3.4.

4.5.4. $a_{35} \neq 0$. We replace $a_{35} = 1, a_{25} = a_{45} = a_{55} = 0$ and we obtain 4.4.4.

5. $a_{15} \neq 0, a_{16} = 0$. We can suppose $a_{15} = 1$, $a_{25} = a_{35} = a_{45} = a_{55} = 0$.

5.1. $a_{22} = a_{23} = a_{24} = a_{26} = 0$. Therefore $a_{21} \neq 0$ and choosing $a_{21} = 1, a_{11} = a_{31} = a_{41} = a_{51} = 0$ we have 1.4.

5.2. $a_{22} \neq 0, a_{23} = a_{24} = a_{26} = 0$. We put $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = a_{52} = 0$ and we get 2.4.

5.3. $a_{23} \neq 0, a_{24} = a_{26} = 0$. Taking $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = a_{53} = 0$ we obtain 3.4.

5.4. $a_{24} \neq 0, a_{26} = 0$. We assume $a_{24} = 1$, $a_{14} = a_{34} = a_{44} = a_{54} = 0$ and we arrrive to 4.4.

5.5. $a_{26} \neq 0$. Now we suppose $a_{26} = 1, a_{36} = a_{46} = a_{56} = 0$.

5.5.1. $a_{32} = a_{33} = a_{34} = 0$. Then $a_{31} \neq 0$ and putting $a_{31} = 1, a_{11} = a_{21} = a_{41} = a_{51} = 0$ we get 1.4.4.

5.5.2. $a_{32} \neq 0, a_{33} = a_{34} = 0$. We choose $a_{32} = 1$, $a_{12} = a_{22} = a_{42} = a_{52} = 0$ and we obtain 2.4.4.

5.5.3. $a_{33} \neq 0, a_{34} = 0$. Taking $a_{33} = 1$, $a_{13} = a_{23} = a_{43} = a_{53} = 0$ we have 3.4.4.

5.5.4. $a_{34} \neq 0$. We put $a_{34} = 1, a_{14} = a_{24} = a_{44} = a_{54} = 0$ and we get 4.4.4.

6. $a_{16} \neq 0$. Now we assume $a_{16} = 1, a_{26} = a_{36} = a_{46} = a_{56} = 0$.

6.1. $a_{22} = a_{23} = a_{24} = a_{25} = 0$. Therefore $a_{21} \neq 0$ and choosing $a_{21} = 1, a_{11} = a_{31} = a_{41} = a_{51} = 0$ we obtain 1.5.

6.2. $a_{22} \neq 0, a_{23} = a_{24} = a_{25} = 0$. We put $a_{22} = 1$, $a_{12} = a_{32} = a_{42} = a_{52} = 0$ and we find 2.5.

6.3. $a_{23} \neq 0, a_{24} = a_{25} = 0$. Taking $a_{23} = 1$, $a_{13} = a_{33} = a_{43} = a_{53} = 0$ we get 3.5.

6.4. $a_{24} \neq 0, a_{25} = 0$. We put $a_{24} = 1$, $a_{14} = a_{34} = a_{44} = a_{54} = 0$ and we obtain 4.5.

6.5. $a_{25} \neq 0$. Then we can replace $a_{25} = 1$, $a_{15} = a_{35} = a_{45} = a_{55} = 0$ and we have 5.5.

Now we may summarize the foregoing results in the following

Theorem 2. *The five-parametric subgroups of B_6 can be reduced to one of the subgroups*

$$B_{51} = \{X_1, X_2, X_3, X_4, X_5\},$$

$$B_{52} = \{X_2, X_3, X_4, X_5, \alpha X_1 + X_6 | \alpha \in \mathbb{R}\}.$$

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