Mathematatica Balkanica

Mathematical Society of South-Eastern Europe
A quarterly published by
the Bulgarian Academy of Sciences – National Committee for Mathematics

New Series Vol.11, 1997, Fasc. 1-2

Addition, Subtraction and Multiplication of Sequences of Fractions by Means of Residue Arithmetic and Mathematical Spectra

M. Stanković, J. Madić, P. Stanimirović

Presented by Ž. Mijajlović

In this paper we define a new kind of mathematical spectra, on sequences of rational numbers. We show applications of this spectra in exact computations on sequences of rational numbers.

We use the mathematical spectra of sequences of integers and residue arithmetic.

Using single or multiple-moduli residue arithmetic we convert given sequences of fractions into corresponding sequences of positive integers. Then we use well known methods on spectra of integers and we get a resulting sequence of integers in spectral form. Finally, each element of the resulting sequence of integers is converted into corresponding fraction.

We also present several applications of the introduced theory.

1. Introduction

The theory of mathematical spectra was introduced by M. Petrović. K. Orlov discovered the practical applications of it ([3],[4]).

Definition 1.1. The mathematical spectrum of a sequence of integers $\alpha_0, ..., \alpha_n$, i.e. the spectrum of the polynomial $P(x) = \alpha_0 x^n + ... + \alpha_n$ is the ordered pair (S, h), where $S = P(10^h)$ is the spectral value, and h is the spectral rhythm, obtained as the first integer such that $10^h > \max\{\|\alpha_i|\}$.

There is a bijection between spectrum (S, h) and the corresponding sequence (polynomial).

Definition 1.2. The mathematical spectrum of a sequence of fixed-point numbers $\alpha_o, ..., \alpha_n$ is the ordered triple (S, h, k), where k is the maximal number of decimal digits, and (S, h) is the spectrum of sequence of integers $b_0, ..., b_n$ obtained from the equalities $b_i = 10^k * a_i$, $i = 1, ..., n$.

K. Orlov defined a mathematical spectrum of a fraction and introduced its use for addition, subtraction, multiplication and division ot two fractions. In [3] the spectrum of a fraction was defined as the spectrum of its numerator and denominator. Mathematical operations of two fractions were defined as follows:

Algorithm U1.

Step 1. Multiply the spectra of the initial fractions.

Step 2. The value of the second strip represents numerator of their sum, and denominator of the sum is equal to the effective value of the third strip.

Similarly, numerator of the product of given sequences is equal to the effective value of the first strip and denominator is equal to the effective value of the third strip.

Remark. Subtraction and division are defined by addition and multiplication respectively.

In this paper we define more general mathematical spectrum on a sequence of fractions and describe its use in exact addition, subtraction and multiplication of the sequences. To perform this we use the single or multiple-moduli residue arithmetic.

The theory of residue arithmetic and its applications is given in [1], [2].

Definition 1.3. For given a, b, and $m > 1$ $a \equiv b \pmod{m}$ if and only if $m|(a-b)$ ($a-b$ is divisible by m, i.e. $b = a \pmod{m}$). In residue arithmetic a residue b is always chosen to satisfy formula

$$0 \leq b < m$$

and we call it the least residue of a module m (we denote it $b = |a|_m$). It is obvious that $|\circ|_m : [] \mapsto []_m = \{0, 1, ..., m-1\}$.

Theorem 1.1. *Let a, b and $m > 1$ be integers.*

(i) The number $|a|_m$ is unique for any a.

(ii) $|a|_M = |b|_m$ if and only if $a \equiv b \pmod{m}$.

Theorem 1.2. $|a \pm_\circ b|_m = ||a|_m \pm_\circ b|_m = |a \pm_\circ |b|_m|_m = ||a|_m \pm_\circ |b|_m|_m$, *for any integers a, b and $m > 1$.*

Theorem 1.3. *If an integer and module m are relatively prime, i.e. $(a, m) = 1$, there exists only one integer $b \in []_m$ such that $|ab|_m = 1$. Obtained number b is called the multiplicative inverse of a module m and we denote it by $a^{-1}(m)$.*

Definition 1.4. If $(b,m)=1$, then $|a/b|_m = |ab^{-1}(m)|_m$.

Definition 1.5 $\hat{Q} = \{a/b \mid (b,m) = 1\}$ denotes the set of rational numbers convertible into $[]_m$.

Theorem 1.4. *Let N be the maximal nonnegative integer satisfying condition*

$$2N^2 + 1 \leq m.$$

If we define sets

$$F_N = \{a/b \mid (a,b) = 1, \quad 0 \leq |a|_m < N, \quad) \leq |b|_m < N \quad \text{and}$$

$$\hat{[]}_m = \{|a/b|_m | a/b \in F_N\},$$

then exists the inverse function $|\circ|_m^{-1}$ of the holomorphism $|\circ|_m \; (F_n, +, \circ) \to (\hat{[]}_m, +, \circ)$ (see [2]).

2. Mathematical spectrum of a sequence of rational numbers

Definition 2.1 The mathematical spectrum of rational numbers $a_0/b_0, \ldots, a_k/b_k$ is the ordered triple (M, S, h), where M is sufficiently big module and (S, h) is the spectrum of corresponding residues $r_i = |a_i/b_i|_M$, $1 \leq i \leq k$.

Using Theorem 1.4 we can immediately conclude that module M could be obtained from the conditions (0) and (1):

(0) $$\circ N = \max\{|a_0|, \ldots, |a_k|, \ldots, |b_0|, \ldots, |b_k|\}$$

(1) $\circ M$ is the first prime number such that $M \geq 2 * N^2 + 1$

If a set of moduli $[m_1, \ldots, m_t]$ is used, then we introduce following definitions.

Definition 2.2. The mathematical spectrum of a sequence of rational numbers is the ordered triple

(2) $$(M, S, h),$$

where

- $m_1, m_2, \ldots, m_t$ are the least prime numbers such that $M = \prod_{i=1}^{t} m_i \geq 2 * N^2 + 1$.

- S is the spectral value whose j-th spectral strip is obtained by the mixed-radix conversion process with radices $m_1, \ldots, m_t$ on the j-th spectral strips of the values of the spectra (m_k, S_k, h_k), $1 \leq k \leq t$ which are formed according to definition 2.1.

- The uniform rhythm h is the least integer such that

$$10^h > 2 * \max\{|S_i|\}.$$

Definition 2.3 The set of spectra $\{(m_j, S_j, h_j)\}$ used in Definition 2.2 is called residue representation of the resulting spectrum (2).

The introduced binary spectral operations (addition, subtraction and multiplication) are carried out with the rhythm common for the operators and results ([3], [6]), and with the module which suffices for all operators and results. Symbolically we can write

$$(m, S_1, h) \ominus (m, S_2, h) = (m, S, h), \quad \text{where} \quad \ominus \in \{+, -, *\},\ S = S_1 \ominus S_2.$$

3. Choice of module

We choose module m so that values of resulting sequence of numbers, given in a spectral form, could be converted into corresponding fractions. Each element of the resulting sequence depends on all elements of given sequence. Therefore, both sequences should be entered before the corresponding module is calculated.

Let the following two sequences of fractions are entered:

$$p = \left(\frac{a_i}{b_i}\right)_{i=1}^{r} \quad \text{and} \quad q = \left(\frac{c_i}{d_i}\right)_{i=1}^{s}. \tag{3}$$

Algorithm $M1$ gives a module which allows exact addition and subtraction on these sequences, and algorithm $M2$ gives a sufficiently big module for exact multiplication.

Algorithm M1.

Step 1. Compute values:

$$\max 1 = \max\{|a_i * d_i \pm b_i * c_i|,\ i = 1, \ldots, \max\{r, s\}\};$$

$$\max 2 = \max\{1cd(|b_i d_i|,\ i = 1, \ldots, \max\{r, s\}\};$$

$$N = \max\{max1, max2\};$$

where $1cd(x, y)$ represents the lowest common denominator of x and y. If $r < s$ we use $a_k = 0$, $b_k = 1$, $k = r + 1, \ldots, s$, and if $s < r$ we use $c_k = 0$, $d_k = 1$, $k = s + 1, \ldots, r$.

Step 2. Module m satisfies condition (1).

Theorem 3.1. *Module m given according to algorithm M1 suffices for the exact computation of sum and difference of two sequences of fractions by means of defined mathematical spectra.*

Proof. Consider sequences (3). Denominators of the sum and the difference of these sequences are equal to $lcd(|b_i|, |d_i|) \leq max2 \leq N$. Also, absolute values of all numerators and denominators is equal to N. Using Theorem 1.4 and inequality (1) we can conclude that all the elements of the resulting sequence are representable with module m. ■

Algorithm M2.
Step 1. Compute $iml = lcd\{|b_1 * d_1|, ..., |b_1 * d_2|, ..., |b_r * d_s|\}$.
Step 2. Compute values

$$\max 1 = \max\{|a_1|, ..., |a_r|\}\ ; \ \max 2 = \max\{|c_1|, ..., |c_s|\}\ ;$$

$$p1 = \min\{|b_1|, ..., |b_r|\}\ ; \ p2 = \min\{|d_1|, ..., |d_s|\}\ ; \quad \text{and}$$

$$brl = l * \max 1 * \max 2 * iml/(p1 * p2), \quad l = \min\{r, s\}.$$

Step 3. Set $N = \max\{iml,\ brl\}$. *Module m is obtained from the condition (1).*

Theorem 3.2. *Product of sequences (3) can be obtained using single-modulus residue arithmetic with module given according to algorithm M2 and defined mathematical spectra.*

Proof. The i-th element of the product of sequences (3) is

$$\frac{a_1}{b_1} * \frac{c_i}{d_i} * \frac{a_2}{b_2} * \frac{a_{i-1}}{b_{i-1}} + ... + \frac{a_i}{b_i} * \frac{c_1}{d_1}, \ 1 \leq i \leq \max\{r, s\}.$$

It is evident that denominator is equal to

$$lcd\,(|a_1 * d_i|, ..., |a_i * d_1|) \leq iml \leq N.$$

Also, numerator is equal to

$$a_1 * c_i * iml/(b_1 * d_i) + a + 2 * c_{i-1} * iml/(b_2 * d_{i-1}) + ...+$$
$$a_i * c_1 * iml/(b_i * d_1) \leq \min\{r, s\} * \max 1 * \max 2 * iml/(p1 * p2) = brl \leq N.$$

In this way, the absolute values of all numerators and denominators of the product are less than or equal to N. Since $m \geq 2 * N^2 + 1$, we easily conclude that given numerators and denominators can be transformed into corresponding residue representations.

4. Addition, subtraction and multiplication by means of single-modulus arithmetic and mathematical spectra

First we form the mathematical spectra of given sequences using Definition 2.1, where the module M is obtained according to algorithm $M1$ or algorithm $M2$.

Then we use mathematical spectra in order to perform addition, subtraction or multiplication of obtained spectra.

In this way, a resulting sequence is obtained in the spectral and residual form, i.e. residue representations of its elements are equal to the effective values of the spectral strips of the resulting spectrum.

Performing conversion back (the computed residue representations to corresponding fractions), we obtain the resulting sequence of rational numbers.

Algorithm 01.

Step 1. Shorten all the given fractions.

Step 2. Select a prime module m according to algorithm $M1$ or algorithm $M2$.

Step 3. Transform given sequences (3) into corresponding sequences of residual representations module m

$$\mathcal{N}_i = |a_i/b_i|_m, \quad i = 1, \dots, r$$

$$\mathcal{P} = |c_i/d_i|_m, \quad i = 1, \dots, s. \tag{4}$$

Step 4. The uniform rhythm h is the least integer such that

$$10^h > 2 * \max\{\mathcal{N}_1, \dots, \mathcal{N}_r, \mathcal{P}_1, \dots, \mathcal{P}_s\}$$

(for addition and subtraction), or

$$10^h > 2 * \min\{r, s\} * \max\{\mathcal{N}_1, \dots, \mathcal{N}_r\} * \max\{\mathcal{P}_1, \dots, \mathcal{P}_s\}$$

(for multiplication).

Step 5. Using standard methods form corresponding mathematical spectra of the sequences (4) with uniform rhythm h and perform condensation on the left. In this way, we get mathematical spectra of the initial sequences by Definition 2.1.

Step 6. After performing necessary mathematical operations on the spectra, we obtain the resulting spectrum, i.e. the solution in the spectral form.

Step 7. Conversely, from the resulting spectrum, using uniform rhythm and computing the effective values of the spectral strip we obtain the sequence of residual representations

$$\mathcal{R}_1, ..., \mathcal{R}_l, \ l = \max(r, s). \tag{5}$$

Step 8. For all the integers in (5) perform conversion back into corresponding fraction using module m (see [1],[2]).

Example 1. In this example we perform subtraction of sequences

$$[-857/510\ 2465/187\ 225/333 - 115/60] \quad \text{and} \quad [2103/198 - 38/57 - 263/240].$$

First we determine a module m applying algorithm $M1$.

$$\begin{aligned} \max 1 &= \max\{175, 169, 149, 197\} = 197; \\ \max 2 &= \max\{102, 66, 111, 240\} = 240; \end{aligned}$$

Applying steps 3,4 and 5 of Algorithm 01, we obtain the following spectra:

$$\begin{aligned} s1 &= (115201, +06889301048602490910 5599, 6); \\ s2 &= (115201, +06889207680001103 9, 6). \end{aligned}$$

According to step 6, we get the resulting spectrum

$$s = s1 - s2 = (115201, +068892945892948109094560, 6).$$

Using rhythm $h = 6$ we get effective values of the spectral strips (step 7);

$$68893, -54107, -51891, 94560,$$

which are converted into resulting sequence of fractions using single-modulus arithmetic with module m (step 8):

$$[-175/102\ 169/66\ 149/111\ - 197/240].$$

5. Applying multiple-moduli arithmetic

First we choose a base-vector $\beta = [m_1, ..., m_t]$ consisting of prime moduli $m_1, ..., m_t$ which allows conversion of all the elements of the resulting sequence. Main criterion follows from Theorem 1.4: if absolute values of all the resulting numerators and denominators are less than or equal to N, then $m_1, ..., m_t$ are least primes such that

$$M = \prod_{i=1}^{t} m_i \geq 2 * N^2 + 1. \tag{6}$$

for given sequences (3), a base vector β corresponding to their sum, difference or product is selected according to algorithm $M3$.

Algorithm M3.

Step 1. Compute N according to Algorithm $M1$ or Algorithm $M2$.

Step 2. Select moduli $m_1, ..., m_t$ according to condition (6).

Theorem 5.1. *Sum, difference and product of sequences (3) can be obtained using multiple-moduli arithmetic with a base vector β given according to Algorithm $M3$.*

P r o o f. According to Theorem 3.1 or Theorem 3.2, absolute values of all the numerators and denominators of the resulting sequence are less than or equal to N. Knowing that the base vector β satisfies condition (6) and using Theorem 1.4, we easily conclude that the given numerators and denominators have corresponding residue representations. ■

A mathematical operation applied to fractions (3) can be exactly performed using multiple-moduli arithmetic according to Algorithm 02.

Algorithm 02.

Step 1. Shorten all given fractions.

Step 2. Choose a base vector $\beta = [m_1, ..., m_t]$ according to Algorithm $M3$.

Step 3. Apply Algorithm 01 (except the step 1) for any module from the chosen base vector. This way, we get t resulting sequences of integers

$$\mathcal{R}_1^1, \mathcal{R}_2^1, ..., \mathcal{R}_l^1, \qquad \text{for module } m_1$$

$$\mathcal{R}_1^1, \mathcal{R}_2^2, ..., \mathcal{R}_l^2, \qquad \text{for module } m_2$$

$$\mathcal{R}_1^t, \mathcal{R}_2^t, ..., \mathcal{R}_l^t, \qquad \text{for module } m_t,$$

where $1 = \max(r, s)$.

Then $\{\mathcal{R}_i^1, ..., \mathcal{R}_i^t\}$, $i = 1, ..., l$ represent a multiple residue representation of the $i-th$ element of the resulting sequence. A mixed-radix system or the Chinese remainder theorem could be used during conversion from the residue system to the mixed-radix system. The result obtained by this conversion process is the residue representation module M of this element, where $M = \prod_{i=1}^t m_i$. We will use a mixed-radix system associated with chosen residue system β, i.e. the set of radices $R_1, ..., R_t$ is chosen so that $R_i = m_i$.

E x a m p l e 2. Let sequences be

$$[-23/4 \ 12/5 \ 22/11] \quad \text{and} \quad [1/5 \ -32/16 \ 4/5 \ 2/9].$$

We illustrate the exact computation of the difference of these sequences with a base vector $\beta = [m_1, m_2]$. Applying Algorithms $M1$ and $M3$, we get

$$N = 414; \ m_1 = 631, \ m_2 = 641.$$

Applying Algorithm 01 with module m_1, we get the following spectra of given fractions:

$$s1 = (631, +152381002, 3); \quad s2 = (631, 0505062901270491, 4).$$

Difference of this spectra is the spectrum

$$s' = (631, -0505047697460489, 4).$$

Effective values of the spectral strips are

$$-505, \ -477, \ 254, \ -489.$$

If we use module m_2, we obtain the rhythm $h = 4$ and following spectra:

$$s3 = (641, +475387002, 3); \ s4 = (641, +0513063901290570, 4).$$

Their difference is

$$s'' = (641, -0513016397420568, 4)$$

and the effective values of the spectral strips are

$$-513, \ -164, \ 258, \ -568.$$

Using mixed-radix system with radices m_1 and m_2 we get the following mixed-radix representations, i.e. residue representations module $M = m_1 * m_2 = 404471$

$$80894, \ 101114, \ 161790, \ 44943.$$

Thus, the resulting spectrum is

$$(404471, +080894101114161790044943, 6).$$

After the conversion of the effective values of its spectral strips into fractions with module $M = m_1 * m_2$ we get the following resulting sequence of fractions:

$$[-1/5, \ -15/4, \ 8/5, \ 16/9].$$

Example 3. This example shows multiplication of entered sequences

$$[12/30 \ -16/56 \ 135/117] \quad \text{and} \quad [-5/17 \ 1215/270 \ 11 \ -5/20].$$

with a base vector $\beta = [m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8]$.

Using Algorithm $M2$, we get $N = 3063060$. Also, accomplishing Algorithm $M3$, we get

$$m_1 = 37,\ m_2 = 41,\ m_3 = 43,\ m_4 = 47,\ m_5 = 53,\ m_6 = 59,\ m_7 = 61,\ m_8 = 67.$$

Using module m_1 we have the following spectra:

$$s_1^1 = (37, +300504, 2);\ s_1^2 = (37, +28231109, 2).$$

Product of this spectra is

$$s_1 = (37, +084008300557041700089, 4).$$

Also applying module m_2 we get the following spectra:

$$s_2^1 = (41, +252939.2) \quad \text{and} \quad s_2^2 = (41, +1925110, 2)$$

$$s_2 = s_2^1 * s_2^2 = (41, +0475117617411154407190390, 4).$$

Similarly, using the rest of the moduli we get:

$$s_3^1 = (43, +091221, 2) \quad \text{and} \quad s_3^2 = (43, +25261132, 2)$$

$$s_3 = (43, +0225053409360966061506 72, 4);$$

$$s_4^1 = (47, +384012, 2)\ ;\ s_4^2 = (47, +08281135, 2)$$

$$s_4 = (47, +030413841634210061532, 4);$$

$$s_5^1 = (53, +113046, 2)\ ;\ s_5^2 = (53, +34311113, 2)$$

$$s_5 = (53, +0374136126151899089605 98, 4);$$

$$s_6^1 = (59, +242542, 2)\ ;\ s_6^2 = (59, +24341144, 2)$$

$$s_6 = (59, +057614162122275915621848, 4);$$

$$s_7^1 = (61, +037052034, 3)\ ;\ s_7^2 = (61, +32351115, 2)$$

$$s_7 = (61, +1184295933152317115405 10, 4);$$

$$s_8^1 = (67, +054038, 063, 3)\ ;\ s_8^2 = (67, +047038011050, 3)$$

$$s_8 = (67, +0253803838049990551202593031 50, 5).$$

The set of spectra $(s_1, ..., s_8)$ represents multiple residue representation of the resulting spectra (Definition 2.3).

The standard residue representations of elements of the resulting sequence, i.e. of the spectral strips of the resulting spectrum are

(26, 24, 10, 22, 3, 45, 25, 59); (16, 28, 18, 21, 36, 0, 31, 19);

(2, 19, 33, 36, 18, 57, 21, 41); (10, 27, 20, 38, 44, 45, 60, 18);

(15, 22, 13, 28, 48, 28, 56, 47); (36, 21, 27, 44, 15, 19, 22, 1).

Using mixed-radix system with radices $m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8$ we get the following mixed-radix representations

32267366918700, 262090004150293, 21690559677534

31517669853715, 15285208740543, 2260488616557.

The resulting spectrum is (291818802686993,
022673669187002620900041502930216905596775340151766985317500152852208740543002260488616557, 15),
where $M = m_1 * m_2 * m_3 * m_4 * m_5 * m_6 * m_7 * m_8 = 391818026869993$. Converting the effective values of the spectral strips into corresponding fractions using module M we get the result:

$$[-2/17\ 1121/595\ 21464/7735\ 887/455\ 2323/182\ -15/52].$$

Example 4. In this example we compute the sum of the fractions

$$[218792/56217\ 253649/96371\ -51493/59143\ 676/1524] \quad \text{and}$$

$$[38025/8031\ -180252/60039\ 12059535/2601900\ 201/406653].$$

Using Algorithm $M3$, we get

$$N = 58444631, \quad \text{and}$$

$m1 = 79$, $m2 = 83$, $m3 = 89$, $m4 = 97$,
$m5 = 101$, $m6 = 103$, $m7 = 107$, $m8 = 109$.

According to Algorithm 02 we get the following standard residue representations of the resulting sequence:

$$(33, 19, 40, 77, 22, 49, 29, 29);\ (64, 3, 62, 51, 20, 77, 103, 75);$$

$$(64, 49, 1, 4, 28, 50, 91, 47); (57, 71, 60, 86, 4, 49, 52, 19).$$

Mixed-radix representations of the elements of the resulting sequence are:
41451243062919994 ; 6032934837430087 ;
15005082343384496 ; 6062299981932533334.

Accomplishing the conversion back using module $79 * 83 * 89 * 97 * 101 * 103 * 107 * 109 = 688087171373809$, we get the resulting sequence
[57778507/8810007 − 21638719/58444631
46359139/12315660 22933646/51644931].

6. Comparison of the algorithm $U1$ and algorithms 01, 02

The resulting sequence in Example 3 can be obtained after 12 multiplications and 11 additions. This way, we must apply Algorithm $U1$ 23 times. Also, each application of Algorithm $U1$ requires increase of the rhythm and the resulting fractions are not shortened. Entries of a new elements in the given fractions increase consequently. Applying Algorithm 02 we must perform only 8 multiplications, independently of the number of the elements of the initial sequences, and the rhythm does not increase.

The main advantage to multiple moduli arithmetic is that smaller moduli can be used at each repetition. Also, smaller rhythms and effective values of the spectral strips can be used in the residue representation of the resulting spectra.

We choose the number of moduli in a base vector in accordance with the computed value of N.

7. Applications

Defined mathematical spectra could be applied in exact computations with polynomials and vectors. Introduced operations are equivalent with the corresponding operations on polynomials. Several others applications are presented below.

7.1. Internal product of two vectors

Internal product of two sequences of integers $a = [a_1...a_n]$ and $b = [b_1...b_n]$, i.e. $\sum_{i=1}^{n} a_i * b_i$ is equal to the effective value of the central strip (i.e. n-th strip) of the spectrum which is obtained after multiplication of the spectrum of sequence a and the spectrum of the reversed sequence b ([6]). In this way, internal product of two sequences of fractions could be obtained as follows:

Algorithm A1.

Step 1. Compute product of spectrum of first sequence and the spectrum of reversed second sequence and the effective value into corresponding fraction with module (or moduli) used in Step 1.

7.2. Value of polynomial

Value of polynomial $P(x) = a_1x^n + ... + a_n$ could be obtained as the internal product of vectors $[a_1...a_n]$ and $[x^n...1]$, using Algorithm A1.

Example 5. Value of the polynomial $\frac{135}{7}x^3 - \frac{11}{5}x^2 + \frac{175}{2}x + \frac{3}{7}$ for $x = 1/2$ could be obtained using modules 59, 61, 71, 73, 79. The resulting sequence after multiplication of the sequences $\left[\frac{135}{7}\ \frac{11}{5}\ \frac{175}{2}\ \frac{3}{7}\right]$ and $\left[1\ \frac{1}{2}\ \frac{1}{4}\ \frac{1}{8}\right]$ is $\left[\frac{135}{7}\ \frac{521}{70}\ \frac{12771}{140}\ \frac{12891}{280}\ \frac{1527}{70}\ \frac{1237}{112}\ \frac{3}{56}\right]$. The value of the central strip, i.e. the value of the polynomial is $\frac{12891}{280}$.

References

[1] N. Szabo, R. Tanaka. *Residue Arithmetic and its Applications to Computer Technology.* McGraw-Hill, 1967.

[2] R. Gregory, E. Krishnamutthy. *Bezošibočnie Vičislenija, Metodi i Priloženija.* Moskva, Mir, 1988.

[3] K. Orlov. *Numerička Spektralna Aritmetika i Algebra.* Društvo Matematičara Fizičara i Astronoma SR Srbije, Beograd, 1973.

[4] K. Orlov. Nove računske operacije inspirisane teorijom matematičkih spektara. *Matematički Vesnik (Beograd)*, **5(20)**, 1968, 393-398.

[5] B. Mihajlović. The program of multiplication of two spectra in FORTRAN IV and its application on the solution of numerical algebraic and differential equations. *Matematički Vesnik (Beograd)*, **21**, 1969, 151-155.

[6] J. Madić. *Prilog Teoriji i Primeni Matematičkih Spektara*, Doktorska Teza, Filozofski Fakultet, Niš, 1987.

[7] Max Arie Wotulo. A new kind of irregular spectra. *Matematički Vesnik (Beograd)*, **8 (23)**, 357-362.

Faculty of Philosophy
Department of Mathematics
Ćirila i Metodija 2
18000 Niš
YUGOSLAVIA

Received 29.12.1992