Mathematica Balkanica

Mathematical Society of South-Eastern Europe
A quarterly published by
the Bulgarian Academy of Sciences – National Committee for Mathematics

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New Series Vol.11, 1997, Fasc. 1-2

Continuous Dependence of the Solution of a System of Differential Equations with Impulses on the Initial Condition and on the Right-Hand Side of the System [1]

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In the present paper an initial value problem is considered for systems of ordinary differential equations with impulses where the impulses are realised in the moments when the integral curve of the system meets some of the previously given hypersurfaces called impulse hypersurfaces. Definitions and sufficient conditions for the continuous dependence of the solution on the initial condition and on the right-hand side of the system are given. The results are applied to one mathematical model of biology.

1. Introduction.

In the present paper systems of differential equations with impulses are considered such that the inpulses are realised by the intersection of the integral curve of the system with some of the previously given hypersurfaces

$$\sigma_i : \; t = t_i(x), \quad i = 1, 2, \ldots, \tag{1}$$

where $t_i : D \mapsto \mathbb{R}^+$, D is domain in $\mathbb{R}^n$.

In the paper the following initial value problem of systems of differential equations with impulses is considered:

$$\dot{\mathbf{x}} = f(t, x), \quad t \neq \tau_i \tag{2}$$

$$\Delta x(t) \mid_{t=\tau_i} = x(\tau_i + 0) - x(\tau_i) = I j_i(x(\tau_i)), \; i = 1, 2, \ldots \tag{3}$$

[1] This research has been partially supported by the Bulgarian Ministry of Education and Science under Grant MM 422

with initial condition

$$x(\tau_0) = x_0, \tag{4}$$

where $f : \mathbb{R}^+ xD \mapsto \mathbb{R}^n; \mathbb{I}_i : D \mapsto \mathbb{R}^n$, $i = 1, 2, \ldots;$ $\tau_0 \geq 0;$ $x_0 \in D$.

Here and further on by

$$\tau_i,\ i = 1, 2, \ldots, \quad 0 < \tau_1 < \tau_2 < \ldots, \tag{5}$$

we denote the moments when the integral curve $(t, x(t))$ of the problem (2),(3),(4) meets some hypersurfaces from (1); j_i is the number of the hypersurface which is met by the integral curve in the moment τ_i.

The solution of problem (2),(3),(4) is determined in the following way:

a) for $\tau_0 \leq t \leq \tau_i$ it coincides with the solution of the problem (2),(4);

b) for $\tau_i < t \leq \tau_{i+1}$ it coincides with the solution of problem (2) with initial condition $x(\tau_i + 0) = x_i^+ = x_i + I j_i(x_i)$, $x_i = x(\tau_i)$, $i = 1, 2, \ldots;$

c) the integral curve $(t, x(t))$ of the problem (2),(3),(4) meets the hypersurfaces (1) only in the moments τ_i, which satisfy (5).

Remark 1. In general it is possible that $j_i \neq i$ (see Example 1, [4]).

2. Auxiliary results and notations

Further we use the following notations: $x_f(t; \tau_0, x_0)$ is the solution of the problem (2), (3), (4); $x_i = x_f(\tau_i; \tau_0, x_0)$; $x_i^+ = x_i + I_{j_i}(x_i)$; $\Omega_i = \{(t, x); t_{i-1}(x) < t < t_i(x), x \in D\}$; $i = 1, 2, \ldots,$ $t_0(x) = 0$. $x \in D$; $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^m$, $m = 2, 3, \ldots;$ $\rho(A, B)$ is the Euclidean distance between the non empty sets $A, B \subset \mathbb{R}^m$; $B_\varepsilon(x) = \{ x'; x' \in \mathbb{R}^m, \|x' - x\| < \varepsilon \}$, where $\varepsilon > 0$; $M(\tau_0, x_0, T, \varepsilon, f) = \{ (t, x); t \in [\tau_0 - \varepsilon, \tau_0 + T + \varepsilon], x \in B_\varepsilon(x_f(t; \tau_0, x_0)) \}$, where T is a positive constant or $+\infty$; $C[X, Y]$ is the set of all continuous functions $f : X \mapsto Y$ and $F(\tau_0, x_0, T, \varepsilon, f)$ is the set of all functions g such that:

1) $g \in C[M(\tau_0, x_0, T, \varepsilon, f), \mathbb{R}^n]$;
2) $g \in \mathrm{Lip}_x[K, M(\tau_0, x_0, T, \varepsilon, f)]$.

We denote by (A) the following conditions:

A1. $f \in \mathrm{Lip}_x[L, \mathbb{R}^+ x D]$;
A2. $\exists M = \mathrm{const} > 0 : ||f(t,x)|| \leq M$ for $(t,x) \in \mathbb{R}^+ x D$;
A3. For each point $(\tau_0, x_0) \in \mathbb{R}^+ x D$ the solution $x_f(t, \tau_0, x_0)$ of the problem (2), (4) (without impulses) does not leave the domain $D$for $t > \tau_0$;
A4. Functions $(I + I_i) : D \mapsto D,\ i = 1, 2, \ldots$, where I is the identity in $\mathbb{R}^n$;
A5. Functions $t_i \in \mathrm{Lip}_x[L_i, D],\ i = 1, 2, \ldots,\ L_i < 1/M$;
A6. $0 < t_1(x) < t_2(x) < \ldots$ for $x \in D$;
A7. $t_i(x + I_i(x)) \leq t_i(x)$ for $x \in D,\ i = 1, 2, \ldots$;
A8. Uniformly on $x \in D$ $\lim\limits_{i \mapsto +\infty} t_i(x) = +\infty$;
A9. Functions $I + I_i,\ i = 1, 2, \ldots$ defined in A4 are biuniques.

Further the following lemmas are used:

Lemma 1 [4]. *Let conditions A1 - A7 hold. Then for every point (τ_0, x_0) the integral curve of the problem (2), (3), (4) meets every one of the hypersurfaces not more than once.*

Lemma 2 [4]. *Let conditions A1 - A6 hold and let the point $(\tau', x_f(\tau'; \tau_0, x_0)) \in \Omega_i$. Then, if for $t > \tau_i'$ the integral curve of the problem (2), (3), (4) meets hypersurfaces of (1), then the first of them has a number not greater than i.*

Lemma 3 [5]. *Let conditions A1 - A6 hold and let the point $(\tau', x_f(\tau'; \tau_0, x_0)) \in \Omega_i,\ \tau' > \tau_0$. Then, the integral curve $(t', x_f(t'; \tau_0, x_0))$ of the problem (2),(3),(4) for $t > \tau'$ meets first the hypersurface σ_i.*

Lemma 4 [5]. *Let the conditions A hold. Then for any point $(\tau_0, x_0) \in \mathbb{R}^+ x D$ we have:*
i) the integral curve $(t', x_f(t'; \tau_0, x_0))$ meets infinitely many hypersurfaces from (1);
ii) if $\tau_1, \tau_2, \ldots$ are the moments of these meetings, then $\tau_i \mapsto +\infty$ for $i \mapsto \infty$;
iii) the solution of the problem (2),(3),(4) is unique and defined for all $t \geq \tau_0$.

Denote by (B) the following conditions:
B1. The functions $I_i,\ i = 1, 2, \ldots$ are continuous on D.
B2. The inequalities $\tau_i \neq t_k(x_i^+)$ hold for $i = 1, 2, \ldots$ and $k = 1, 2, \ldots$.
We shall use the following system:

$$\dot{x} = g(t, x),\ t \neq \tau_i',\ g \in C[\mathbb{R}^+ x D, \mathbb{R}^n] \tag{6}$$

(7) $$\Delta x(t)\,|_{t=\tau_i'} = I_{S_i}(x(\tau_i')),\ i=1,2,\dots$$

called a perturbation system.

Theorem 1. *Let conditions A1 - A6, B1 hold and* $0 < \tau_0 < t_1(x_0)$, *then* $(\forall \varepsilon > 0)(\exists \partial = \partial(\varepsilon, \tau_0, x_0) > 0)$ *such that*

$$(\forall(\tau_0', x_0') \in \mathbb{R}^+ x D,\ |\tau_0' - \tau_0| < \delta,\ ||x_0' - x_0|| < \delta)$$
$$(\forall g \in C\ [M(\tau_0, x_0, \infty, \varepsilon, f), \mathbb{R}^n],\ ||g(t,x) - f(t,x)|| < \delta,$$
$$(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f))$$

the following relations hold:

i) *The integral curve* $(t, x_g(t;\ \tau_0', x_0'))$, *where* $x_g(t;\ \tau_0', x_0')$ *is any solution of the system (6),(7), with initial condition* $x(\tau_0') = x_0'$ *meets for* $t > \tau_0'$ *first the hypersurface* σ_1 *in the moment* τ_1'(σ_1 *is the first hypersurface which is met by integral curve* $(t, x_f(t;\ \tau_0, x_0))$*when* $t > \tau_0$);

ii) $|\tau_1' - \tau_1| < \varepsilon$;

iii) $||x_1' - x_1|| < \varepsilon$ *where* $x_1' = x_g(\tau_1';\ \tau_0', x_0')$;

iv) $||x_1'^{+} - x_1^{+}|| < \varepsilon$ *where* $x_1'^{+} = x_1' + I_1(x_1')$;

v) $x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \infty, \varepsilon, f)$ *for* $\tau_0' \le t \le \max(\tau_1', \tau_1)$.

Proof.

i) In fact by Lemma 3 we see that the integral curve $(t, x_f(t;\ \tau_0, x_0))$ for $t > \tau_0$ meets first (in the moment τ_1) the hypersurfaces σ_1. Since $(\tau_0, x_0) \in \Omega_1$ and Ω_1 is an open set, then for $\delta = \delta(\tau_0, x_0)$ small enough, the including $(\tau_0', x_0') \in \Omega_1$ holds.

Let us assume that i) is not true. The following two possible cases will be considered.

1st case. The first hypersurface which is met by the integral curve $(t, x_g(t;\ \tau_0', x_0'))$ for $t > \tau_0'$ is σ_k, $k > 1$ and the meeting is realised in the moment τ_1'. We shall consider the continuous function $\varphi(t) = t_1(x_g(t;\ \tau_0', x_0')) - t$ for $\tau_0' \le t \le \tau_1'$. It can be easily seen that the inequalities $\varphi(\tau_0') > 0$ and $\varphi(\tau_1') < 0$ are satisfied. Hence there exists a point τ, $\tau_0' < \tau < \tau_1'$ such that $\varphi(t) = 0$, i.e. $t_1(x_g(\tau;\ \tau_0', x_0')) = \tau$. It means that the integral curve $(t, x_g(t;\ \tau_0', x_0'))$ meets the hypersurface σ_1 in the moment τ which contradicts the assumption.

2nd case. The integral curve $(t, x_g(t;\ \tau_0', x_0'))$ for $t > \tau_0'$ meets no hypersurface from (1). It means that

$$(t, x_g(t;\ \tau_0', x_0')) \in \Omega_1, \quad t > \tau_0'.$$

Let $D \neq \mathbb{R}^n$. We shall consider the sets $\mathbb{R}^n / D \neq \emptyset$ and $Q = \{x;\ x = x_f(t;\ \tau_0, x_0),\ \tau_0 \le t \le \tau_1\}$. The set $\mathbb{R}^n \setminus D$ is closed and the set Q is closed and bounded. It is clear that $(\mathbb{R}^n \setminus D) \cap Q = \emptyset$, so $\rho(\mathbb{R}^n \setminus D, Q) = r > 0$. If $D \equiv R^n$ we shall take for r whatever positive number. Let ε be a positive number and $\varepsilon < r(1 - ML_1)/4$. Having in mind (8) and the theorem for continuous dependence of the solution of a differential equation (without impulses) on the initial condition and on the right-hand side (further we shall call it theorem for continuous dependence) we find, that for $\delta = \delta(\varepsilon, \tau_0, x_0)$ small enough is satisfied

(10)

$$||x_g(t;\ \tau_0', x_0') - x_f(t;\ \tau_0, x_0)|| < \varepsilon < r(1 - ML_1)/4 < r, \max(\tau_0', \tau_0) \le t \le \tau_1.$$

The last inequatlity means, that the solution $x_g(t;\ \tau_0', x_0')$ belongs to D for $\tau_0' \le t \le \tau_1$. By inequalities (9) for $t = \tau_1$ we obtain

$$||x_g(\tau_1;\ \tau_0', x_0') - x_1|| < r(1 - ML_1)/4. \tag{11}$$

We shall show, that for all $t > \tau_1$, the following estimate is satisfied

$$||x_g(t; \tau_0', x_0') - x_1|| < r/2. \tag{12}$$

Let us assume the contrary, i.e. that there exists a point $\tau^* > \tau_1$ such that

$$||x_g(\tau^*;\ \tau_0', x_0') - x_1|| \ge r/2. \tag{13}$$

The point τ^* satisfies the inequality

$$\tau^* < t_1(x_g(\tau^*;\ \tau_0', x_0')). \tag{14}$$

By the condition A2 and the inequalities (8) we find the estimation

$$||g(t, x)|| \le M + \delta.$$

Using the last inequality, (11) and (14) we obtain the estimation

$$\begin{aligned}
&||x_g(\tau^*;\ \tau_0', x_0') - x_1|| \\
&\le ||x_g(\tau^*;\ \tau_0', x_0') - x_g(\tau_1;\ \tau_0', x_0')|| + ||x_g(\tau_1;\ \tau_0', x_0') - x_1|| \\
&\le (M + \delta)(\tau^* - \tau_1) + r(1 - ML_1)/4 \\
&\le (M + \delta)(t_1(x_g(\tau^*;\ \tau_0', x_0')) - t_1(x_1)) + r(1 - ML_1)/4 \\
&\le (M + \delta)L_1||x_g(\tau^*;\ \tau_0', x_0') - x_1|| + r(1 - ML_1)/4.
\end{aligned}$$

Thus we obtain the estimate

$$||x_g(\tau^*;\ \tau_0', x_0') - x_1|| \le \frac{r(1 - ML_1)}{4(1 - ML_1 - L_1\delta)}.$$

If $0 < \delta < (1 - ML_1)/(2L_1)$ is satisfied, then the last inequatility contradicts (13).

Hence for $t > \tau_1$ the estimation (12) holds. If $\tau^* > t_1(x)$, $x \in \overline{B}_{r/2}(x_1)$, then having in mind inequality (12) we deduce that the point $(\tau^*, x_g(\tau^*;\ \tau_0, x_0)) \notin \Omega_1$ which contradicts (9), so the case 2 is impossible, i.e. assertion i) is true.

ii) Let us consider the initial problems

$$x = f(t, x), \quad x(\tau_0) = x_0 \tag{15}$$

$$x = g(t, x), \quad x(\tau_0') = x_0' \tag{16}$$

with solutions respectively φ_f and φ_g. It is clear, that

$$\begin{aligned} \varphi_f(t) &= x_f(t;\ \tau_0, x_0), \quad \tau_0 \le t \le \tau_1, \\ \varphi_g(t) &= x_g(t;\ \tau_0, x_0), \quad \tau_0' \le t \le \tau_1'. \end{aligned} \tag{17}$$

The problem (2),(3),(4) turns into the problem (15) if $I_i(x) = 0$, $x \in D$, $i = 1, 2, \ldots$. Moreover for the problem (15) the condition A7 holds. Then using Lemma 1 we deduce that the integral curve $(t, \varphi_f(t))$ meets each of the hypersurfaces (1) at most once. Let the meetings be realised at the moments $\tau_{1\varphi}, \tau_{2\varphi} \ldots$. By (17) it is seen that $\tau_{1\varphi} = \tau_1$. So we obtain

$$(t, \varphi_f(t)) \in \begin{cases} \Omega_1, & \tau_0 \le t \le \tau_1 \\ \Omega_2, & \tau_1 < t < \tau_{2\varphi}. \end{cases}$$

Let τ' and τ'' are points satisfying the inequalities:

$$\tau_0 < \tau' < \tau_1, \quad \tau_1 < \tau'' < \tau_{2\varphi}, \quad \tau'' - \tau' < \varepsilon. \tag{18}$$

Hence $(\tau', \varphi_f(\tau')) \in \Omega_1$ and $(\tau'', \varphi_f(\tau'')) \in \Omega_2$. Having in mind the last two includings and the theorem for continuous dependence, we find that for small enough $\delta = \delta(\varepsilon, \tau_0, x_0) > 0$:

$$(\tau, \varphi_g(\tau)) \in \Omega_1, \ \tau_0' \le t \le \tau'; \ (\tau'', \varphi_g(\tau'')) \in \Omega_2. \tag{19}$$

From (19) and the second of inequalities (17), we get $\tau' < \tau_1' < \tau''$, and using that $\tau' < \tau_1 < \tau''$ we obtain $|\tau_1' - \tau_1| < \tau'' - \tau' < \varepsilon$; thus ii) is proved.

iii) Let the inequality $\tau_1' \ge \tau_1$ holds (the case $\tau_1'' < \tau_1$ is considered analogously). Then

$$\begin{aligned} ||x_1' - x_1|| &\le ||x_g(\tau_1';\ \tau_0', x_0') - x_g(\tau_1;\ \tau_0', x_0')|| + ||x_g(\tau_1;\ \tau_0', x_0') - x_1|| \\ &\le \int_{\tau_1}^{\tau_1'} ||g(\tau;\ x_g(\tau;\ \tau_0', x_0'))|| d\tau + \varepsilon \\ &\le (\tau_1' - \tau_1)(M + \delta) + \varepsilon \le \varepsilon(M + \delta + 1). \end{aligned}$$

iv) It follows immediately from condition B1 and iii).

v) The following four cases are possible:

$$\begin{aligned}
&v1)\ \tau_0' \le \tau_0, \quad \tau_1' \ge \tau_1;\\
&v2)\ \tau_0' \le \tau_0, \quad \tau_1' < \tau_1;\\
&v3)\ \tau_0' > \tau_0, \quad \tau_1' \ge \tau_1;\\
&v4)\ \tau_0' > \tau_0, \quad \tau_1' < \tau_1.
\end{aligned}$$

We shall consider only the case v1). The other cases are considered analogously. For δ small enough the estimation (10) holds which means that

$$x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \infty, \varepsilon, f),\ \tau_0 \le t \le \tau_1$$

For $\tau_0' \le t < \tau_0$ we obtain

$$\begin{aligned}
&||x_g(t; \tau_0', x_0') - x_0|| \le ||x_g(t;\ \tau_0', x_0') - x_0'|| + ||x_0' - x_0||\\
&\le \int_{\tau_0'}^{t} ||g(\tau, x_g(\tau;\ \tau_0', x_0'))||d\tau + \delta\\
&\le (t - \tau_0')(M + \delta) + \delta \le \delta(M + \delta + 1),
\end{aligned}$$

whence we find that for ∂ small enough:

$$||x_g(t;\ \tau_0', x_0') - x_0|| < \varepsilon, \quad \text{i.e.}$$

$$x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \infty, \varepsilon, f),\ \tau_0' \le t < \tau_0.$$

Finally, for $\tau_1 < t \le \tau_1'$ from the inequalities

$$\begin{aligned}
&||x_g(t;\ \tau_0', x_0') - x_f(\tau_1; \tau_0, x_0)||\\
&\le ||x_g(t; \tau_0', x_0') - x_g(\tau_1;\ \tau_0', x_0')|| + ||x_g(\tau_1; \tau_0', x_0') - x_f(\tau_1;\ \tau_0, x_0)||\\
&\le \int_{\tau_1}^{t} ||g(\tau;\ x_g(\tau;\ \tau_0', x_0'))||d\tau + \varepsilon_1\\
&\le (t - \tau_1)(M + \delta) + \varepsilon_1 \le \varepsilon_1(M + \delta + 1),
\end{aligned}$$

where ε_1 is so small, that $\varepsilon_1(M + \partial + 1) < \varepsilon$, we deduce that $x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \infty, \varepsilon, f)$, $\tau_1 < t \le \tau_1'$. Theorem 1 is proved. ■

Remark 2. Theorem 1 holds even if we change set $M(\tau_0, x_0, \infty, \varepsilon, f)$ with the set $M(\tau_0, x_0, \max(\tau_1' - \tau_0, \tau_1 - \tau_0), \varepsilon, f)$.

3. Main results

Definition 1. We shall say that the solution $x_f(t;\ \tau_0, x_0)$ of the problem (2),(3),(4) depends continuously on the initial condition, if

$$(\forall \varepsilon, \eta, T > 0)(\forall (\tau_0, x_0) \in \mathbb{R}^+ x D)(\exists \delta = \delta(\varepsilon, \eta, T, \tau_0, x_0) > 0)$$

$$(\forall (\tau_0', x_0'), (\tau_0', x_0') \in \mathbb{R}^+ x D \cap B_\delta(\tau_0, x_0)):\ \ ||x_f(t;\ \tau_0', x_0') - x_f(t;\ \tau_0', x_0)|| < \varepsilon$$

for $\tau_0 \le t \le \tau_0 + T$ and $|t - \tau_i| > \eta$, $i = 0,\ 1, \ldots$.

Definition 2. We shall say that the solution $x_f(t;\ \tau_0, x_0)$ of the problem (2),(3),(4) depends continuously on the right-hand side of the system, if

$$\begin{aligned}&(\forall \varepsilon, \eta, T > 0)(\forall (\tau_0, x_0) \in \mathbb{R}^+ x D)(\exists \delta = \delta(\varepsilon, \eta, T, \tau_0, x_0) > 0)\\ &(\forall g \in C[M(\tau_0, x_0, T, \varepsilon, f), \mathbb{R}^n],\ ||g(t,x) - f(t,x)|| < \delta\\ &\quad \text{for } (t,x) \in M(\tau_0, x_0, T, \varepsilon, f)):\\ &||x_g(t;\ \tau_0, x_0) - x_f(t;\ \tau_0, x_0)|| < \varepsilon \text{ for } \tau_0 \le t \le \tau_0 + T,\\ &|t - \tau_i| > \eta, \quad i = 1,\ 2, \ldots,\end{aligned}$$

where $x_g(t;\ \tau_0, x_0)$ is any of the solution of system (6),(7) with an initial condition $x(\tau_0) = x_0$.

Denote by (C) the following condition:

C1. $0 \in D$ and $f(t,0) = 0$ for $t \in \mathbb{R}^+$;

C2. $I_i(0) = 0$, $i = 1,\ 2, \ldots$.

R e m a r k 3. It is clear that if conditions (C) are satisfied, then for any $\tau_0 > 0$ the equality $x_f(t; \tau_0, 0) = 0$ holds for $t \ge \tau_0$. Moreover, if $\tau_0 < t_1(x_0)$, then the integral curve of the zero solution meets each of the hypersurfaces $\sigma_1,\ \sigma_2, \ldots$ just once. Further we shall use the following notations $\tau_i^0 = t_i(0)$, $i = 1,\ 2, \ldots$ i.e. $\tau_1^0,\ \tau_2^0, \ldots$ are the moments in which the intergral curve $(t, x_f(t; \tau_0, 0)) = (t, 0)$ meets respectively the hypersurfaces $\sigma_1, \sigma_2, \ldots$.

Definition 3. We shall say that the zero solution of system (2),(3) depends continuously on the initial condition, if

$$(\forall \varepsilon, T, \tau_0 > 0)\quad (\exists \delta = \delta(\varepsilon, T, \tau_0) > 0)\quad (\forall (\tau_0', x_0')\ (\tau_0', x_0') \in \mathbb{R}^+ x D \cap B_\delta(0,0)):$$

$$||x_f(t;\ \tau_0', x_0')|| < \varepsilon \quad \text{for} \quad \tau_0' \le t \le \tau_0 + T.$$

Definition 4. We shall say that the zero solution of system (2),(3) depends continuously on the right-hand side if $(\forall \varepsilon, T, \tau_0 > 0)(\exists \delta = \delta(\varepsilon, T, \tau_0) > 0)$,

$$\begin{aligned}&(\forall g \in C[M(\tau_0, 0, T, \varepsilon, f), \mathbb{R}^n],\ ||g(t,x) - f(t,x)|| < \delta,\\ &(t,x) \in M(\tau_0, 0, T, \varepsilon, f)):\ ||x_g(t;\ \tau_0, 0)|| < \varepsilon\end{aligned}$$

for $\tau_0 \leq t \leq \tau_0 + T$, where $x_g(t;\ \tau_0, 0)$ is any of the solutions of system (6),(7) with an initial condition $x(\tau_0) = 0$.

Theorem 2. *Let conditions (A) and (B) hold. Then the solution of the problem (2),(3),(4) depends continuously on the initial condition and on the right-hand side of the system.*

Proof. Let ε, η are positive numbers. Let $(\tau_0, x_0) \in \mathbb{R}^+ x D$. By condition A8 we deduce that there exists a number k such that $t_{k-1}(x_0) \leq \tau_0 < t_k(x_0)$. It is clear that the integral curve $(t, x_f(t;\ \tau_0, x_0))$ for $t > t_0$ does not meet the hypersurfaces $\sigma_1,\ \sigma_2, \ldots,\ \sigma_{k-1}$ so we shall change the number of hypersurfaces $\sigma_k,\ \sigma_{k+1}, \ldots$, respectively in $\sigma_1,\ \sigma_2, \ldots$. So we obtain the inequality $\tau_0 < t_1(x_0)$.

From Lemma 4 follows that the solution $x_f(t;\ \tau_0, x_0)$ is defined in the interval $[\tau_0,\ \tau_0 + T]$ and that there exists a number k such that $\tau_0 < \tau_1 < \ldots < \tau_{k-1} \leq \tau_0 + T < \tau_k$ which means that the integral curve $(t, x_f(t;\ \tau_0, x_0))$ meets finitely many hypersurfaces of (1) for $\tau_0 \leq t \leq \tau_0 + T$. Let us suppose that function $g:\ M(\tau_0, x_0, \infty, \varepsilon, f) \mapsto \mathbb{R}^n$ (not like in D2 $g):\ M(\tau_0, x_0, T, \varepsilon, f) \mapsto \mathbb{R}^n$). Let $\tau_1', \tau_2', \ldots$ be the moments in which the integral curve $(t, x_g(t;\ \tau_0', x_0'))$ meets hypersurfaces of (1), $x_i' = x_g(\tau_i', \tau_0', x_0')$, $x_i'^{+} = x_i' + I_{j_i}(x_i')$, $i = 1,\ 2, \ldots$. From Theorem 1 follows that there exists a constant $\delta_1 > 0$ such that if $|\tau_0' - \tau_0| < \delta_1$, $||x_0' - x_0|| < \delta_1$ and $||g(t,x) - f(t,x)|| < \delta_1$ for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$ and $g \in C[m(\tau_0, x_0, \infty, \varepsilon, f), \mathbb{R}^n]$, then:

1 a) The integral curve $(t, x_g(t;\ \tau_0', x_0'))$ meets for $t > \tau_0'$ first the hypersurface σ_1;

1 b) $|\tau_1' - \tau_1| < \delta_2 \leq \eta$;

1 c) $||x_1'^{+} - x_1^{+}|| < \delta_2$, where the constant δ_2 is defined further;

1 d) $x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \tau_1' - \tau_0, \varepsilon, f)$ for $\tau_0' \leq t \leq \tau_1'$.

From the theorem for continuous dependence it follows that we can choose δ_1 so small that

$$\text{1e)}\quad ||x_G(t; \tau_0', x_0') - x_f(t; \tau_0, x_0)|| < \varepsilon \text{ for } \max(\tau_0', \tau_0) \leq t \leq \min(\tau', \tau_1).$$

Having in mind condition B1, Lemma 3 and the fact that the integral curve $(t,\ x_f(t;\ \tau_0, x_0))$ for $t > \tau_1$ meets first hypersurface σ_{j2}, we find that the point $(\tau_1, x_1^{+}) \in \Omega_{j_2}$. Since Ω_{j_2} is an open set, using 1 b) and 1 c) we find that the constant δ_2 can be chosen so small, that

$$\text{1f)}\quad (\tau_1', x_1'^{+}) \in \Omega_{j_2}.$$

According the condition A5, $L_i < 1/M$, $i = 1,\ 2, \ldots$, if the constant δ_1 is small enough then:

$$\text{1g)}\quad L_1(M + \delta_1) < 1.$$

Assume that $\tau_2' = t_1(x_g(\tau_2';\ \tau_0', x_0'))$, then

$$\begin{aligned}\tau_2' - \tau_1' = \ & t_1(x_g(\tau_2';\ \tau_0', x_0')) - t_1(x_g(\tau_1';\ \tau_0', x_0')) \\ & \leq L_1 \| x_g(\tau_2';\ \tau_0', x_0') - x_g(\tau_1';\ \tau_0', x_0') \| \\ & \leq L_1(\tau_2' - \tau_1') \sup[\|g(t,x)\|;\ (t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)] \\ & \leq L_1(\tau_2' - \tau_1') \sup[\|g(t,x) - f(t,x)\| + \|f(t,x)\|; \\ & \quad (t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)] \\ & < \tau_2' - \tau_1'.\end{aligned}$$

The found condition means that the second hypersurface which is met by the integral curve $(t, x_g(t;\ \tau_0', x_0'))$ is not σ_1. Then from Theorem 1 follows that there exists a constant $\delta_2 > 0$ (the same which was used in 1 b) and 1 c)) small enough, such that, if $|\tau_1' - \tau_1| < \delta_2$, $\|x_1'^+ - x_1^+\| < \delta_2$ and $\|g(t,x) - f(t,x)\| < \delta_2$ for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$ and $g \in C[M(\tau_0, x_0, \infty, \varepsilon, f), \mathbb{R}^n]$, then:

2a) The integral curve $(t, x_g(t;\ \tau_0', x_0'^+))$, which for $t > \tau_1'$ coincides with the integral curve $(t, x_g(t;\ \tau_0', x_0'))$, meets for $t > \tau_1'$ first the hypersurface σ_{j_2} (the second hypersurface which is met by the integral curve $(t, x_f(t;\ \tau_0, x_0))$);

2b) $|\tau_2' - \tau_2| < \delta_3 \leq \eta$;

2c) $\|x_2'^+ - x_2^+\| < \delta_3$, where the constant δ_3 is defined further;

2d) $x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \tau_2' - \tau_0, \varepsilon, f)$ for $\tau_1' < t \leq \tau_2'$.

From theorem for continuous dependence we obtain that

2e) $\|x_g(t;\ \tau_0', x_0') - x_f(t;\ \tau_0, x_0)\| < \varepsilon$ for $\max(\tau_1', \tau_1) < t \leq \min(\tau_2', \tau_2)$.

For δ_3 small enough by 2b) and 2c) we find, that

2f) $(\tau_2', x_2'^+) \in \Omega_{j_3}$.

Moveover δ_2 can be chosen such that

2g) $L_2(M + \delta_2) < 1$.

Using induction we deduce that for all $i = 1, 2, \ldots, k$ there exists a constant δ_i small enough such that, if

$$|\tau_{i-1}' - \tau_{i-1}| < \delta_i,\ \|x_{i-1}'^+ - x_{i-1}^+\| < \delta_i \text{ and } \|g(t,x) - f(t,x)\| < \delta_i$$

for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$ and $g \in C[M(\tau_0, x_0, \infty, \varepsilon, f), \mathbb{R}^n]$, then:

ia) The integral curve $(t, x_g(t;\ \tau_0', x_0'))$ first meets for $t > \tau_{i-1}'$ the hypersurface σ_{j_i};

ib) $|\tau_i' - \tau_i| < \delta_{i+1} \leq \eta$;

ic) $||x_i'^{+} - x_i'|| < \delta_{i+1}$;

id) $x_g(t;\ \tau_0', x_0') \in M(\tau_0, x_0, \tau_i' - \tau_0, \varepsilon, f)$, $\tau_{i-1}' < t \leq \tau_i'$;

ie) $||x_g(t;\ \tau_0', x_0') - x_f(t;\ \tau_0, x_0)|| < \varepsilon$, $\max(\tau_{i-1}', \tau_{i-1}) < t \leq \min(\tau_i', \tau_i)$;

if) $(\tau_i', x_i'^{+}) \in \Omega_{j_{i+1}}$;

ig) $L_i(M + \delta_i) < 1$.

First we choose the constant δ_{k+1} such that $0 < \delta_{k+1} < \tau_k - \tau_0 - T$. After this we define the constant $\delta_k = \delta_k(\varepsilon, \eta, \delta_{k+1}])$ from the conditions ka) - kg). We choose $\delta_{k-1} = \delta_{k-1}(\varepsilon, \eta, \delta_k)$ from conditions (k-1)a) - (k-1)g) and so on.

Finally we choose δ_1 from the respective conditions.

Let $\delta = \min(\delta_1, \ldots, \delta_k)$, then by the inequalities ie), $i = 1, 2, \ldots, k$ follows that if $|\tau_0' - \tau_0| < \delta$, $||x_0' - x_0|| < \delta$ and $||g(t,x) - f(t,x)|| < \delta$ for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$ and $g \in C[M(\tau_0, x_0, \infty, \varepsilon, f), \mathbb{R}^n]$, then

$$||x_g(t;\ \tau_0', x_0') - x_f(t; \tau_0, x_0)|| < \varepsilon \text{ for } t \in A,$$

where $A = \{t; \max(\tau_{i-1}', \tau_i) < t \leq \min(\tau_i', \tau_i), i = 1, 2, \ldots, k, \tau_0' = \tau_0\}$. Having in mind that $A \supset \{t;\ \tau_0 < t < \tau_0 + T,\ |t - \tau_i| > \eta,\ i = 1, 2, \ldots\}$ we deduce that the solution $x_f(t;\ \tau_0, x_0)$ depends continuously on the initial condition and on the right-hand side of the system. The proof does not change essentially is the function $g(t,x)$ is defined in the set $M(\tau_0, x_0, T, \varepsilon, f)$. In this case instead of function $g(t,x)$ we are to consider the function

$$\bar{\mathbf{g}}(t,x) = \begin{cases} g(t,x), (t,x) \in M(\tau_0, x_0, T, \varepsilon, f); \\ f(t,x), (t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f); t \geq \tau_0 + T + h; \\ (1 - (t - \tau_0 + T)/h)\, g(\tau_0 + T, x_f(\tau_0 + T, \tau_0, x_0) \\ \quad + x - x_f(t;\ \tau_0, x_0)) + ((t - \tau_0 + T)/h)\, f(\tau_0 + T + h, \\ \quad x_f(\tau_0 + T + h, \tau_0, x_0) + x - x_f(t; \tau_0, x_0)), \\ \quad (t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f),\ \tau_0 + T < t < \tau_0 + T + h \end{cases}$$

where h is a positive constant and $h < \tau_{k+1} - T$. It is immediately seen that the function $\bar{\mathbf{g}}$ is a continuous extension of g in $M(\tau_0, x_0, \infty, \varepsilon, f)$. Moreover if the function g is "close" to f for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$, then for h small enough the function $\bar{\mathbf{g}}$ is "close" to f too for $(t,x) \in M(\tau_0, x_0, \infty, \varepsilon, f)$. Theorem 2 is proved.

■

Theorem 3. *Let conditions A and C hold, then the zero solution of system (2),(3) depends continuously on the initial condition and on the right-hand side of the system.*

Proof. Like in the proof of Theorem 1, after eventual re-numbering of the hypersurfaces, we can consider that $\tau_0 < t_1(0)$. Let $B_r(0) \subset D$. Let $\varepsilon < r$ and T be arbitrary positive numbers. The integral curve $(t, 0) = (t, x_f(t; \tau_0, 0))$ meets any of the hypersurfaces (1) just once. According to A8 the moments $\tau_1^0 = t_1(0), \tau_2^0 = t_2(0), \dots$ of this meeting satisfy $\tau_i^0 \mapsto \infty$ when $i \mapsto \infty$. It means that there exists a number k such that $\tau_{k-1}^0 \le \tau_0 + T < \tau_k^0$. Here as in the proof of Theorem 1 we suppose that the function $g \in C[M(\tau_0, 0, \infty, T, \varepsilon, f), \mathbb{R}^n]$. Let $x_g(t;\ \tau_0', x_0')$ be any solution of system (6),(7) with initial condition $x(\tau_0') = x_0'$. Let the integral curve of this solution meet the hypersurface (1) at the moments $\tau_1',\ \tau_2', \dots$. By Theorem 1 using induction, we obtain that for any $i = 1,\ 2, \dots,\ k$ and for each positive constant δ_{i+1} there exists a constant $\delta_i > 0$ such that if $|\tau_{i-1}' - \tau_{i-1}^0| < \delta_i$, $||x_{i-1}'^+|| = ||x_{i-1}' + I_{i-1}(x_{i-1}')|| < \delta_i$, $(\tau_{i-1}', x_{i-1}'^+) \in \Omega_i$, $L_{i-1}(M + \delta_{i-1}) < 1$, $||g(t,x) - f(t,x)|| < \delta_i$ for $(t,x) \in M(\tau_0, 0, \infty, \varepsilon, f)$ and $g \in C[m(\tau_0, 0, \infty, \varepsilon, f), \mathbb{R}^n]$, then:

ia) The integral curve $(t, x_g(t;\ \tau_0', x_0'))$ meets first for $t > \tau_{i-1}'$ the hypersurface σ_i;

ib) $|\tau_i' - \tau_i^0| < \delta_{i+1}$;

ic) $||x_i'^+|| < \delta_{i+1}$.

From the theorem for continuous dependence (fot system without impulses) it follows that

$$\text{id)}\quad ||x_g(t;\ \tau_0', x_0')|| < \varepsilon,\ \tau_{i-1}' < t \le \tau_i'.$$

By condition A7 we obtain $\tau_i' \le t_i(x_i'^+)$ which implies that

$$(\tau_i', x_i'^+) \in \bigcup_{k=1}^{\infty} \Omega_{i+k} \cup \sigma_i.$$

Using ib) and ic) we obtain the estimate $\rho((\tau_i', x_i'^+), \sigma_i) \le \rho((\tau_i', x_i'^+), (\tau_i^0, 0)) \le \sqrt{2}\delta_{i+1}$. From the last inequality and (20) it follows that for δ_{i+1} small enough

$$\text{ie)}\quad (\tau_i', x_i'^+) \in \Omega_{i+1}.$$

Because of A5,

$$\text{if)}\quad L_i(M + \delta_i) < 1.$$

First, set $\delta_{k+1} = \tau_k^0 - T - \tau_0$ (from inequality kb) and from this choise of δ_{k+1} we conclude that $\tau_k' > T + \tau_0$, i.e. the integral curve $(t, x_g(t; \tau_0', x_0'))$ meets for $t < T + \tau_0$ right $k-1$ hypersurfaces). Then we define consecutively the constants $\delta_k,\ \delta_{k-1}, \dots,\ \delta_1$ for $(t,x) \in M(\tau_0, 0, \infty, \varepsilon, f)$ and $g \in c[M(\tau_0, 0, \infty, \varepsilon, f), \mathbb{R}^n]$, then relations ia)-if) hold for each $i = 1,\ 2, \dots,\ k$.

The assertion of the theorem comes from the inequalities id) $i = 1, \dots, k$. Theorem 3 is proved.

■

4. Application

We shall apply the received results to one model of biology. Let us consider a system of two species "predator - pray". The fight between the species and the changes in the quantity in each species are given by the following system of differential equations

$$(21)\qquad \left|\begin{array}{l} \frac{dN_1}{dt} = \varepsilon_1(t)N_1 - \gamma_1(t)N_1N_2 - \gamma(t)N_1^2 \\ \frac{dN_2}{dt} = \gamma_2(t)N_1N_2 - \varepsilon_2(t)N_2, \end{array}\right.$$

where the functions $N_k(t), k = 1, 2$, describe the quantitative changes in the species: N_1 - of the pray and N_2 - of the predator; the functions $\varepsilon_k(t)$, $k = 1, 2$ are the coefficients of the growth of the species of the pray and the predator correspondingly; the functions $\gamma_k(t)$, $k = 1, 2$ are coefficients characterizing the interfaction between the species; the inner competition inside the pray species is expressed by the addend $\gamma(t)N_1^2$.

We shall assume that after influence form outside, the quantities of the two species change with jumps in the moments $\tau_1, \tau_2, \ldots$ which satisfy the equalities

(22)

$$\Delta N_k \mid_{t=\tau_i} = N_k(\tau_i + 0) - N_k(\tau_i) = \alpha_{ki}(N_k(\tau_i) - N_k^*),\ k = 1, 2 \quad i = 1, 2, \ldots,$$

where α_{ki} and N_k^* are real constants.

At this moment biomass quantitative is added or taken away both from the predator and from the pray.

The changes of biomass ΔN_1 and ΔN_2 of the pray and the predator respectively depend not only on the quantities of the species at the moments of influence but also on the moments of influence themselves.

We shall assume also that the moments of influence are these for which the points $(\tau_i, N_1(\tau_i), N_2(\tau_i))$ satisfy one of the following equations

$$(23)\qquad \chi_j + \beta_{1j}|N_1(t) - N_1^*| + \beta_{j2}|N_2(t) - N_2^*| = t,\ j = 1, 2, \ldots,$$

where χ_j, $\beta_{k_j} \in \mathbb{R}^+$, $k = 1, 2$.

The system of differential and algebraic equations (21)-(23) is a mathematical model of the biological process. The continuous dependence of the solution of the problem (21)-(23) on the initial condition and on the right-hand side is basic for the optimal yield of biomass.

If we equate the right-hand sides of the system (21) to zero, we obtain an algebraic system with a solution

$$(24)\qquad N_1^*(t) = \frac{\varepsilon_2(t)}{\gamma_2(t)},\ N_2^*(t) = \frac{\varepsilon_1(t)\,\gamma(t) - \varepsilon_2(t)\,\gamma(t)}{\gamma_1(t)\,\gamma(t)}.$$

The biologists usually assume that the quantitative changes of the species, the interfaction between the species and the inner competition are determined by the following functions: $\varepsilon_k(t) = e_k\Pi(t)$, $\gamma_k(t) = g_k\Pi(t)$ and $\gamma(t) = g\Pi(t)$, where e_k, g_k and g, $k = 1,\ 2$ are positive constants; function $\Pi \in C[\mathbb{R}^+, \mathbb{R}^+]$ and $\lim_{t\mapsto\infty} \Pi(t) = 1$, then turns (24) into

$$N_1^*(t) = \frac{e_2}{g_2},\ N_2^* = \frac{e_1g_2 - e_2g}{g_1g_2}.$$

In this way we find the constants N_1^* and N_2^* taking part in equalities (22) and (23). The point (N_1^*, N_2^*) is a fixed point for the sytem (21).

We shall assume also that $-1 < \alpha_{ki} \le 0$, $\beta_{ki} = \beta_k$, $\beta_k > 0, 0 < \chi_1 < \chi_2 < \ldots$, $\chi_i \mapsto \infty$ when $i \mapsto \infty$, $k = 1,\ 2, \ldots$, $i = 1,\ 2, \ldots$.

Now we are able to give new, easier to investigate form to problem (21),(22),(23):

$$(25)\qquad \frac{dN_1}{dt} = (r_1N_1 - g_1N_1N_2 - gN_1^2)\Pi(t)$$

$$(26)\qquad \frac{dN_2}{dt} = (g_2N_1N_2 - e_2N_2)\Pi(t)$$

$$(27)\qquad \Delta N_k\mid_{t=\tau_i} = \alpha_{ki}(N_k(\tau_i) - N_k^*),\ k = 1,\ 2,\ i = 1,\ 2, \ldots.$$

where τ_i, $i = 1,\ 2, \ldots$ are the moments in which the integral curve $(t, N_1(t), N_2(t))$ of the system (25),(26),(27) meets some of the hypersurfaces

$$\sigma_i : t = t_i(N_1, N_2) = \chi_i + \beta_1|N_1(t) - N_1^*| + \beta_2|N_2(t) - N_2^*|.$$

Further, by $N_k(t) = N_k(t;\ 0, N_1^0, N_2^0)$, $k = 1, 2$ we denote the solutions of system (25)-(27) with initial condition

$$(28)\qquad N_k(0) - N_1^0,\ k = 1,\ 2,$$

where the points N_1^0 and N_2^0 satisfy the inequalities

$$(29)\qquad N_k^0 > 0,\ k = 1,\ 2,\ N_1^0 < e_1/g.$$

First we show that the solutions of the problem (25)-(28) are boundedly limited. In fact, having in mind (29) we obtain that the right-hand side of (25) is negative, then from (27) follows that the solution $N_1(t)$ is bounded above, i.e.

$$N_1(t) < e_1/g, \; t \geq 0. \tag{30}$$

A particular case of (30) is the interquality

$$N_1(t;\; 0, N_1^*, N_2^*) < e_2/g_2 < e_1/g. \tag{31}$$

Assume that the solution $N_2(t)$ is not upper-bounded.

Let the constant $N \geq \max(e_1/g, N_2^*)$. Denote by $T_N, T_N > 0$ the first moment in which the solution $N_2(t+0) \geq N$ and by t_N, $0 \leq t_N \leq T_N$ denote the last moment in which $N_2(t) \leq \max(e_1/g, N_2^*)$. So for $t_N \leq t \leq T_N$ the inequalities $N_2(t) \geq \max(e_1/g_1, N_2^*)$ and $N_2(t) - N_2^* \geq 0$ hold. Then using (27) we find that for every moment τ_i, $t_N \leq \tau_i \leq T_N$

$$N_2(\tau_i + 0) \leq N_2(\tau_i).$$

From (26),(27) and (31) we obtain the estimate

$$N_2(t) \leq N_2(t_N)\exp(\int_{t_N}^{t}(g_2 N_1(\tau) - e_2)\Pi(\tau)d\tau), \; t_N \leq \tau \leq T_N \tag{32}$$

or

$$N_2(T_N) \leq N_2(t_N)\exp[\max_{t_N \leq t \leq \infty} \Pi(t) g_2(\frac{e_1}{g_1} - \frac{e_2}{g_2})(T_N - t_N)].$$

Solving the last inequality we find

$$T_N - t_N \geq \ln(\frac{N}{\max(\frac{e_1}{g_1}, N_2^*)})|(\max_{t_N \leq t < \infty} \Pi(t) g_2(\frac{e_1}{g_1} - \frac{e_2}{g_2})).$$

It is clear that $T_N - t_N \mapsto \infty$ if $N \mapsto \infty$. Let us suppose that for every constant $N > \max(\frac{e_1}{g}, N_2^*)$ for $t_N \leq t \leq T_N$ the inequality $N_1(t) > N_1^*$ is satisfied. Then from (25) and (27) we obtain

$$\frac{dN_1}{dt} \leq -g(N_1^*)^2\Pi(t), \; t_N \leq t \leq T_N$$

$$N_1(\tau_i + 0) \leq N_1(\tau_i), \; t_N \leq \tau_i \leq T_N.$$

The last two inequalities for N big enough (which means that the interval $[t_N, T_N]$ is big enough) contradicts the assumption. So, if the constant N is big enough, then exists a point θ_n, $t_N \leq \theta_n \leq T_N$ such that $N_1(t) < N_1^*$ for

$\theta_n \le t \le T_N$, or $g_2 N_1(t) - e_2 < 0$, $\theta_n \le t \le T_N$. But using (32) we obtain that for $\theta_n \le t \le T_N$ the solution $N_2(t)$ is monotonously increasing which contradicts the definition of the constant T_N. So it means that the solution $N_2(t)$ is upper-bounded. Now we are able to definite the domain D,

$$(N_1(t), N_2(t)) \in ((0, e_1/g)x(0, N)) \equiv D \subset \mathbb{R}^2, t \ge 0.$$

Denote by f_1 and f_2 respectively the right-hand sides of the equations (25) and (26), i.e.

$$\begin{aligned} f_1(t, N_1, N_2) &= (e_1 N_1 - g_1 N_1 N_2 - g N_1^2)\Pi(t) \\ f_2(t, N_1, N_2) &= (g_2 N_1 N_2 - e_2 N_2)\Pi(t). \end{aligned} \tag{33}$$

Since the function $\Pi(t)$ is continuous and limited for $t \ge 0$ and $(N_1, N_2) \subset D$ is a limited domain, then the function (33) and their first partial derivative

$$\frac{\partial f_1}{\partial N_1} = (e_1 - g_1 N_2 - 2g N_1)\Pi(t), \ \frac{\partial f_1}{\partial N_2} = -g_1 N_1 \Pi(t)$$
$$\frac{\partial f_2}{\partial N_2} = (g_2 N_1 - e_2)\Pi(t), \ \frac{\partial f_2}{\partial N_2} = -g_1 N_1 \Pi(t)$$

are continuous and limited. It means that the conditions A1 and A2 hold. Let $||f(t, N_1, N_2)|| \le M_1$, $(t, N_1, N_2) \in \mathbb{R}^+ x D$, $f = (f_1, f_2)$. Conditions A3 and A4 follow from the definition of the domain D. Condition A5 holds if $\beta_k M < 1$, $k = 1, 2$, conditions A6-A9 follow of determining from the constants χ_i, α_{ki} and β_{ki}, $k = 1, 2$, $i = 1, 2, \ldots$. So all the conditions of Theorem 3 are satisfied and it means that the solution of system (21) depends continuously on the initial condition and on the right-hand side of the system.

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Received 11.04.1995