Mathematica Balkanica

Mathematical Society of South-Eastern Europe
A quarterly published by
the Bulgarian Academy of Sciences – National Committee for Mathematics

Mathematica Balkanica

New Series Vol. 12, 1998, Fasc. 1-2

Izomorphizm of Functional Spaces

Mitrofan M. Choban

Presented by P. Kenderov

Let G be a linear topological space. Consider the functor F with the properties:

- for every space X the object $F(X)$ is a linear subspace of the linear space G^X and $\{f \in G^X : f \text{ is continuous and } cl_G f(X) \text{ is compact } \} \subseteq F(X)$;

- every linear space $F(X)$ is equipped with a concrete topological structure;

- for every continuous mapping $g : X \mapsto Y$ the mapping $\Phi_g : F(Y) \mapsto F(X)$, where $\Phi_g(f) = f\ g$ for every $f \in F(Y)$, is a continuous homomorphism;

- if Y is a closed countable subspace of a metric space X, then there exists a continuous linear extender $u : F(Y) \mapsto F(X)$.

The main results are connected with the questions:
1. When $F(X)$ and $F(Y)$ are linearly homeomorphic?
2. When $F(X)$ and $F(X+1)$ are linearly homeomorphic?
3. When $F(X)$ and $F(X+S)$ are linearly homeomorphic?
4. When $F(X)$ and $F(X+X)$ are linearly homeomorphic?
5. Let $P(X)$ be a linear subspace of a space $F(X)$. When $P(X)$ is linearly homeomorphic with $F(Y)$ for some space Y?

These questions for $F(X) = C^*_u(X)$ and $F(X) = C_p(X)$ were formulated by A. V. Arhangel'skii, A. Pelczynski and Z. Semadeni.

AMS Subj. Classification: 54XX (54A05,54A20,54C15), 46AXX

Key Words: linear topological spaces, topological structures, continuous homeomorphic mappings, functors, extenders

0. Introduction

The first attempts to solve the problem of the linear topological classification of Banach spaces $C_U(X)$ for compact spaces X go back to the works [19,23,54]. K. Borsuk [23] showed that $C_U(I) \sim C_U(I+S) \sim C_U(I+I) \sim C_U(I:x)$. In the proofs he has used the linear extension operators.

S. Banach [19] has proved that $C_U(S) \sim C_U(S:0) \sim C_U(S+S)$ and raised the question whether $C_U(I) \sim C_U(I^2)$. The Banach problem was solved by A.A. Miljutin in 1952. He proved that $C_U(X) \sim C_U(Y)$ for all non-countable metrizable compact spaces X and Y (see [94,107,117]). The crucial point in the Miljutin's proof is his lemma. The Miljutin's lemma was generalized by S.Z. Ditor [46]. The classification of Banach spaces $C_U(X)$ for countable compact spaces X was given by C. Besaga and A. Pelczynski (see [107,117]).

The problem of the linear topological classification of Banach spaces $C_U(X)$ for compact spaces X remains open and seems to be very complex. Moreover, the study of this problem has given the possibility to formulate some new interesting and natural questions.

In the 1970's A.V. Arhangel'skii has begun the systemical investigation of spaces $C_p(X)$ with the topology generated by pointwise convergence (see [7,9,10,17]).

In virtue of A.V. Arhangel'skii's [6] and S.D. Pavlovskii's [104] theorem the C_p-classification of compact spaces is more fine than its C_u-classification. Therefore in the C_p-theory we have more various problems. In particular, the problems of classification of spaces $C_p(X)$ for X metrizable separable and X metrizable compact spaces are open. Essential results in that direction were obtained in [16,17,47,64,103,124,125].

The profile of this paper was determined in 1987-1993 due to enjoyble and fruitful contacts established at the Universities of Moscow, Munchen, Tartu, Tbilisi and the Mathematical Institute of Bulgarian Acadamy of Sciences in Sofia. Conversations with Professors A.V. Arhangel'skii, W. Roelcke, S. Nedev, P. Kenderov, M. Abel and L. Zambahidze have been of great value to me. I would like to express my deep gratitude to everybody by whom I was influenced.

1. Basic Definitions

1. We consider only completely regular spaces. We shall use the notation and terminology from [43,53,63,85,117]. In particular, βX is the Stone-Čech compactification of the space X, νX is the Hewitt real compactification of the space X, ωX is the weight of the space X, clA or $cl_X A$ denotes the closure of a set A in X, $|X|$ is the cardinality of X, $N = \{0, 1, 2, \ldots\}$ is a discrete space of the natural numbers, the symbol R will denote the topological field of real numbers. By $X + Y$ we denote the topological sum of disjoint spaces X and Y.

2. The topological sum of the spaces $\{E_n : n \in N\}$ is denoted by E and is called a signature of a set of fundamental operations. For every $n \in N$ the set E_n is a set of operations of type n.

We say that an E-algebra or an algebra G of a signature E is given, if the set G is non-empty and there are mappings $\{e_{nG} : E_n \times G^n \mapsto G : n \in N\}$. The mappings $\{e_{nG} : n \in N\}$ are called the structure of the E-algebra in the set G.

Subalgebras, homomorphism, isomorphism and Cartesian products of E-algebras are defined as in [21,26,33,63].

An E-algebra G together with a given topology on it is called a topological E-algebra, if all the mappings e_{n_G} are continuous.

A topological product of topological E-algebras is a topological E-algebra (see [21,26]).

3. Fix a topological E-algebra G. Let $B(X,G)$ be the space of all mappings of a space X in $G, C(X,G)$ be the space of all continuous mappings of a space X in G, $B^*(X,G) = \{f \in B(X,G) : \ cl_g f(X) \text{ is compact}\}$ be the space of all bounded mappings of X in G and $C^*(X,G) = C(X,R) \cap B^*(X,R)$. The sets $C^*(X,G)$, $C(X,G)$ and $B^*(X,G)$ are E-subalgebras of the E-algebra $B(X,G) = G^X$.

If $Q \subset B(X,G)$, then T_Q is the topology on X generated by Q and it has a base consisting of all sets of the form $\cap\{f_i^{-1}U_i : \ i = 1,2,\ldots,n\}$, where $n \in N$, $f_1,\ldots,f_n \in Q$ and $U_1,\ldots,U_n$ are open subsets of G.

Any mapping $\psi : \ X \mapsto Y$ induces the homomorphism $\Phi_\psi : \ B(Y,G) \mapsto B(X,G)$ defined by letting $\Phi_\psi(f) = \psi f$ for $f \in B(Y,G)$.

The set $B(X,G)$ naturally can be provided with various algebrical and topological structures:

- the discrete topology;
- the topology of pointwise convergence. In that topology $B(X,G) = G^X$;
- if $E' \subseteq E$, then every E-algebra is an E'-algebra too. Therefore every $B(X,G)$ admits the structure of an E'-algebra for each subset E' of E.

4. The topological E-group is a topological E-algebra G for which:

a^0. there exist the operations $0 \in E_0$, $- \in E_1$, $+ \in E_2$ such that relatively to the operations $\{0,-,+\}$ the space G is a topological group with the neutral element 0;

b^0. if $n \geq 1$ and $\omega \in E_n$, then $e_{n_G}(\omega,0,\ldots,0) = 0$;

c^0. $e_{0_G}(E_0 \times G^0) = \{0\}$;

d^0. if $\omega \in E_n$, $n \geq 1$ and $x_1, y_1, \ldots, x_n, y_n \in G$, then
$e_{n_G}(\omega, x_1 + y_1, \ldots, x_n + y_n) = e_{n_G}(\omega, x_1, \ldots, x_n) + e_{n_G}(\omega, y_1, \ldots, y_n)$.

Remark 1.1. If G is a topological E-group with a neutral element 0, then $\{0\}$ is a subalgebra of G and an E-group.

Remark 1.2. Let G be an E-algebra. Then:

- if $\omega \in E_0$, then $0_\omega = e_{0_G}(\{\omega\} \times G_0)$;
- if $n \geq 1$ and $\omega \in E_n$, then $e_{n_G}(\omega, x_1, \ldots, x_n) = \omega(x_1, \ldots, x_n)$.

Example 1.3. Every linear topological space is a topological E-group for $E = \{0,-,+\} + P$, $E_0 = \{0\}$, $E_1 = \{-\} + P$, $E_2 = \{+,\}$, where P is a scalar field.

2. Functional functors

Fix a signature E and a topological E-group G.

Definition 2.1. By a function G-functor we mean a mapping F that associates with each space X some E-subalgebra $F(X)$ of $B(X, G)$ satisfying the following conditions:

1. $C^*(X,G) \subseteq F(X)$.
2. Every $F(X)$ is equipped with a concrete topological structure.
3. For every space X the space $F(X)$ is a topological E-group.
4. For every continuous mapping $\phi : X \mapsto Y$ we have $\Phi_\phi(F(\phi)) \subseteq$, where $\Phi_\phi(f) = f_\phi$ for every $f \in F(Y)$, and $\Phi_\phi : (F(\phi)) \mapsto F(X)$ is a continuous homomorphism.

Every functional G-functor F is contravariant. Each functional G-functor is a Banach-Stone functor in sence of A. Pelczynski [107].

Fix a functional G-functor F.

Definition 2.2. Let Y be a subspace of a space X. An extension F-operator or an F-extender is a continuous homomorphism $u : F(Y) \mapsto F(X)$ such that for each $g \in F(Y)$ the mapping $u(g)$ is an extension of g, i.e. $g = u(g)|Y$.

Lemma 2.3. *If $\phi : X \mapsto Y$ is a retraction , then $\Phi_\phi : F(Y) \mapsto F(X)$ is an F-extender.*

Definition 2.4. Let $\phi : X \mapsto Y$ be a continuous mapping of X onto Y. An F-averaging operator is a continuous homomorphism $u : F(X) \mapsto F(Y)$ such that $u(\Phi_\phi(g)) = g$ for each $g \in F(Y)$.

Definition 2.5. The functor F is called a Dugundji G-functor, if F satisfies the following conditions:

1. F is a functional G-functor.
2. For every closed countable subspace Y of a metric space X there exists an F-extender $u : F(Y) \mapsto F(X)$.

3. On complimented algebras

Fix a signature E and an E-topological group H.

Definition 3.1. The subalgebra H_1 of H is complimented in H if there exists a subalgebra H_2 of H such that the continuous homomorphism $\omega : H_1 \times H_2 \mapsto H$, where $\omega(x, y) = x + y$ for every $(x, y) \in H_1 \times H_2$, is a homeomorphism of $H_1 \times H_2$ onto H. The subalgebra H_2 is called a complement to H_1.

Lemma 3.2. *If H_1 has a complement H_2 in H, then H and $H_1 H_2$ are topological isomorphic E-groups.*

Definition 3.3. Let H_1 be a subalgebra of H. The continuous homomorphism $u : H \mapsto H_1$ is a projection, if $u(x) = x$ for every $x \in H_1$.

Every projection is an open mapping.

Proposition 3.4. *If the subalgebra H_1 is complement in H, then there exists a projection from H onto H_1.*

Proof. Let H_1 have a complement H_2 in H. Consider a topological isomorphism ω : $H_1 \times H_2 \mapsto H_1$, where $\omega(x, y) = x + y$, and a continuous homomorphism $u : H_1 \times H_2 \mapsto H_1$, where $w(x, y) = x$. Then u : $H \mapsto H_1$, where $u(x) = u(\omega^{-1}(x))$, is a projection. ■

Proposition 3.5. *Let H' and H'' be the complement of subalgebra H_1 in H. Then H' and H'' are topologically isomorphic algebras.*

Proof. There exist the projections $u : H \mapsto H'$ and ν : $H \mapsto H''$ such that $u^{-1}(0) = \nu^{-1}(0) = H_1$. Then $\omega = u|H''$: $H'' \mapsto H'$ is a topological isomorphism of H'' onto H'. ■

Definition 3.6. The E-topological group H is direct decomposition of E-subgroups $H_1, H_2, \ldots, H_n$ if the continuous homomorphism ω : $H_1 \times H_2 \times \ldots \times H_n \mapsto H$, where $\omega(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \ldots + x_n$, is a topological isomorphism.

4. Main properties of extenders

Fix a signature E, a topological E-group G and a functional G-functor F.

If Y is a subspace of a space X, then $F(X : Y) = \{f \in F(X) : Y \subseteq f^{-1}(0)\}$, where 0 is the neutral element of G.

Lemma 4.1. *$F(X : Y)$ is an E-subalgebra of the E-algebra $F(X)$.*

Proof. Follows from Remark 1.1 and the definition of E-group. ■

Theorem 4.2. *Let Y be a subspace X and u : $F(Y) \mapsto F(X)$ be an F-extender. Then $F(X)$ and $F(Y) \times F(X : Y)$ are the topologically isomorphic algebras and $F(X : Y)$ is closed in $F(X)$.*

Proof. If $\alpha \in G$, then we put $\alpha_X(x) = \alpha$ for every $x \in X$. We have $u(O_Y) = O_X$.

Consider the continuous homomorphism s_Y : $F(X) \mapsto F(Y)$, where $s_Y(f) = f \;\; Y$ for every $f \in F(X)$. It is obvious that $S_Y^{-1}(O_Y)) = F(X : Y)$. Therefore, $F(X : Y)$ is closed in $F(X)$.

For every $f \in F(Y)$ and $g \in F(X : Y)$ we put $\omega(f, g) = u(f) + g$. Then $\omega : F(Y) F(X : Y) \mapsto F(X)$ is a continuous homomorphism. By construction, $\omega^{-1}(f) = (s_Y(f), f - u(s_Y(f)))$. Therefore $\omega^{-1} = F(X) \mapsto F(Y) F(X : Y)$ is a continuous mapping and ω is a topological isomorphism. ■

Corollary 4.3. *Let $Y \subseteq X$ and $r : X \mapsto Y$ be a retraction. Then $F(Y) \times F(X : Y)$ and $F(X)$ are topologically isomorphic E-groups.*

Corollary 4.4. *Let $X = Y + Z$. Then $F(X)$ and $F(Y) \times F(Z)$ are topologically isomorphic E-groups.*

Corollary 4.5. *Let $Y \subseteq X$ and $u : F(Y) \mapsto F(X)$ be an F-extender. Then u is an embedding isomorphism of $F(Y)$ into $F(X)$ and $u(F(Y))$ is a complement of $F(X : Y)$ in $F(X)$.*

Remark 4.6. In the case when $F(X)$ denotes the Banach space of all bounded continuous real or complex valued functions on X with supremum norm the notions of projection and extender have been used by different authors [1,2,3,4,5,11,12,13,14,15,18,20,25,30,42,44,45, 48,61,70,71,72,73,83,86,88,91,92, 93,95,105,106,107,109,114,116,117, 124,125,127,129]. For more information and further references see Z. Semadeni [117], A. Pelczynski [107] and A.V. Arhangel's-kii and M.M. Choban [11-15].

5. On one-point subspaces

Fix the signature E, a topological E-group G and a functional G-functor F.

Denote by D_τ the discrete space of cardinality $\tau \geq 1$ and $1 = D_1$. Let $x \in X$. We put $F(X : x) = F(X : \{x\}) = \{f \in F(X) : f(x) = 0\}$ and $K(X, G) = \{f \in B(X, G) : f$ is a constant $\}$. By construction $K(X, G) \subset C^*(X, G) \subset F(X)$. By $FK(X)$ we denote the set $K(X, G)$ with the subspace topology of the space $F(X)$.

Proposition 5.1. *Let $x, y \in X$. Then:*
1. $F(X : x)$ and $F(Y : y)$ are topologically isomorphic.
2. $F(X)$ and $F(X : x) \times FK(X)$ are topologically isomorphic.

Proof. Let $\phi : X \mapsto \{x\}$ be a retraction. Then $\Phi_\phi : F(\{x\}) \mapsto F(X)$ is an F-extender and $\Phi_\phi(F(\{x\})) = FK(X)$. Therefore $FK(X)$ is a complement of $F(X : x)$ for every $x \in X$. Lemma 3.2 and Proposition 3.5 complete the proof. ■

Corollary 5.2. *The subalgebra $F(X : Y)$ is closed in $F(X)$ for every subspace Y of X.*

Proof. We have $F(X:Y) = \cap\{F(X:y): y \in Y\}$. From Theorem 4.2 the sets $F(X:y)$ are closed in $F(X)$. The proof is complete. ∎

Corollary 5.3. *For every point $x \in X$ the E-groups $F(X)$ and $F(X:x) \times F(1)$ are topologically isomorphic.*

Remark 5.4. The mapping $u: F(1) \mapsto G$, where $u^{-1}(\alpha)(1) = \alpha$ for every $\alpha \in G$, is an isomorphism. The isomorphism u is not obligatory continuous or topological. Therefore $F(1)$ is the E-algebra G with another topology on it.

6. On finite subspaces

Let G be a linear topological space over the field P of real or complex numbers.

Fix a functional G-functor F. Then $F(X)$ is a linear topological space for every space X.

Lemma 6.1. *If G is a finite-dimensional space and X is a finite space, then $F(X) = G^X$.*

Proof. In this case $F(X)$ is a finite-dimensional linear space. Therefore on $F(X)$ there exists a unique separated topology and $F(X) = G^X$. ∎

Theorem 6.2. *Let G be a finite-dimensional space, Y and Z be finite subspaces of a space X and $|Y| = |Z|$. Then:*
1. The linear topological spaces $F(X:Y)$ and $F(X:Z)$ are topologically isomorphic.
2. There are F-extenders $\nu: F(Y) \mapsto F(X)$ and $\omega: F(Z) \mapsto F(X)$ such that $\nu(F(Y)) = \omega(F(Z))$.

Proof. Let $|Y| = |Z| = n \geq 1$ and $|Y \cap Z| = m$. Then we consider $Y = \{y_1, y_2, \dots, y_n\}$ and $Z = \{z_1, z_2, \dots, z_n\}$, where $y_i = z_i$ for every $i \leq m$. Then there exist the open in X sets $U_1, U_2, \dots, U_n$ and the continuous functions $\{h_i : X \mapsto [0,1] : i \leq n\}$ for which:
1. $U_i \cap U_j = \emptyset$ if $i \neq j$.
2. $\{y_i, z_i\} \subseteq U_i$ for every $i \leq n$.
3. $h_i(y_i) = h_i(z_i) = 1$ and $X/U_i \subseteq h_i^{-1}(0)$ for every $i \leq n$.

Consider the homeomorphism $\phi: Y \mapsto Z$, where $\phi(y_i) = z_i$. Then $\Phi_\phi: F(Y) \mapsto F(Z)$ is a topological isomorphism.

For every $f \in F(Y)$ and $g \in F(Z)$ we put $\nu(f) = \sum\{f(y_i).h_i : i \leq n\}$ and $\omega(g) = \sum\{f(z_i).h_i : i \leq n\}$. Then, by construction, we have:
4. ν and ω are F-extenders.
5. If $\Phi_\phi(f) = g$, then $\nu(f) = \omega(g)$.

The assertion 2 is proved.

By construction, $F(X:Y)$ and $F(X:Z)$ are complements of the finite-dimensional linear subspace $\nu(F(Y)) = \omega(F(Z))$. Proposition 3.5 completes the proof. ∎

Proposition 6.3. *Let G be a direct decomposition of the linear subspaces $G_1, G_2, \dots, G_n$. Then for every G-functor F and every $i \leq n$ there exists the G_i-functor F_i such that:*
1. For every space X and $i \leq n$ there exists an open continuous linear mapping $u_i : F(X) \mapsto F_i(X)$.
2. $F(X)$ is a subspace of a space $F_1(X) \times F_2(X) \times \dots \times F_n(X)$.
3. If $Y \subseteq X$ and $\nu : F(Y) \mapsto F(X)$ is an F-extender, then for every $i \leq n$ there exists an F_i-extender $\nu_i : F_i(Y) \mapsto F_i(X)$.

Proof. Fix $i \leq n$. Denote $L_i = \{x_1 + \dots + x_{i-1} + x_{i+1} + \dots + x_n : x_j \in G_j,\ j \neq i\}$. Then L_i is a complement of G_i and there exists a projection $\omega_i : G \mapsto G_i$ for which $\omega_i^{-1}(0) = L_i$. For every mapping $f : X \mapsto G$ we put $u_i(f) = \omega_i f$. Since $G_i \subseteq G$ we have $u_i(B^*(X,G)) = B^*(X,G_i)$ and $u_i(C(X,G)) = C(X,G_i)$. Denote $F_i(X) = u_i(F(X))$. On $F_i(X)$ we consider the quotient topology. Then $u_i : F(X) \mapsto F_i(X)$ is an open continuous linear mapping. From construction $F_i(X) \subseteq B(X,G_i) \subseteq B(X,G)$. If $f \in F(X)$, then $f = u_1(f) + \dots + u_n(f)$ and this decomposition is unique. The mapping $u : F(X) \mapsto F_1(X) \times \dots \times F_n(X)$, where $u(f) = (u_1(f) + \dots + u_n(f))$ is an isomorphic embedding. If W is an open subset of the space $F(X)$, then $u(W) = u(F(X)) \cap (u_1(W) \times \dots \times u_n(W))$. If $i \leq n$ and W_i is open in $F_i(X)$ then $u^{-1}(W_1 \times \dots \times W_n) = u^{-1}(W_1) \cap \dots \cap u^{-1}(W_n)$. If $Y \subseteq X$ and $\nu : F(Y) \mapsto F(X)$ is an F-extender, then $\nu_i = u_i \nu$ is an F-extender. The proof is complete. ∎

Definition 6.4. The G-functor is called homogeneous if for every topological isomorphism $\phi : G \mapsto G$ and every space X we have $\{\phi f : f \in F(X)\} = F(X)$.

Proposition 6.5. *Let $n \geq 1$ and Φ be a homogeneous G^n-functor. Then there exists a homogeneous G-functor F such that $\Phi(X) \subseteq F(X)^n$ for every space X.*

Proof. Consider the projections $\omega_i : G^n \mapsto G$, where $\omega_i(x_1, \dots, x_n) = x_i$. For every mapping $f : X \mapsto G^n$ we put $u_i(f) = \omega_i f$. Denote $F_i(X) = u_i(\Phi(X))$. From Proposition 6.3 F_i is a G-extender. Since Φ is homogeneous, then $F_1(X) = F_2(X) = \dots = F_n(X) = F(X)$ and $F(X)$ is a homogeneous G-functor. By construction $\Phi(X) \subseteq F(X)^n = F_1(X) \dots F_n(X)$. The proof is complete. ∎

Theorem 6.6. *Let F be an P-functor, i.e. $G = P$, X be a space, $F(X)$ be a direct decomposition of subspaces L and H and H be isomorphic with P^n*

for some $n \geq 1$. Then there exists a finite subspace Y of X such that:

1. *$Y = \{y_1, y_2, \ldots, y_n\}$.*
2. *There exists an F-extender $u : F(Y) \mapsto F(X)$ such that $u(F(Y)) = H$.*
3. *L is topologically isomorphic to $F(X : Y)$.*

P r o o f. There exists a Hamel base $f_1, f_2, \ldots, f_n$ in H. By induction on $i \leq n$ we construct the point y_i and the function $g_i \in H$ such that:

C1. $f_1(y_1) \neq 0, g_1 = a_{11} f_1,\ g_1(y_1) = 1$

C2. $g_2 = a_{21} g_1 + a_{22} f_2,\ g_2(y_1) = 0, g_2(y_2) = 1$

Ci. $g_i = a_{i1} g_1 + \ldots + a_{i(i-1)} g_{i-1} + a_{ii} f_i,\ g_i(y_1) = 0, g_i(y_2) = 0, \ \ldots,\ g_i(y_{i-1})$ $0,\ g_i(y_i) = 1$.

We put $h_n = g_n, h_{n-1} = g_{n-1} - g_{n-1}(y_n) h_n, \ \ldots, h_1 = g_1 - g_1(y_2) h_2 - \ldots - g_1(y_n) h_n$. In general, $h_i = g_i - g_i(y_{i+1}) h_{i+1} - \ldots - g_i(y_n) h_n$. The functions $h_i, h_2, \ldots, h_n$ form a Hamel basis in H, $h_i(y_i) = 1$ and $h_i(y_i) = 0$ for every $i, j \leq n$ and $i \neq j$.

The mapping $u : F(Y) \mapsto F(X)$, where $u(f) = f(y_1)h_1 + \ldots + f(y_n)h_n$ for f in H, is an F-extender and $u(F(Y)) = H$. The assertions 1 and 2 are proved. The assertion 3 follows from the assertion 2, Corollary 4.5 and Proposition 3.5. ■

R e m a r k 6.7. When $F(X)$ is the Banach space $C^*(X, R)$ the assertion 3 of Theorem 6.6 is proved in [44], Theorem 3.

E x a m p l e 6.8. Let $G = R^2$. Consider the functor F, where $F(X)$ is the Banach space $C^*(X, G)$ of bounded mappings of X to R^2 with the sup-norm. Fix an infinite space X, the two-point subspace $Y = \{y_1, y_2\} \subset X$ and the functions $h_1, h_2 \in F(X)$ for which:

1. $h_1(y_1) = h_2(y_2) = (1, 0)$
2. $h_1(y_2) = h_2(y_1) = (0, 0)$
3. $h_1(X), h_2(X) \subseteq R \times \{0\}$.

Denote $H = \{\alpha h_1 + \beta h_2 :\ \alpha, \beta \in R\}$. Then no subset Z of X and F-extender $u :\ F(Z) \mapsto F(X)$ for which $u(F(Z)) = H$ exist.

Theorem 6.9. *Let $m \geq 1$, $G = P^m$, F be a P-functor and $\Phi(X) = F(X)^m$ for every space X. Then:*

1. *Φ is a G-functor.*
2. *For every space X we have $\Phi(X) = F(X \times D_m)$.*
3. *If $\Phi(X)$ is a direct decomposition of subspaces L and H, $n \geq 1$ and H is isomorphic with G^n, then there exists a finite subspace $Y = \{y_1, \ldots, y_n\}$ of X such that L is topologically isomorphic to $\Phi(X : Y)$.*

P r o o f. The assertion 1 is obvious. The assertion 2 follows from Corollary 4.4.

By Theorem 6.6 there exists a subset $Z = \{z_i : i \leq mn\}$ of $X \times D_m$ such that L is topologically isomorphic to $F(X \times D_m : Z)$. Fix a subset $Y = \{y_1, \ldots, y_n\}$ of X. By Theorem 6.2 the subspace $F(X \times D_m : Z)$ and $F(X \times D_m : Y \times D_m)$ are topologically isomorphic. It is obvious that $\Phi(X : Y)$ is topologically isomorphic to $F(X \times D_m : Y \times D_m)$. The proof is complete. ∎

Theorem 6.10 *Let G be a linear topological space, F be a G-functor and for every space X the topology of uniform convergence on $F(X)$ is finer than the topology on $F(X)$. Let Y and Z be the finite subspaces of a space X and $|Y| = |Z|$. Then:*

1. The linear topological spaces $F(X : Y)$ and $F(X : Z)$ are topologically isomorphic.

2. There exist the F-extenders $\nu : F(Y) \mapsto F(X)$ and $\omega : F(Z) \mapsto F(X)$ such that $\nu(F(Y)) = \omega(F(Z))$.

Proof. See the proof of Theorem 6.2. ∎

7. Examples of functors

Fix a signature E and arcwise connected topological E-group G.

Let X be a space. We put $B_0(X, G) = C(X, G)$. Inductively define the Baire class $B_\alpha(X, G)$ for each ordinal $\alpha \leq \Omega$, where Ω is the first uncountable ordinal, to be the space of pointwise limits of sequences of mappings in $\cup\{B_\mu(X, G) : \mu < \alpha\}$. Denote $B^*_\alpha(X, G) = B_\alpha(X, G) \cap B^*(X, G)$ (see [18,26,27,31,34,35,36,42,49,55,56,66,74,75,76,85,87,90,110]).

Example 7.1. For every $\alpha < \Omega$ the object $B_{d,\alpha}(X, G)$ denotes the E-algebra $B_\alpha(X, G)$ with the discrete topologies on E and $B_\alpha(X, G)$. Let $B^*_{d,\alpha}(X, G)$ be the discrete subsalgebra of the algebra $B_{d,\alpha}(X, G)$. Then $B_{d,\alpha}$ and $B^*_{d,\alpha}$ are functional functors.

Example 7.2. For every $\alpha \leq \Omega$ the object $B_{p,\alpha}(X, G)$ denotes the E-algebra $B_\alpha(X, G)$ with the topology of pointwise convergence and $B^*_{p,\alpha}(X, G)$ is a subalgebra $B^*_\alpha(X, G)$ of $B_{p,\alpha}(X, G)$. The $B_{p,\alpha}$ and $B^*_{p,\alpha}$ are funtional functors.

Example 7.3. Let E be a scalar field of real numbers R or of complex numbers C. Fix a Banach space G. Then $B^*_{u,\alpha}(X, G)$ denotes the linear space $B^*_\alpha(X, G)$ with the topology of uniform convergence; $B^*_{u,\alpha}(X, G)$ is a Banach space and $B^*_{u,\alpha}$ is a functional functor.

If G is a locally convex linear topological space, then $B^*_{u,\alpha}(X, G)$, $B^*_{p,\alpha}(X, G)$ and $B_{p,\alpha}(X, G)$ are locally convex linear topological spaces. Therefore $B^*_{u,\alpha}$, $B^*_{p,\alpha}$ and $B_{p,\alpha}$ are functional functors.

The following theorem for a closed subspace Y of a metric space X is proved by J. Dugundji, H. Arens, E. Michael (see [53]).

Theorem 7.4. *(M. Choban and S. Nedev [41]). Let d be a continuous pseudometric on a space X, Y be a non-empty subspace of X, the subspace Y be metrizable by a metric d on Y and if $d(x, Y) = \inf\{d(x,y) : y \in Y\} = 0$, then $d(x,y) = 0$ for some $y \in Y$. Then for every locally convex linear topological space G there exists a linear extender $u : C(Y,G) \mapsto C(X,G)$ with the properties:*

1. $u(f)(X)$ is contained in the convex null of the set $f(Y)$ for every $f \in C(Y,G)$.

2. u is continuous whenever both $C(Y,G)$ and $C(X,G)$ have compact-open topology, topology of pointwise convergence, or topology of uniform convergence.

Proof. Consider the natural mapping $\pi_d : X \mapsto X/d$, where $\pi_d^{-1}(\pi_d(x)) = \{y \in X : d(x,y) = 0\}$, onto a metric space $(X/d, d)$, where $d(z,t) = \inf\{d(x,y) : x \in \pi_d^{-1}(z)\}, y \in \pi_d^{-1}(t)$. Then π_d is continuous, $\pi_d|Y$ is a homeomorphism and $\pi_d(Y)$ is closed in X/d. To conclude the proof it is sufficient to apply the Dugundji - Arens - Michael theorem [53]. ■

Corollary 7.5. *Let G be a locally convex linear topological space. Then C_p^*, C_p, C_u^* are Dugundji functors.*

8. On the test spaces

Fix a signature E, an arcwise connected topological E-group G and a functional G-functor F.

Denote by D_τ the discrete space of cardinality τ, by $A_\tau = \{\alpha\} \cup D_\tau$ - the one-point compactification of the space D_τ, $S = A_\omega$, $D = D_2 = \{0,1\}$, $I = [0,1]$.

By $X + Y$ we denote the topological sum of disjoint spaces X and Y. In particular $X + 1 = X + D_1$ is the topological sum of the space X and the one-point space D_1.

Lemma 8.1. *Let $m \le \tau$ and τ be an infinite cardinal. Then:*

1. $D_\tau = D_\tau + D_m = D_\tau \times D_m$.

2. If m is a finite cardinal, then $A_\tau = A_\tau + D_m$.

Definition 8.2. The spaces X and Y are F-equivalent and we denote $X \overset{F}{\sim} Y$ or $F(X) \sim F(Y)$, if the algebras $F(X)$ and $F(Y)$ are topologically isomorphic.

The C_u^*-equivalent spaces X and Y are called B-equivalent or u-equivalent and denoted as: $X \overset{B}{\sim} Y$ or $X \overset{u}{\sim} Y$. The C_p-equivalent spaces X and Y are called l-equivalent and denoted $X \overset{l}{\sim} Y$ (see [6-10,17,117]).

Lemma 8.3. *Let τ, m be the infinite cardinals, $m \le \tau$ and $n \in N$. Then:*

1. $D_\tau \overset{E}{\sim} D_\tau + D_m \overset{E}{\sim} D_\tau + D_n$

2. $A_\tau \overset{E}{\sim} A_\tau + A_m \overset{E}{\sim} A_\tau + D_n$.

Proof. The equivalences $D_\tau \overset{E}{\sim} D_\tau + D_m \overset{E}{\sim} D_\tau + D_n$ and $A_\tau \overset{E}{\sim} A_\tau + D_n$ are obvious.

We consider that $X = A_\tau + A_m = \{(x,0): x \in A_\tau\} + \{(x,1): x \in A_m\}$. We put $Y = \{(\alpha,0),(\alpha,1)\} \subset X$ and $Z = \{\alpha\} \subset A_\tau$. It is obvious that the algebras $F(X:Y) = \{f \in F(X) : Y \subset f^{-1}(0)\}$ and $F(A_\tau : Z) = \{f \in F(A_\tau) : f(\alpha) = 0\}$ are topologically isomorphic. Since $X = X + D_2$ and $A_\tau = A_\tau + 1$ we have $F(A_\tau : Z) = F(A_\tau : Z) + F(D_1) = F(A_\tau)$ and $F(X:Y) = F(X : Y) + F(D) = F(X)$. Therefore $F(X)$ and $F(A_\tau)$ are topologically isomorphic. The proof is complete. ■

Lemma 8.4. *Let τ be an infinite cardinal and $m \leq \tau$. Then $D^\tau \overset{E}{\sim} D^\tau + D^m \overset{E}{\sim} D^\tau + 1$ and $D^\tau \overset{E}{\sim} D^\tau + S$.*

Proof. The spaces D^τ and $D^\tau + D^\tau$ are homeomorphic, $D^m \subset D^\tau$ and D^m is a retrack of F^τ. If m is an infinite cardinal then

$$\begin{aligned} F(D^\tau) &= F(D^\tau + D^\tau) = F(D^\tau) \times F(D^\tau) \\ &= F(D^\tau) \times F(D^\tau : D^m) \times F(D^m) \\ &= F(D^\tau) \times (F(D^\tau : D^m) \times F(D^m)) \times F(D^m) \\ &= F(D^\tau) \times F(D^\tau) \times F(D^m) = F(D^\tau) \times F(D^m) = F(D^\tau + D^m). \end{aligned}$$

Therefore $F(D^\tau) \sim F(D^\tau) + F(D^m)$.
We consider that $S \subset D^\tau$. Then S is a retract of D^τ. Hence

$$\begin{aligned} F(D^\tau) &= F(D^\tau + D^\tau) = F(D^\tau) \times F(D^\tau) \\ &= F(D^\tau) \times F(D^\tau : S) \times F(S) \\ &= F(D^\tau) \times (F(D^\tau : S) \times F(S)) \times F(S) \\ &= F(D^\tau) \times F(D^\tau) \times F(S) = F(D^\tau) \times F(S) = F(D^\tau + S) \end{aligned}$$

and $D^\tau \overset{E}{\sim} D^\tau + S$, $D^\tau \overset{E}{\sim} D^\tau + S \overset{E}{\sim} D^\tau + (S+1) \overset{E}{\sim} (D^\tau + S) + 1 \overset{E}{\sim} D^\tau + 1$. The proof is complete. ■

Lemma 8.5. *Let F be a Dugundji functor, τ be a cardinal number, $n \in N$ and $1 \leq n \leq \tau$. Then $I^\tau \overset{E}{\sim} I^\tau + S$, $I^\tau \overset{E}{\sim} I^\tau + D_n \overset{E}{\sim} I^\tau + 1$ and $I^\tau \overset{E}{\sim} I^\tau + I^n$. In particular, $I^n \overset{E}{\sim} I^n + I^n$.*

Proof. Assume $S \subset I^\tau$. Then there exists an F-extender $u : F(S) \mapsto F(I^\tau)$. In virtue of Theorem 4.2 and Lemma 8.3 we have $F(I^\tau) = F(I^\tau : S) \times F(S) \sim F(I^\tau : S) \times F(S) \times F(S) \sim F(I^\tau) \times F(S) \sim F(I^\tau + S)$. Hence $I^\tau \overset{E}{\sim} I^\tau + S \overset{E}{\sim} I^\tau + D_n$. We have $F(I) = F(I) \times F(D_1) \sim F(I : \{2^{-1}\}) \times F(\{2^{-1}\}) \times F(D_1) \sim$

$F(I : \{2^{-1}\}) \times F(D)$ and $F(I \times D) = F(I \times D : \{(1,0),(0,1)\}) \times F(D)$. The algebras $F(I : \{2^{-1}\})$ and $F(I \times D : \{(1,0),(0,1)\})$ are topologically isomorphic. Hence $I \sim I \times D = I + I$. Suppose that $I^{n-1} \sim I^{n-1} + I^{n-1}$. The algebras $F(I^n : (\{2^{-1}\} \times I^{n-1}))$ and $F((I^n \times D) : ((I^{n-1} \times (0,0)) \cup ((I^{n-1} \times (0,1))))$ are topologically isomorphic. Since $F(I^n) = F(I^n : (\{2^{-1}\} \times I^{n-1})) \times F(I^{n-1})$ and $F(I^n \times D) = F((I^n \times D) : ((I^{n-1} \times \{(0,0),(0,1)\}))) \times F(I^{n-1} \times D)$ the algebras $F(I^n)$ and $F(I^n \times D) = F(I^n + I^n)$ are topologically isomorphic. Therefore $I^n \overset{E}{\sim} I^n + I^n$ for every $n \in N$. We consider that $I^n \subseteq I^\tau$. Then I^n is a retract of I^τ and $F(I^\tau) = F(I^\tau : I^n) \times F(I^n) \sim F(I^\tau : I^n) \times F(I^n) \times F(I^n) \sim F(I^\tau) \times F(I^n)$. The proof is complete. ■

9. General problems

Fix a signature E, an arcwise connected E-group G and a functional G-functor F.

Many concrete questions are particular cases of the following general problems.

First general problem. Which topological properties of the spaces X are characterized in terms of the objects $F(X)$?

Second general problem. Which properties of the objects $F(X)$ are characterized by the properties of the spaces X?

The program of matching "interesting" topological properties of compact spaces X with "interesting" properties of Banach spaces $C_u(X)$ was formulated by S. Eilenberg [52]. The references concerning Eilenberg program can be found in the books [9,17,23,43,59,107,117].

L. Gillman, M. Jerison and M. Henriksen investigated some problems in the way: to find properties of spaces X corresponding to ring conditions on $C(X)$. This topic is covered in the monograph [59]. The isomorphism problem of functional rings is studied in [26,27,29,30,34,36,57,59,67,68,69,75,76,78,79,81,82, 98,99,110,118,119,120]. The cases of functional semirings, almost rings and functional lattices were investigated in [21,24,37,62,78,79,98,99,100,101,105,122,126].

Definition 9.1. A property P of spaces is called F-stable, if there exists a property Q of subalgebras of $F(X)$ and for every space X there exists a subalgebra $F_P(X)$ of $F(X)$ such that the space X has property P if and only if $F_P(X)$ is a subalgebra with the property Q. If $F_P(X) = F(X)$, then (P, Q) is called an F-pair.

Third general problem. (A.V. Arhangel'skii [7,8,9] for l-equivalence).

a^0. Find F-stable properties.

b^0. Find F-pairs.

It is not always easy to establish that the given property is F-stable or that the given pair (P, Q) is the F-pair (see [7] for l-equivalence). In terms of subrings of $C(X)$ M. Katetov [81,82] has characterized the Lebesque dimension of the compact spaces.

Problem 1. *(M. E. Henriksen [68]). Characterize local connectedness of a space X in terms of the ring $C(X)$.*

Problem 2. *(P. R. Misra [96], J. Nagata [98]). Characterize metrizability of a space X in terms of the ring $C(X)$.*

Fourth general problem. Which topological properties are preserved by F-equivalence?

A topological property will be called an F-property, if it is preserved by the F-equivalence. Every F-stable property is an F-property.

Problem 3. *(Z. Semadeni [117], p. 381 for $F = C_u^*$). Let X be an infinite compact space.*

1. Is it true that $X \overset{F}{\sim} X + 1$?

2. Does there exist a 0-dimensional compact space Y such that $X \overset{F}{\sim} Y$?

The Problem 3 for $F = C_u^*$ was discussed by A. Pelczynski [107]. A. A. Miljutin has proved that the answer is "yes" for metrizable compact spaces [94,107].

Problem 4. *(A. V. Ahangel'skii [6-10] for $F = C_p$). Let X be an infinite space.*

1. Is it true that $X \overset{F}{\sim} X + 1$?

2. When $X \overset{F}{\sim} X + S$?

3. When $X \overset{F}{\sim} X + N$?

4. When $X \overset{F}{\sim} X + \beta N$?

5. When $X \overset{F}{\sim} X + X$?

6. When $X \overset{F}{\sim} X \times S$?

7. When $X \overset{F}{\sim} X \times N$?

Example 9.2. Let $m \geq 1$ be a cardinal number, $G = R^m$ and $F = C_p$. Then $F(X) = C_p(X, R^m) \sim (C_p(X))^m \sim C_p(X \times D_m)$. Hence $X \overset{F}{\sim} Y$ if and only if $X \times D_m \overset{F}{\sim} Y \times D_m$. In particular, if $X \overset{F}{\sim} Y$ then:

1. $\dim X = \dim Y$
2. If $m \leq \aleph_0$ and X is σ-compact, then Y is σ-compact too.
3. If $m \in N$ and X is compact, then Y is compact too.
4. If $m \in N$ and X is pseudocompact, then Y is pseudocompact too.
5. If m is infinite, then $X \overset{F}{\sim} X \times D_m$ and $X \overset{F}{\sim} X \times N$.

Example 9.3. Let $m \geq 1$ be a cardinal number, $G = C_u^*(D_m)$ and $F = C_u^*$. Then:

1. $F(X) = C_u^*(X \times D_m)$.
2. If m is infinite, then $X \overset{F}{\sim} X \times D_m$ and $X \overset{F}{\sim} X \times N$.

Example 9.4. For any ordinal number α let $T(\alpha)$ denote the set of all ordinal numbers less than α with the topology induced by the natural order. Z. Semadeni (see [117], p. 381) proved that for $X = T(\omega + 1)$ the Banach spaces $C_u(X)$ and $C_u(X + X)$ are not isomorphic, i.e. the spaces X and $X + X$ are not B-equivalent. A. V. Arhangel'skii [6] and D. S. Pavlovskii [102,103,104] have proved that the l-equivalent compact spaces are B-equivalent. Hence the spaces X and $X + X$ are not l-equivalent.

Problem 5. *(A. V. Arhangel'skii [7,10]). Is the countable compactness an l-property?*

Problem 6. *(A. V. Arhangel'skii [7,10]). Let $X \overset{l}{\sim} Y$, where X is Lindelöf. Is it true that Y is also Lindelöf?*

A homeomorphism $\sigma : X \mapsto X$ is involutory if $\sigma(\sigma(x)) = x$ for each $x \in X$. Denote $\sigma F(X) = \{f \in F(X) : f(\sigma(x)) = -f(x)\}$. Then $\sigma F(X)$ is the subalgebra of the algebra $F(X)$ and $\sigma F(X)$ is the subalgebra of odd functions of $E(X)$.

Problem 7. *(A. Pelczynski [106] for $F = C_u^*$). Let $\sigma : X \mapsto X$ be a non-identity involution. Is it true that $\sigma F(X) \overset{E}{\sim} F(Y)$ for some space Y?*

Problem 8. *Let Y be a subspace X. When $F(X : Y) \sim F(Z)$ for some space Z?*

The Banach space $\sigma C_u^*(X)$ was introdused by M. Jerison [72] and studied in [43,86,106].

Proposition 9.5. *Let for every space X and for every non-identity involution $\sigma : X \mapsto X$ there exists a space $Z = Z(X, \sigma)$ such that $\sigma F(X)$ and $F(Z)$ are homeomorphic. Then for every proper closed subspace Y of X there exists a space Z such that $F(X : Y)$ and $F(Z)$ are isomorphic.*

Proof. In space $X \times D$ identify $(x, 0)$ and $(x, 1)$ for every $x \in Y$ and we obtain the space $D(X : Y)$ and an involution $\sigma : D(X : Y) \mapsto D(X : Y)$, where $\sigma(x, 0) = (x, 1)$ and $\sigma(x, 1) = (x, 0)$ for each $x \in X$. By construction $\sigma F(D(X : Y)) \sim F(X : Y)$. The proof is complete. ∎

Proposition 9.6. *Let $\sigma : X \mapsto X$ be a non-identity involution and G be a linear topological space. Then there exists a projection $u : F(X) \mapsto \sigma F(X)$.*

Proof. We put $u(f)(x) = 2^{-1}(f(x) - f(\sigma(x)))$ for each $f \in F(X)$ and $x \in X$. The proof is complete. ■

10. On a Baire functors

Fix a signature E and arcwise connected topological E-group G.

Definition 10.1. The mapping $\phi : X \mapsto Y$ is a Baire mapping if $\Phi_\phi(C(Y,G)) \subseteq B_1(X,G)$.

Definition 10.2. The functional G-functor F is called a Baire functor, if $\Phi_\phi(F(Y)) \subseteq F(X)$ and the homomorphism $\Phi_\phi : F(Y) \mapsto F(Y)$ is continuous for every Baire mapping $\phi : X \mapsto Y$ onto a countable discrete space Y.

Theorem 10.3. *Let F be a Baire G-functor. Then for every infinite space X we have $X \overset{E}{\sim} X + N \overset{E}{\sim} X + S \overset{E}{\sim} X + 1$.*

Proof. There are sequences of points $\{x_n \in X : n \in N\}$ and of open subsets $\{U_n : n \in N\}$ for which:

1. $x_n \in U_n$ and $X \backslash \{U_i : i \leq n\}$ is infinite for every $n \in N$.
2. $U_n\ U_m = 0$ if $n \neq m$.
3. For every $n \in N$ there exists a continuous function $f_n : X \mapsto [0,1]$ such that $X \backslash U_n = f_n^{-1}(0)$ and $f(x_n) = 1$.

We put $Y = \{x_n : n \in N\}$ with the discrete topology. Consider the mapping $\phi : X \mapsto Y$, where $\phi(U_n) = x_n$ and $\phi(X \backslash \cup \{U_n : n \in N\}) = x_0$. Then Y is a discrete subspace of a space X and ϕ is a Baire mapping. The mapping $\Phi_\phi : F(Y) \mapsto F(X)$ is an F-extender. In virtue of Theorem 4.2 we have $F(X) = F(Y) \times F(X : Y)$. From Lemma 8.3 we have $F(Y) = F(Y) \times F(Y)$. Therefore $F(X) = F(Y) \times F(X)$ and $X \overset{E}{\sim} X + N$. It is obvious that $N \overset{E}{\sim} S$. The proof is complete. ■

Theorem 10.4. *Suppose that the G-functor F satisfies one of the following conditions:*

1. $B_1^*(X,G) \subseteq F(X) \subseteq B^*(X,G)$ and the topology on $F(X)$ is the topology of uniform convergence.

2. $B_1^*(X,G) \subseteq F(X) \subseteq B^*(X,G)$ and the topology on $F(X)$ is the topology of pointwise convergence.

3. $B_1^*(X,G) \subseteq F(X)$ and the topology on $F(X)$ is the topology of pointwise convergence.

Then F is a Baire functor.

Proof. Let $\phi : X \mapsto Y$ be a Baire mapping onto a discrete space Y. Then $\Phi_\phi : F(Y) \mapsto F(X)$ is continuous in the topology of uniform convergence and in the topology of pointwise convergence. The proof is complete. ■

11. On the problems of Semadeni and Arhangels'skii

Fix a signature E, an arcwise connected E-group G and a Dugundji G-functor F.

Theorem 11.1. *Let X be a non-pseudocompact space. Then $X \overset{E}{\sim} X + N$ and $X \overset{E}{\sim} X + 1$.*

Proof. There exists a continuous function $f_0 : X \mapsto R$ and a sequence $Y = \{x_n \in X : n \in N\}$ such that $f(x_{n+1}) > f(x_n) + 4$. Then $d(x, y) = |f_0(x) - f_0(y)|$ is a continuous pseudometric on X and if $d(x, Y) = 0$ then $d(x, y) = 0$ for some $y \in Y$. From Theorem 3.4 there exists an F-extender $u : F(Y) \mapsto F(X)$. In virtue of Theorem 2.7 we have $F(Y) = F(Y) \times F(N)$ and $F(X) \sim F(X) \times F(Y)$. The proof is complete. ■

Theorem 11.2. *If X contains a non-trivial converging sequence S, then $X \overset{E}{\sim} X + S$ and $X \overset{E}{\sim} X + 1$.*

Proof. By Theorem 7.4 there exists an F-extender $u : F(S) \mapsto F(X)$. By Theorem 2.7 and Lemma 8.3 we have $X \overset{E}{\sim} X + S$ and $X \overset{E}{\sim} X + 1$. The proof is complete. ■

A space is scattered, if every non-empty subspace has an isolated point (see [38,85]).

Theorem 11.3. *Let X be an infinite scattered space. Then*

1. $X \overset{E}{\sim} X + 1$.

2. $X \overset{E}{\sim} X + N$ or $X \overset{E}{\sim} X + S$.

Proof. Let $Z = \{x \in X : x \text{ is isolated in } X\}$. If $X = Z$, then X and $X + N$ are homeomorphic and $X \overset{E}{\sim} X + 1 \overset{E}{\sim} X + N$. Let $X \neq Z$ and $a \in X \backslash Z$ be an isolated point of $X \backslash Z$. Then there exists an open and closed subspace Y of X such that $a \in Y \subseteq Z \cup \{a\}$. If Y is pseudocompact, then Y contains a non-trivial converging sequence S and $X \overset{E}{\sim} X + S$. Let Y be non-pseudocompact. Then $Y \overset{E}{\sim} Y + N$ and $X \overset{E}{\sim} X + N$. The proof is complete. ■

A family B of subsets of a space X is called a π-base, if for every non-empty open subset U of X there exists an open non-empty set $V \in B$ such that $V \subseteq U$. The π-weight $\pi - \omega(X)$ of the space X is the smallest cardinal number τ such that there exists a π-base B of X of cardinality τ.

The space is extremally disconnected, if the closure of each open subset is open. Every compact space X is a continuous irreducible image of some extremally disconnected compact space pX. The space pX is called the absolute or the projective resolution of the space X (see [60,53,117]).

Theorem 11.4 *Let X be an infinite extremally disconnected compact space. Then:*

1. $X\overset{F}{\sim}X+1$ *and* $X\overset{F}{\sim}X+\beta N$.

2. If $\pi-\omega(X)=\aleph_0$, *then* $X\overset{F}{\sim}X+X$ *and* $X\overset{F}{\sim}\beta N$.

Proof. There exists a sequence $\{U_n : n \in N\}$ of non-empty open and closed subsets of X such that $U = \cup\{U_n : n \in N\}$ is dense in X. Then $N = \{x_n \in U_n : n \in N\} \subseteq X$, $\beta N = cl_X N \subseteq X$ and there exists a retract $\phi : X \mapsto \beta N$. Therefore $X\overset{F}{\sim}X+\beta N$. The assertion 1 is proved. ■

Let X be an infinite extremally disconnected compact space of a countable π-weight and X_1 be a dense countable subset of X. There exists a continuous mapping $\psi : \beta N \mapsto X$ such that $X_1 = \psi(N)$. Then there exists a closed subspace X_2 of βN such that $\psi(X_2) = X$ and $\phi = \psi|X_2 : X_2 \longrightarrow X$ is irreducible. Since X is extremally disconnected the mapping ϕ is a homeomorphism (see [53], p. 464). Hence $X = X_2 \subseteq \beta N$, $\psi : \beta N \longrightarrow X$ is a retraction, $F(\beta N) \sim F(X) \times F(\beta N : X)$ and $F(X) \sim F(X) \times F(\beta N)$.

Denote by H the set of all isolated points of X and put $Y = clH$. From Assertion 1 we can assume that the set H is infinite and $Y = \beta N$.

Case 1. $Z = X\backslash Y = \emptyset$.

In that case $X = Y = \beta N$ and the assertion 2 is proved.

Case 2. $Z \neq \emptyset$.

In this case Z is homeomorphic to the absolute of a Cantor set D^ω. Therefore $Z\overset{F}{\sim}Z+Z$ and $X = Z + \beta N$. The proof is complete.

Remark 11.5. If X is an infinite extremally disconnected compact space, then $C_u(X)$ and $C_u(X+S)$ are not isomorphic Banach spaces (D. Amir [1,2,3], A. Pelczinski [106]). Moreover, X and $X+S$ are not l-equivalent spaces [14].

Theorem 11.6. *Let the pseudocharacter* $\psi(x, X) = \aleph_0$ *for every point* $x \in X$. *Then:*

1. $X\overset{F}{\sim}X+1$.

2. $X\overset{F}{\sim}X+S$ *or* $X\overset{F}{\sim}X+N$.

Proof. If X is a non-pseudocompact space, then the theorem follows by Theorem 11.1. If X is pseudocompact, then X contains the copy of S. The proof is complete. ■

Remark 11.7. For $F \in \{C_u^*, C_p, C_p^*\}$ and $G = R$, Theorems 11.1, 11.2, 11.3 and 11.4 are proved in [14].

Theorem 11.8. *Let X be an infinite suborderable space, i.e. X be a subspace of an orderable space. Then:*

1. $X\overset{F}{\sim}X+1$.
2. $X\overset{F}{\sim}X+S$ or $X\overset{F}{\sim}X+N$.

Proof. If X is pseudocompact then X contains the copy of S. The proof is complete. ■

From Proposition 2 in [115] it follows:

Corollary 11.9. *Let X be an infinite heriditarily normal k-space. Then:*

1. $X\overset{F}{\sim}X+1$.
2. $X\overset{F}{\sim}X+S$ *or* $X\overset{F}{\sim}X+N$.
3. *If X is non-discrete, then $X\overset{F}{\sim}X+S$ and X contains a copy of S.*

From Theorem 10.3 it follows.

Corollary 11.10. *Let $F\in\{B_{p,\alpha},B^*_{p,\alpha},B^*_{u,\alpha}:\ \alpha\geq 1\}$. Then:*

1. $S\overset{F}{\sim}N$.
2. $X\overset{F}{\sim}X+S$ *and* $X\overset{F}{\sim}X+N$ *for every infinite space X.*

12. On the functor C_u

Fix a Banach space G. Consider the G-functor $F=C^*_u$. Then for every space X the space $F(X)$ is a Banach space with the norm $\|f\|=\sup\{\|f(x)\|:x\in X\}$.

Let E_1 and E_2 be linear topological spaces. The space E_2 is a factor of E_1 (and we write $E_2|E_1$), if $E_1\sim E_2\times E_3$ for some linear topological space E_3.

Lemma 12.1. *Let E_1 and E_2 be linear topological spaces, $E_1\sim E_1\times E_1$, $E_2\sim E_2\times E_2$, $E_1|E_2$ and $E_2|E_1$. Then $E_1\sim E_2$.*

Proof. Let $E_1\sim E_2\times E_4$ and $E_2\sim E_1\times E_3$. Then $E_1\sim E_2\times E_4\sim E_2\times E_2\times E_4\sim E_2\times E_1\sim E_1\times E_1\times E_3\sim E_1\times E_3\sim E_2$. ■

Lemma 12.2. *Let E_1 and E_2 be linear topological spaces, $E_1\sim E_1\times E_1$, $E_1|E_2$ and $E_1\sim E_1\times E_2$. Then $E_1\sim E_2$.*

Proof. We have $E_2\sim E_1\times E_3\sim E_1\times E_1\times E_3\sim E_1\times E_2\sim E_1$. The proof is complete. ■

Remark 12.3. $X\overset{F}{\sim}\beta X$ for each space X.

Lemma 12.4. *Let $X\overset{F}{\sim}Y$ and X, Y be compact spaces. Then $X\times Z\overset{F}{\sim}Y\times Z$ for every compact Z.*

Proof. Let $u:C^*_u(X,G)\mapsto C^*_u(Y,G)$ be a linear homeomorphism. Let $f\in C^*_u(X\times Z)=F(X\times Z)$. For every $z\in Z$ we put $f_z(x)=f(x,z)$ and

$\omega(f)(y,z) = u(f_z)(y)$. Then $\omega : F(X \times Z) \mapsto F(Y \times Z)$ is a one-to-one linear mapping. We have $||w|| = ||u||$. Therefore, w is a linear homeomorphism.

For every Banach space E we put $(E \times E \times \ldots)_\omega = \{(x_n \in E : n \in N) : \lim ||x_n|| = 0\}$ with norm $||(x_n : n \in N)|| = \sup\{||x_n|| : n \in N\}$(see A. Pelczynski [106,107] or [117]). ■

Lemma 12.5. *For every compact space X we have $(F(X) \times F(X) \times \ldots)_\omega \sim F(X \times S) \sim F((X \times S) : (X \times \{\alpha\}))$.*

Proof. Let $S = \{\alpha, \alpha_n : n \in N\}$ and $\alpha = \lim \alpha_n$. It is obvious that $(F(X) \times F(X) \times \ldots)_\omega \sim F(X \times S) \sim F((X \times S) : (X \times \{\alpha\}))$. The proof is complete. ■

Lemma 12.6. *(A. Pelczynski [106,107,117]). Let E_1 and E_2 be Banach spaces, $E_1 \sim (E_1 \times E_1 \times \ldots)_\omega$, $E_1|E_2$ and $E_2|E_1$. Then $E_1 \sim E_2$.*

Proof. Let $E_1 \sim E_2 \times E_4$ and $E_2 \sim E_1 \times E_3$. Then $E_1 \sim (E_1 \times E_1 \times \ldots)_\omega \sim ((E_2 \times E_4) \times (E_2 \times E_4) \times \ldots)_\omega \sim E_2 \times ((E_2 \times E_4) \times (E_2 \times E_4) \times \ldots)_\omega \sim E_2 \times E_1$ and $E_1 \sim E_1 \times E_1$. By Lemma 12.2 we have $E_1 \sim E_2$. ■

Lemma 12.7. *Let X be a compact space, $|X| \geq 2$ and τ be an infinite cardinal. Then $X^\tau \overset{E}{\sim} X^\tau \times S$.*

Proof. Let $E_1 \sim E_2 \times E_4$ and $E_2 \sim E_1 \times E_3$. Then $E_1 \sim (E_1 \times E_1 \times \ldots)_\omega \sim ((E_2 \times E_4) \times (E_2 \times E_4) \times \ldots)_\omega \sim E_2 \times ((E_2 \times E_4) \times (E_2 \times E_4) \times \ldots)_\omega \sim E_2 \times E_1$ and $E_1 \sim E_1 \times E_1$. By Lemma 12.2 we have $E_1 \sim E_2$. ■

Lemma 12.7. *Let X be a compact space, $|X| \geq 2$ and τ be an infinite cardinal. Then $X^\tau \overset{E}{\sim} X^\tau \times S$.*

Proof. Let $Y = X^\tau$. Then $Y^\tau = X^\tau$, $Y = Y^\tau = Y \times Y$ and Y contains the copy of S, i.e. $S \subset Y$. Let $u : F(S) \mapsto F(Y)$ be an F-extender. Therefore there exists an F-extender $v : F(S \times Y) \mapsto F(Y \times Y)$ and $F(S \times Y)|F(Y)$. By construction, $F(S \times Y) \sim F(Y + S \times Y) \sim F(Y) \times F(S \times Y)$ and $F(S \times Y) \sim F(S \times Y) \times F(S \times Y)$. From Lemma 12.2 we have $F(S \times Y) \sim F(Y)$. The proof is complete. ■

Corollary 12.8. *Let τ be an infinite cardinal. Then $D^\tau \overset{E}{\sim} D^\tau \times S$, $D^\tau \overset{E}{\sim} D^\tau + D^\tau$, $I^\tau \overset{E}{\sim} I^\tau \times S$, $I^\tau \overset{E}{\sim} I^\tau + I^\tau$.*

Theorem 12.9. *(A. Pelczynski [107] for $G = R$). Let for some continuous mapping $\phi : D^\tau \mapsto X$ there exists an F-averaging operator $u : F(D^\tau) \mapsto F(X)$ and the space X satisfy one of the following conditions:*

1. $\omega(X) = \tau$ and τ is not a sequential cardinal.

2. X is not a union of a countable family of closed subspaces of weight less than τ.

Then $D^\tau \overset{E}{\sim} X$.

Proof. We have $F(X)|F(D^\tau)$. From Gerlits - Hagler - Efimov theorem [51,58,65] the space X contains the copy of D^τ. Therefore $F(D^\tau)|F(X)$. From Corollary 12.8 and Lemma 12.6 we have $F(D^\tau) \sim F(X)$. ∎

Corollary 12.10. $I^\tau \overset{E}{\sim} D^\tau$ *for every infinite cardinal* τ.

Theorem 12.11. *Let* X *be a compact space of weight* τ *with the properties:*

1. X *contains the copy of* D^τ.

2. For some embeding of X *in* I^m *there exists an* F*-extender. Then* $X \overset{E}{\sim} D^\tau$.

Proof. Follows from Lemma 12.6 and Corollaries 12.8 and 12.10. ∎

Corollary 12.12. *(A. A. Miljutin [94,107] for* $G = R$*). Let* X *be a non-countable compact metric space. Then* $X \overset{E}{\sim} D^\omega$.

Let Y be a closed sybset of a space X. Shrink the subset Y into a point and obtain the set X/Y and the natural projection $i_X : X \mapsto X/Y$. In X/Y we consider the finest completely regular topology with the continuous i_Y. This is the R-topology on X/Y. If X is normal then i_Y is a quotient and closed mapping.

Theorem 12.13. *Let* $\sigma : X \mapsto X$ *be a non-identity involution of a suborderable space* X. *Denote* $Y = \{\min\{x, \sigma(x)\} : x \in X\}, Z = \{\max\{x, \sigma(x)\} : x \in X\}$ *and* $X_\sigma = \{x \in X : x = \sigma(x)\} = Y \cap Z$. *Then:*

1. The mapping $\phi = \sigma|Y$ *is a homeomorphism of* Y *onto* Z.

2. The subsets Y *and* Z *are closed in* X.

3. $\sigma F(X) = F(Y : Z_\sigma)$.

4. $\sigma F(X) \sim F(X_1)$ *for some space* X_1.

5. If the set X/X_σ *is infinite then* $\sigma F(X) \sim F(Y/X_\sigma)$.

6. If the set $X \setminus X_\sigma$ *is infinite then* $Y/X_\sigma \overset{E}{\sim} N + Y/X_\sigma$ *or* $Y/X_\sigma \overset{E}{\sim} S + Y/X_\sigma$.

7. The space Y/X_σ *is collectionwise normal and if* Y/X_σ *is infinite and pseudocompact then* Y/X_σ *contains the copy of* S.

8. If $X \setminus X_\sigma$ *is finite then* $\sigma F(X) \overset{E}{\sim} F(Y \setminus X_\sigma)$.

9. If X_σ *is finite and* X *is infinite then* $\sigma F(X) \overset{E}{\sim} F(Y)$.

Proof. The assertions 1-2 are obvious.

If $X \setminus X_\sigma$ is finite then $Y_1 = Y \setminus X_\sigma$ is open and closed in X and $\sigma F(X) \overset{E}{\sim} \sigma(Y_1)$. The assertion 3 is proved.

Let $Y_2 = i_{X_\sigma}(X_\sigma) \subseteq Y/X_\sigma$. Then $\sigma F(X) \overset{E}{\sim} F(Y : X_\sigma) \sim F(Y/X_\sigma : Y_2)$. The mapping $i_{X_\sigma} : Y \mapsto Y/X_\sigma$ is closed. Therefore Y and Y/X_σ are collectionwise normal spaces (see [53]). Suppose that Y/X_σ is pseudocompact and infinite. If every point $y \in Y\backslash X_\sigma$ is isolated in Y, then Y/X_σ is homeomorphic with A_τ for $\tau = |Y/X_\sigma|$. Let $a \in Y\backslash X_\sigma$ be a non-isolated point of the space Y. Then there exists a sequence $\{y_n \in Y\backslash X_\sigma : n \in N\}$, where $y_n < y_{n+1} < \alpha$ and $[y_0, \alpha] \subset Y\backslash X_\sigma$ or $\alpha < y_{n+1} < y_n$ and $[y_0, \alpha] \subseteq Y\backslash X_\sigma$ for every $n \in N$. Then in $Y\backslash X_\sigma$ there exists $\lim y_n = b \in Y\backslash X_\sigma$. The assertion 7 is proved.

The assertions 5,6 and 9 follow from the assertion 7. The assertion 8 is obvious. The proof is complete. ■

Remark 12.14. For a compact orderable space X and $|X_\sigma| \leq 1$ the assertion 4 is proved in [86].

Theorem 12.15. *Let X be an extremally disconnected space and $\sigma : X \mapsto X$ be a non-identity involution. Then $\sigma F(X) \sim F(Y)$ for some open and closed subspace Y of X.*

Proof. Let $X_\sigma = \{x \in X : x = \sigma(x)\}$. There are open subsets U and V of X for which $U \cap V = \emptyset$, $V = \sigma(U)$ and $U \cup V \cup X_\sigma$ is a dense subset of X. We put $Y = \mathrm{cl}_X U$ and $Z = \mathrm{cl}_X V$. Then Y and Z are open and closed subsets of X, $Y \cap Z = \emptyset$, $\sigma(Y) = Z$ and $X = Y \cup Z \cup X_\sigma$. Therefore $Y \cap X_\sigma = Z \cap X_\sigma = \emptyset$ and X_σ is open and closed in X. The proof is complete. ■

Theorem 12.16. *Let X be a hereditarily paracompact space,* $\dim X = 0$ *and $\sigma : X \mapsto X$ be a non-identity envolution. Then:*

1. There exist the closed subspaces Y and Z of X such that $X = Y \cup Z$ and $Y\backslash Z = X_\sigma = \{x \in X : x = \sigma(x)\}$.

2. $\sigma F(X) \overset{E}{\sim} F(Y/X_\sigma)$.

3. If X/X_σ is non-compact then $\sigma F(X) \overset{E}{\sim} F(Y/X_\sigma)$ and $Y/X_\sigma \overset{E}{\sim} N + Y/X_\sigma$.

4. If $\psi(x, X) \leq \aleph_0$ for every $x \in X$ and $X\backslash X_\sigma$ is an infinite set then $\sigma F(X) \sim F(Y/X_\sigma)$.

5. If $Y/X_\sigma \overset{E}{\sim} Y/X_\sigma + 1$ then $\sigma F(X) \sim F(Y/X_\sigma)$.

Proof. For every point $x \in X\backslash X_\sigma$ there exists an open set U_x such that $x \in U_x \subseteq X\backslash X_\sigma$ and $U_x \cap \sigma(U_x) = \emptyset$. There exists some discrete cover $\{V_\alpha : \alpha \in A\}$ of $X\backslash X_\sigma$ such that $V_\alpha \subset U_{x_\alpha}$ for some $x_\alpha \in X$. Therefore $V_\alpha \cap \sigma(V_\alpha) = \emptyset$ for every $\alpha \in A$. Denote $W_{\alpha,\beta} = V_\alpha \cap \sigma(V_\beta)$ for every $(\alpha, \beta) \in A \times A$. From construction $\sigma(W_{(\alpha,\beta)}) = W_{(\alpha,\beta)}, W_{(\alpha,\alpha)} = \emptyset, V_\alpha = \cup\{W_{(\alpha,\beta)} : \beta \in A\}$ and $\{W_{(\alpha,\beta)} : (\alpha, \beta) \in A \times A\}$ is a discrete cover of the space $X\backslash X_\sigma$. On A we consider some well ordering. We put $B = \{(\alpha, \beta) \in A \times A : \alpha < \beta\}$, $U =$

$\cup\{W_{(\alpha,\beta)} : (\alpha,\beta) \in B\}$ and $V = \sigma(U) = \cup\{W_{(\alpha,\beta)} : (\alpha,\beta) \in (A \times A)\backslash B\}$. Then $U \cap V = \emptyset$ and $X = U \cup V \cup X_\sigma$. Denote $Y = U \cup X_\sigma$ and $Z = V = X_\sigma$. The spaces Y and Y/X_σ are hereditarily paracompact. By construction $\sigma F(X) \sim F(Y : X_\sigma)$ and $F(Y : X_\sigma) \sim F(Y/X_\sigma : \sigma(X_\sigma))$. The assertions 1,2 and 5 are proved. If X/X_σ is non-compact then Y/X_σ is non-pseudocompact. Hence the assertion 3 follows from Theorem 11.1. The assertion 4 follows from Theorems 11.1 and 11.6. The proof is complete. ∎

Corollary 12.17. *Let $\sigma : X \mapsto X$ be a non-identity involution of a perfectly normal paracompact space X and* $\dim X = 0$. *Then there exists a perfectly normal paracompact space Y such that $\sigma F(X) \sim F(Y)$ and* $\dim Y = 0$. *If the set X_σ is finite then Y is a closed subspace of X.*

Remark 12.18. Theorem 12.16 is true if $\dim(X\backslash X_\sigma) = 0$ and $X\backslash X_\sigma$ is paracompact.

For every Banach space E we put $(E \times E \times \ldots)^\omega = \{(x_n \in E : n \in N) : \sup\{||x_n|| : n \in N\} < \infty\}$ with the norm $||(x_n : n \in N)|| = \sup\{||x_n|| : n \in N\}$.

Lemma 12.19. *Let E_1 and E_2 be Banach spaces, $E_1 \sim (E_1\ E_2\ \ldots)^\omega$, $E_1|E_2$ and $E_2|E_1$. Then $E_1 \sim E_2$.*

Proof. Is similar to the proof of Lemma 12.6. ∎

Lemma 12.20. *Let G be a finite dimensional space. Then $(F(X) \times F(X) \times \ldots)^\omega \sim F(\beta(X \times N))$.*

Proof. Is obvious. ∎

Theorem 12.21. *Let G be a finite dimensional space and X be an infinite separable extremally disconnected compact space. Then $X \overset{F}{\sim} X + X$, $X \overset{F}{\sim} X + \beta N$ and $X \overset{F}{\sim} \beta N$.*

Proof. In the proof of Theorem 11.4 we proved that $F(X)|F(\beta N)$ and $F(\beta N)|F(X)$. Since $\beta(\beta N \times N)$ and βN are homeomorphic compact spaces from Lemma 12.20 we have $(F(\beta N) \times F(\beta N) \times \ldots)^\omega \sim F(\beta N)$. In virtue of Lemma 12.19 $F(\beta N) \sim F(X)$. The proof is complete. ∎

13. On the functor C_p

Fix a finite dimentional Banach space G and the G-functor $F = C_p$.

Lemma 13.1. *If $u : F(X) \mapsto F(Y)$ is a continuous homomorphism and X, Y are compact spaces, then u is continuous with respect to the topology of uniform convergence on $F(X)$ and $F(Y)$.*

Proof. If $n = \dim G$, then $F(X) = C_p(X \times D_n)$. Therefore Lemma 13.1 follows from Arhangel'skii-Pavlovskii theorem [6,104]. ■

In the same way from Theorem 2.4 [8] it follows

Lemma 13.2. *Let X and Y be F-equivalent compact spaces. Then $X \times Z \overset{F}{\sim} Y \times Z$ for every compact space Z.*

The proof of Lemma 13.2 is similar to the proof of Lemma 12.4.

Lemma 13.3. *Let X be a compact space, $|X| \geq 2$ and τ be an infinite cardinal. Then $X^\tau \overset{F}{\sim} X^\tau \times S$.*

Proof. See the proof of Lemma 12.7. ■

Corollary 13.4. *Let τ be an infinite cardinal and $m \leq \tau$. Then $D^\tau \overset{F}{\sim} D^\tau \times S$, $D^\tau \overset{F}{\sim} D^\tau \times D^m$, $I^\tau \overset{F}{\sim} I^\tau \times S$ and $I^\tau \overset{F}{\sim} I^\tau + I^m$.*

Proposition 13.5. *Let X be a compact space, τ be an infinite cardinal, $Y \subset X^\tau = Z$, $F(Y) \sim F(Z) \times E$, where E is a linear topological space, and $u: F(Y) \mapsto F(Z)$ be an F-extender. Then $Y \overset{F}{\sim} Z$.*

Proof. By Lemma 13.3 $F(Z) \sim F((Z \times S) : (Z \times \{\alpha\}))$, where $S = \{\alpha, \alpha_n : n \in N\}$ and $\alpha = \lim \alpha_n$. We have $F((Z \times S) : (Z \times \{\alpha\})) = \{(f_n \in F(Z) : n \in N) : \lim ||f_n|| = 0\}$. Let $f = (f_n \in F(Z) : n \in N)$ and $f_\alpha = (f_{n,\alpha} \in F(Z) : n \in N)$, where $\alpha \in A$ and A is a directed set. We have $f = \lim f_\alpha$ if and only if $f_n = \lim f_{n,\alpha}$ for every $n \in N$. Since $Y \times S \subset Z \times S$, there exists a continuous linear F-extender $v : F((Y \times S) : (Y \times \{\alpha\})) \mapsto F((Z \times S) : (Z \times \{\alpha\}))$, where $v(f_n : n \in N) = (u(f_n) : n \in N)$. Hence $F((Z \times S) : (Z \times \{\alpha\})) \sim F(Y) \times F((Z \times S) : (Z \times \{\alpha\}))$ and $F(Z) \sim F(Z) \times F(Y)$. By Lemma 12.2, $F(Z) \sim F(Y)$. ■

Definition 13.6. (A. V. Arhangel'skii, M. M. Choban [11,12]). A space X is called extral if whenever a copy of X is a closed subspace of a space Y, then there exists a linear continuous extender $u : C_p(X) \mapsto C_p(Y)$.

Proposition 13.7. *Let X be an extral space. Then:*

1. X is compact.

2. If U is an open non-metrizable subset of X, τ is an infinite cardinal and U is not a union of countable closed subspaces of weight less than τ, then U contains a copy of I^τ.

3. If $X \subset I^\tau$ then there exists an F-extender $u : F(X) \mapsto F(I^\tau)$.

Proof. The assertions 1 and 2 are proved in [11,12]. The assertion 3 is obvious. ■

Theorem 13.8. *(A. V. Arhangel'skii, V. Vilov). Let X be an extral space $\omega(X) = \tau$ and $I^\tau \subseteq X$. Then $X \overset{l}{\sim} I^\tau$.*

Proof. Follows from Proposition 13.5. ■

Theorem 13.9. *Let $\sigma : X \mapsto X$ be a non-identity involution of an extral space X, and every non-empty open subset U of X contains the copy of $I^{\omega(X)}$. Then $\sigma F(X) \sim F(X)$ and $\sigma F(X) \sim F(I^\tau)$.*

Proof. Denote $\sigma_+ F(X) = \{f \in F(X) : f(x) = f(\sigma(x))\}$. The mapping $u : F(X) \mapsto \sigma F(X)$, where $u(f)(x) = 2^{-1}(f(x) - f(\sigma(x)))$, is a continuous projection. Consider $X \subset I^\tau$, where $\tau = \omega(X)$. Then there exists an F-extender $v : F(X) \sim F(I^\tau)$.

Then $F(I^\tau) \sim F(X) \times F(I^\tau : X)$. The mapping $f \mapsto (u(f), f - u(f))$ is a linear homeomorphism of $F(X)$ onto $\sigma F(X) \times \sigma_+ F(X)$. Therefore $F(I^\tau) \sim \sigma F(X) \times \sigma_+ F(X) \times F(I^\tau : X)$. By methods of proof of Proposition 13.5 we obtain $F(I^\tau) \sim F(I^\tau) \times \sigma F(X)$.

We fix a non-empty open subset U of X for which $U \cap \sigma(U) = \emptyset$. Fix in U the copy of I^τ, i.e. $Z_1 = I^\tau \subset U$. Put $V = \sigma(U)$ and $Z_2 = \sigma(Z_1)$. Fix a continuous function $f_0 : Z \mapsto [0,1]$, where $Z_1 \cup Z_2 \subseteq f_0^{-1}(1)$, and $X \backslash (U \cup V) \subseteq f_0^{-1}(0)$. There exists a mapping $\phi : X \mapsto Z_1 \cup Z_2$, where $\phi(x) = x$ for each $x \in Z_1 \cup Z_2$, $\phi|(U \cup V)$ is continuous, $\phi(U) = Z_1$ and $\phi(V) = Z_2$. Let $Z = Z_1 \cup Z_2$ and $\sigma F(Z) = \{f \in F(Z) : f(x) = -f(\sigma(x))\}$. Then $F(I^\tau) \sim \sigma F(Z)$. The mapping $\omega : \sigma F(Z) \mapsto \sigma F(X)$, where $\omega(f)(x) = f_0(x) \cdot f(\phi(x))$ for each $x \in X$, is continuous and linear. Therefore $\sigma F(X) \sim \sigma F(Z) \times E$ for some linear topological space E. Hence $\sigma F(X) \sim F(I^\tau) \times E$. From Lemma 12.2 we have $\sigma F(X) \sim F(I^\tau)$. The proof is complete. ■

Theorem 13.10. *Let $\sigma : X \mapsto X$ be a non-identity involution of a suborderable space X. Then:*

1. There exists a closed subspace Y of X such that $X_\sigma \subseteq Y$ and $\sigma F(X) \sim F(Y : X_\sigma)$.

2. If the set $X \backslash X_\sigma$ is infinite, then $\sigma F(X) \sim F(Y/X_\sigma)$.

3. If the set $X \backslash X_\sigma$ is infinite, then $\sigma F(X) \sim F(S + Y/X_\sigma)$ or $\sigma F(X) \sim F(N + Y/X_\sigma)$.

4. If the space X is infinite and the set X_σ is finite then $\sigma F(X) \sim F(Y)$.

Proof. It is similar to the proof of Theorem 12.13. ■

Theorem 13.11. *Let $\sigma : X \mapsto X$ be a non-identity involution, $X \backslash X_\sigma$ be a paracompact space, $\dim(X \backslash X_\sigma) = 0$ and X/X_σ be non-pseudocompact or contains a copy of S. Then $\sigma F(X) \sim F(Y)$ for some paracompact space Y and Y is a continuous closed image of some closed subspace of the space X.*

Proof. It is similar to the proof of Theorem 12.16. ■

References

[1] D. A m i r, Projections onto continuous function spaces. *Proc. Amer. Math. Soc.*, **15**, 1994, 396-402.

[2] D. A m i r, On projections and simultaneous extensions. *Israel J. Math.*, **2**, 1964, 245-248.

[3] D. A m i r, On isomorphizm of continuous function spaces. *Israel J. Math.*, **3**, 1965, 205-210.

[4] R. D. A n d e r s o n, Hilbert space is homeomorphic to the countable infinite product of lines. *Bull. Amer. Math. Soc.*, **72**, 1966, 515-519.

[5] R. A r e n s, J. L. K e l l y, Characterizations of the space of continuous functions over a compact Hausdorff space. *Trans. Amer. Math. Soc.*, **62**, 1947, 499-508.

[6] A. V. A r h a n g e l's k i i, On linear homeomorphisms of function spaces. *Soviet Math. Dokl.*, **25**, 1982, 852-855.

[7] A. V. A r h a n g e l's k i i, A survey of C_p-theory. *Q and A in General Topology*, **5**, 1987, 1-109.

[8] A. V. A r h a n g e l's k i i, On linear topological classification of spaces $C_p(X)$. *Zbornic Radova Filozofskog Faculteta u Nisu. Serja Matematika*, **3**, 1989, 3-12.

[9] A. V. A r h a n g e l's k i i, *Topological Spaces of Functions.* Moscow University Press, Moscow, 1989.

[10] A. V. A r h a n g e l's k i i, C_p-theory. In: *Recent Progress in General Topology* (Edited by M. Husek and J. van Mill), North Holland, Amsterdam-London-New York-Tokyo, 1993, 1-56.

[11] A. V. A r h a n g e l's k i i, M. M. C h o b a n, The extension property of Tyhonoff spaces and generalized retracts. *Comptes Rendus Acad. Bulgare Sci.*, **41**, No. 12, 1988, 5-7.

[12] A. V. A r h a n g e l's k i i, M. M. C h o b a n, $C(X)$ and other functors in general topology. Continuous extenders. *Categ. Topology and its Relations to Analysis, Algebra and Combinatorics*, London-New York-Singapore, 1989, 432-445.

[13] A. V. A r h a n g e l's k i i, M. M. C h o b a n, On the position of a subspace in the whole space. *Comptes Rendus Acad. Bulgare Sci.*, **43**, 1990, 13-15.

[14] A. V. A r h a n g e l's k i i, M. M. C h o b a n, On functional equivalence of Tychonoff spaces. *Izvestia Acad. Nauk Resp. Moldova*, **3**, 1991, 43-51.

[15] A. V. A r h a n g e l's k i i, M. M. C h o b a n, Extenders of Kuratowski-van Douwen and classes of spaces. *Comptes Rendus Acad. Bulgare Sci.*, **45**, 1992, 5-7.

[16] J. B a a r s, J. de G r o o t, J. van M i l l, J. P e l a n t, On topological and linear homeomorphisms of certain function spaces. *Topology Appl.*, **32**, 1989, 267-277.

[17] J. A. B a a r s, J. A. M. de G r o o t, On topological and linear equivalence of certain function spaces. *Stichting Math. Centrum*, Amsterdam, 1992.

[18] W. S. B a d e, Complementation problems for the Baire classes. *Pacific J. Math.*, **45**, 1973, 1-11.

[19] S. B a n a c h, *Theorie des Operations Lineaires*. Warszawa, 1932.

[20] H. B a n i l o w e r, Isomorphisms and simultaneous extensions in $C(S)$. *Pacif. J. Math.*, **80**, 1979, 305-311.

[21] E. B e c k e n s t e i n, L. N a r i c i, Ch. S u f f e l, *Topological Algebra*. Amsterdam-New York-Oxford, 1977.

[22] C. B e s s a g a, On topological classification of complete linear metric spaces. *Fund. Math.*, **56**, 1965, 251-288.

[23] K. B o r s u k, Uber Isomorphik der Functionalraume. *Bull. Inst. Acad. Polon. Sci.*, 1933, 1-10.

[24] L. I. C a l m u t c h i i, Semirings of functions and charaterization of T_1-spaces. *Dokl. Acad. Nauk Beloruskoi SSR*, **30**, No. 11, 1986, 972-974.

[25] R. C a n t y, L'espace de fonctions continues d'un espace metrique denombrable. *Proc. Amer. Math. Soc.*, **113**, 1991, 493-501.

[26] M. M. C h o b a n, On Baire isomorphism and Baire topologies. Solution of a problem of Comfort. *Soviet. Math. Dokl.*, **30**, 1984, 780-784.

[27] M. M. C h o b a n, Baire mappings and the topological groups. *Fifth Tiraspol Symposium on General Topology and Its Application*. Kishinev, Shtiintsa, 1985, 252-253.

[28] M. M. C h o b a n, On the theory of topological algebric systems. *Trans. Moscow Math. Soc.*, **48**, 1986, 115-159.

[29] M. M. C h o b a n, General conditions of the existence of free objects. *Acta et Commentationes Univ. Tartuensis. Math.*, **836**, 1989, 157-171.

[30] M. M. C h o b a n, On some geometrical problems in topological algebra. *Proced. Scient. Sympos. with the Contribution of Teachers from the Republic of Moldova*, Brasov, 1991, 19-20.

[31] M. M. C h o b a n, Descriptive set theory and topology. In: *Encyclopedia of Mathematical Sciences*, **51**, Springer, 1995, 157-219.

[32] M. M. C h o b a n, The structure of locally compact algebras. *Serdica Bulg. Math. Publ.*, **18**, 1992, 129-137.

[33] M. M. C h o b a n, Some topics in topological algebra. *Topology Appl.*, **54**, 1993, 183-202.

[34] M. M. C h o b a n, Isomorphism problems for rings of functions. *Serdica*

Bulg. Math. Publ., **20**, 1994, 165-185.

[35] M. M. C h o b a n, Well posedness of optimization problems and measurable functions. *Mathematica Balkanica*, **5**, 1995.

[36] M. M. C h o b a n, Realcompactifications, P-spaces and Baire isomorphism of topological spaces. *Serdica Bulg. Math. Publ.*, **20**, 1994, 228-237.

[37] M. M. C h o b a n, L. I. C a l m u t s c h i i, On the Nagata's theorem about the lattices of semicontinuous functions. *Sibirskii Matem. Zh.*, **30**, 1989, 185-191.

[38] M. M. C h o b a n, N. C. D o d o n, *Theory of P-Scattered Spaces.* Stiintsa, Kishinev, 1979.

[39] M. M. C h o b a n, P. S. K e n d e r o v, Dense Gateaux differentiability the sup-norm in $C(T)$ and the topological properties of T. *Comptes Rendus Acad. Bulgare Sci.*, **38**, No. 12, 1985, 1603-1604.

[40] M. M. C h o b a n, P. S. K e n d e r o v, J. P. R e v a l s k i, Generic well-posedness of optimization problems in topological spaces. *Mathematika*, **36**, 1989, 302-324.

[41] M. M. C h o b a n, St. N e d e v, *Strong P-embeddings*, Preprint, September 1993.

[42] F. K. D a s h i e l l, Jr., Isomorphism problems for the Baire classes. *Pacific J. Math.*, **52**, 1974, 29-43.

[43] M. M. D a y, *Normed Linear Spaces.* Berlin, 1958.

[44] D. V. D e a n, Projections in certain continuous function spaces of $C(H)$ and subspaces of $C(H)$ isomorphic with $C(H)$. *Canad. J. Math.*, **14**, 1962, 385-401.

[45] D. W. D e a n, Subspaces of $C(H)$ which are direct factors of $C(H)$. *Proc. Amer. Math. Soc.*, **16**, 1965, 237-242.

[46] S. Z. D i t o r, Averaging operators in $C(S)$ and lower semicontinuous sections of continuous maps. *Trans. Amer. Math. Soc.*, **175**, 1973, 195-208.

[47] T. D o b r o w o l s k i, S. P. G u l 'k o, J. M o g i l s k i, Function spaces homeomorphic to the countable product of l^2_f. *Topology Appl.*, **34**, 1990, 153-160.

[48] E. van D a u w e r, *Simultaneuos Extension of Continuous Functions.* Doctoral Thesis, Free University of Amsterdam, Amsterdam, 1975.

[49] R. M. D u d l e y, On sequential convergence. *Trans. Amer. Math. Soc.*, **112**, 1964, 483-507.

[50] I. J. D u k s t r a, T. D o b r o w o l s k i, W. M a r c i s z e w s k i, J. van M i l l, J. M o g i l s k i, Recent classification and characterization results in geometric topology. *Bull. Amer. Math. Soc.*, **22**, 1990, 277-283.

[51] B. A. E f i m o v, Mappings and embeddings of dyadic spaces. *Matem. Sb.*, **103**, 1977, 52-68.

[52] S. E. E i l e n b e r g, Banach spaces methods in topology. *Ann. of Math.*, **43**, 1942, 568-579.

[53] R. E n g e l k i n g, *General Topology*. PWN, Warszawa, 1977.

[54] M. F r e c h e t, *Les Espaces Abstraits*. Hermann, Paris, 1928.

[55] Z. F r o l i k, Realcompactness is a Baire-measurable property. *Bull. Acad. Polon. Sci. Ser. Math. Astronom. Phys.*, **19**, 1971, 617-621.

[56] Z. F r o l i c, A survey of separable descriptive theory of sets and spaces. *Czech. Math. J.*, **20**, 1970, 406-467.

[57] I. M. G e l f a n d, A. N. K o l m o g o r o f f, On rings of continuous functions on topological spaces. *Doklady AN SSSR*, **22**, 1939, 11-15.

[58] J. G e r l i t s, On subspaces of dyadic compacts. *Math. Inst. Hung. Acad. Sci. Budapest*, 1976, 1-16.

[59] L. G i l m a n, M. J e r s o n, *Rings of Continuous Functions*. Van Nostrand Reinhold, New York, 1960.

[60] A. M. G l e a s o n, Projective topological spaces. *Illinois J. Math.*, **2**, 1958, 482-489.

[61] D. A. G o d n e r, Projection in normed linear spaces. *Trans. Amer. Math. Soc.*, **69**, 1950, 89-108.

[62] H. G o r d o n, Rings of functions determined by zero sets. *Pacific J. Math.*, **36**, 1971, 133-157.

[63] G. G r a t z e r, *Universal Algebra*. Springer Verlag, New York - Heidelberg - Berlin, 1979.

[64] S. G u l 'k o, T. H m u l e v a, The compactnes is not preserved by the relation of t-equivalence. *Matem. Zametki*, **39**, 1986, 895-903.

[65] J. H a g l e r, On the structure of S and $C(S)$ for S duadic. *Trans. Amer. Math. Soc.*, **214**, 1975, 415-426.

[66] H. H a h n, *Reelle Funktionen*. Leipzig, 1932.

[67] M. E. H e n r i k s e n, On the equivalence of the ring. Lattice and semi-group of continuous functions. *Proc. Amer. Math. Soc.*, 1956, 959-960.

[68] M. E. H e n r i k s e n, Unsolved problems on algebric aspects of $C(X)$. In: *Rings of Continuous Functions* (Editor C. E. Aull). Applied Mathematics, **95**, New York - Basel, 1985, 195-202.

[69] E. H e w i t t, Rings of real-valued continuous functions, I. *Trans. Amer. Math. Soc.*, **64**, 1948, 45-99.

[70] K. H o f f m a n, J. W e r m e r, A characterization of $C(X)$. *Pacific J. Math.*, **12**, 1962, 941-944.

[71] J. R. Isbel, Z. Semadeni, Projection constants and spaces of continuous function. *Trans. Amer. Math. Soc.*, **107**, 1963, 38-48.

[72] M. Jerison, Certain spaces of continuous functions. *Trans. Amer. Math. Soc.*, **70**, 1951, 103-113.

[73] M. Jonac, C. Samuel, Sur le sous-espaces complétés de $C(S)$. *Bull. Sci. Math.*, Ser. 2, **94**, 1970, 159-163.

[74] J. E. Jayne, Spaces of Baire functions, I. *Ann. Inst. Fourier, Grenoble*, **24** 4, 1974, 47-76.

[75] J. E. Jayne, The space class α Baire functions. *Bull. Math. Soc.*, **80**, 1974, 1151-1156.

[76] J. E. Jayne, F. Jellet, Anneax de functions de Baire. *C. R. Acad. Sci. Paris*, Ser. 1, **296**, 1983, 311-314.

[77] M. I. Kadec, A proof of topological equivalence of all separable infinite-dimensional Banach spaces. *Funktional. Analiz i Prilogenia*, **1**, 1967, 53-62.

[78] I. Kaplansky, Lattices of continuous functions, I. *Bull. Amer. Math. Soc.*, **53**, 1947, 617-623.

[79] I. Kaplansky, Lattices of continuous functions, II. *Amer. J. Math.*, **70**, 1948, 629-634.

[80] M. Katetov, On rings of continuous functions and the dimension of compact spaces. *Casopis Pest. Mat. Fys.*, **75**, 1950, 1-16.

[81] M. Katetov, A theorem on the Lebesque dimension. *Casopis Pest. Mat. Fys.*, **75**, 1950, 79-87.

[82] M. Katetov, On real-valued functions in topological spaces. *Fund. Math.*, **38**, 1951, 85-91. (Correction, *Ibid.*, **40**, 1953, 203-205).

[83] J. L. Kelley, Banach spaces with the extension property. *Trans. Amer. Math. Soc.*, **72**, 1952, 323-326.

[84] S. V. Kislyakov, Free subalgebras of complete Boolean algebras and spaces of continuous functions. *Siberian Math. J.*, **16**, 1975, 226-231.

[85] K. Kuratowski, *Topology, I.* New York, Academic Press, 1966.

[86] M. S. Labbe, J. Wolfe, Isomorphic classes of the spaces $C(S)$. *Pacific J. Math.*, **47**, 1973, 481-485.

[87] N. Lusin, *Lesons sur les Ensembles Analytiques.* Paris, 1930.

[88] F. J. Lutzer, A survey of Dugundjii. Extension Theory. *Seminar Uniform Spaces 1976-1977 directed by Z. Fronik, Matem. Ustav CSAV*, Praha, 1978, 24-42.

[89] D. J. Lutzer, J. van Mill, R. Pol, Descriptive complexity of function spaces. *Trans. Amer. Math. Soc.*, **291**, 1985, 121-128.

[90] P. R. M e y e r, The Baire order problem for compact spaces. *Duke Math. J.*, **33**, 1966, 33-39.

[91] E. A. M i c h a e l, A linear mapping between function spaces. *Proc. Amer. Math. Soc.*, **15**, 1964, 407-409.

[92] E. A. M i c h a e l, Three mapping theorems. *Proc. Amer. Math. Soc.*, **15**, 1964, 410-415.

[93] E. A. M i c h a e l, A. P e l c y n s k i, A linear extension theorem. *Illinois J. Math.*, **11** 4, 1967, 563-579.

[94] A. A. M i l j u t i n, Isomorphism of spaces of continuous functions on compacta having the power of the continuum. *Teoria Functsii, Functional. Anal. i Prilogenia*, **2**, 1966, 150-156.

[95] J. van M i l l, Topological equivalence of certain function spaces. *Compositio Math.*, **63**, 1987, 159-188.

[96] P. R. M i s r a, A topological view of F-spaces. *Topology Appl.*, **2**, 1972, 349-362.

[97] L. N a c h b i n, On the Hann-Banach theorem. *Ann. Acad. Brasil. Ci.*, **21**, 1948, 151-154.

[98] J. N a g a t a, On lattices of functions on topological spaces and of functions on uniform spaces. *Osaka Math. J.*, **1**, 1949, 166-181.

[99] L. D. N e l, Lattices of lower semi-continuous functions and associated topological spaces. *Pacific J. Math.*, **40** 3, 1972, 667-673.

[100] R. N i e l s o n, C. S l o y e r, Ideals of semicontinuous functions and compactifications of T_1-spaces. *Math. Ann.*, **187**, 1970, 329-331.

[101] L. D. N e l, R. G. W i l s o n, Enireflections in the category of T_0-spaces. *Fund. Math.*, **75**, 1972, 69-74.

[102] D. S. P a v l o v s k i i, Spaces of open sets and spaces of continuous functions. *Soviet Math. Dokl.*, **20**, 1979, 564-568.

[103] D. S. P a v l o v s k i i, On spaces of continuous functions. *Soviet Math. Dokl.*, **22**, 1980, 34-37.

[104] D. S. P a v l o v s k i i, On spaces with linear homeomorphic spaces of continuous functions in the pointwise convergence topology. *Uspchi Matem. Nauk.*, **37**, 1982, 185-186.

[105] E. M. P e c k, Lattice projections on continuous function spaces. *Pacific J. Math.*, **70**, 1976, 477-490.

[106] A. P e l c z y n s k i, Projection in certain Banach spaces. *Studia Math.*, **19**, 1960, 209-228.

[107] A. P e l c z y n s k i, Linear extensions, linear averagings, and their applications to linear topological classification of spaces of continuous functions. *Dissertations Math.*, **58**, 1968.

[108] V. G. P e s t o v, The coincidence of the dimensions in of l-equivalent topological spaces. *Soviet Math. Dokl.*, **26**, 1982, 380-383.

[109] R. S. P h i l l i p s, On linear transformations. *Trans. Amer. Math. Soc.*, **48**, 1940, 516-541.

[110] C. S. R o g e r s, J. E. J a y n e, C. D e l l a s h e r i e, F. T o p s e, J. H o f f m a n - T o r g e n s e n, D. A. M a r t i n, A. S. K e c h r i s, A. H. S t o n e, *Analytic Sets.* London, Academic Press, 1980.

[111] E. A. R e z n i c e n k o, The paracompactness is not preserved by the relation of l-equivalence. *Obsthaija Topologia. Prostranstva i Otobragenia*, Moscow University Press, Moscow, 1990, 155-156.

[112] W. R u d i n, Continuous functions on compact spaces without prefect subsets. *Proc. Amer. Math. Soc.*, **8**, 1957, 39-42.

[113] J. S a i n t - R a y m o n d, Fonctions borelleannes sur un quotient. *Bull. Sci. Math.* (2), **100**, 1976, 141-147.

[114] C. S a m u e l, Sur certains espaces $C(S)$. *Bull. Sci. Math.*, Ser. **2**, **95**, 1971, 65-82.

[115] B. E. S h a p i r o v s k i i, On a π-character and on a π-weight in bi-compacta. *Dokl. Acad. Nauk SSSR*, **223**, 1975, 799-802.

[116] Z. S e m a d e n i, Banach spaces non-isomorphic to their cartesian squares *II*. *Bull. Acad. Polon. Sci. Math.*, **8**, 1960, 81-84.

[117] Z. S e m a d e n i, *Banach Spaces of Continuous Functions.* PWN, Warszawa, 1971.

[118] T. S h i r o t a, A generalization of a theorem of J. Kaplansky. *Osaka Math. J.*, **4**, 1952, 121-132.

[119] M. H. S t o n e, Application of the theory of Boolean rings to general topology. *Trans. Amer. Math. Soc.*, **41**, 1937, 375-481.

[120] M. H. S t o n e, A general theory of spectral, II. *Proc. Nat. Acad.*, **27**, 1941.

[121] R. T e l g a r s k y, Total paracompactness and paracompact dispersed spaces. *Bull. Acad. Polon. Sci. Ser. Math.*, **16**, 1968, 567-572.

[122] W. J. T h r o n, *Topological Structures.* New York, 1966.

[123] H. T o r u n c z y k, Characterizing Hilbert spaces topology. *Fund. Math.*, **111**, 1981, 247-262.

[124] V. M. V a l o v, Linear topological classifications of certain function spaces. *Trans. Amer. Math. Soc.*, **327**, 1991, 583-600.

[125] V. M. V a l o v, Linear topological classifications of certain function spaces *II*. *Math. Pannonica*, **4/1**, 1993, 137-144.

[126] H. W a l l m a n, Lattice and topological spaces. *Ann. Math.*, **39**, 1938, 112-126.

[127] B. B. W e l l s, Jr., *Uncomplemented Spaces of Continuous Functions and Weak Compactness of Measures*. Ph. D. Thesis, University of California, Berkeley, 1967.

[128] R. D. M c W i l l i a m s, A note of weak sequential convergence. *Pacific J. Math.*, **12**, 1962, 333-335.

[129] D. E. W u l b e r t, Some complemented function spaces in $C(X)$. *Pacific J. Math.*, **24**, 1968, 589-602.

[130] D. E. W u l b e r t, A characterization of $C(X)$ for locally connected X. *Proc. Amer. Math. Soc.*, **21**, 1969, 269-272.

[131] L. G. Z a m b a h i d z e, On correlation between dimensional and cardinal functions of spaces imbedded in spaces of special form. *Bull. Acad. Sci. Georgian SSR*, **100**, 1980, 557-580.

Tiraspol State University *Received 12.02.1996*

Mailing address:
Str. Krasnodonskaija 72, ap. 199
278000 Tiraspol, MOLDOVA