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On Para-Kählerian Manifolds with Recurrent Paraholomorphic Projective Curvature

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In the present paper, we study analytic properties of the paraholomorphic projective curvature tensor P of a para-Kählerian manifold (M, J, g). The main result can be formulated in the following way: (i) The manifold (M, J, g) is paraholomorphic projective semi-symmetric ($R \cdot P = 0$) if and only if it is semi-symmetric ($R \cdot R = 0$); (ii) Let $\dim M = 2n \geq 4$. If the manifold (M, J, g) is of recurrent paraholomorphic projective curvature ($\nabla P = \psi \otimes P$, $P \neq 0$), then it is of recurrent curvature (precisely, $\nabla R = \psi \otimes R$, $R \neq 0$). If the manifold (M, J, g) is of recurrent curvature, then it is projectively flat or of recurrent paraholomorphic projective curvature. (iii) The manifold (M, J, g) has parallel paraholomorphic projective curvature ($\nabla P = 0$) if and only if it is locally symmetric ($\nabla R = 0$). Examples of para-Kählerian manifolds with recurrent curvature are discussed and a new example is constructed. These examples are also semi-symmetric.

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1. Introduction

By a para-Kählerian manifold we mean a triple (M, J, g) (see e.g. [3], [4]), where M is a connected differentiable manifold of dimension $m = 2n$, $J = (J_j^i)$ is a $(1,1)$-tensor field and $g = (g_{ij})$ is a pseudo-Riemannian metric on M satisfying the conditions

$$(1) \qquad (a)\ J_a^i J_j^a = \delta_j^i, \qquad (b)\ g_{ia} J_j^a + g_{ja} J_i^a = 0, \qquad (c)\ \nabla_k J_j^i = 0,$$

where ∇ is the Levi-Civita connection of g.

Let (M, J, g) be a para-Kählerian manifold. Denote by $R = (R_{hij}{}^k)$, $S = (S_{ij})$ and r the the Riemann-Christoffel curvature tensor, the Ricci curvature tensor and the scalar curvature, respectively. We assume the folowing convention

$S_{ij} = R_{aij}{}^a$ and $r = g^{ab}S_{ab}$. Since J is parallel, the curvature tensor must satisfy the relation

$$R_{kji}{}^a J_a^h - R_{kja}{}^h J_i^a = 0.$$

As it is well-known (see e.g. [2], [9]), this implies the following additional symmetry properties of the curvature tensors

$$(2)\qquad \begin{aligned} R_{abjk}J_h^a J_i^b &= -R_{hijk}, & R_{aijk}J_h^a &= R_{ahjk}J_i^a, \\ J^{ab}R_{aijb} &= S_{ia}J_j^a, & J^{ab}R_{abij} &= -2S_{ia}J_j^a, \\ S_{ia}J_j^a &= -S_{ja}J_i^a, & S_{ab}J_i^a J_j^b &= -S_{ij}. \end{aligned}$$

where $J^{ij} = J_a^i g^{aj}$ and $R_{hijk} = R_{hij}{}^s g_{sk}$.

Recall that the paraholomorphic projective curvature $(1,3)$-tensor field P of (M, J, g) is defined in the following manner (see [9], [10])

$$(3)\qquad \begin{aligned} P_{hij}{}^k = R_{hij}{}^k - \frac{1}{2(n+1)}\Big(& S_{ij}\delta_h^k - S_{hj}\delta_i^k \\ & + S_{ia}J_j^a J_h^k - S_{ha}J_j^a J_i^k - 2S_{ha}J_i^a J_j^k\Big). \end{aligned}$$

As it is proved in the cited papers, this tensor has strong geometric meaning. Namely, tensor P is an invariant of the paraholomorphic projective transformations; and for a para-Kählerian manifold of dimension $2n \geq 4$, the following conditions are equivalent: (a) $P = 0$, i.e. the manifold is paraholomorphically projectively flat; (b) the manifold is of constant paraholomorphic sectional curvature; (c) the manifold admits paraholomorphic free mobility; (d) the manifold satisfies the axiom of paraholomorphic planes. If $\dim M = 2$, then $P = 0$.

2. The recurrence of the curvature

A psudo-Riemannian manifold is said to be of recurrent curvature, if it is non flat and its curvature tensor satisfies the condition

$$(4)\qquad \nabla_s R_{hijk} = \varphi_s R_{hijk}$$

where $\varphi = (\varphi_i)$ is a 1-form, which is called the recurrence form (cf. e.g. [7], [13], [11]). Note that by the famous Wong's theorem ([14], Theorem 3.8, see also [15]), for a pseudo-Riemannian manifold of recurrent curvature, the curvature tensor does not vanish everywhere on the manifold. The non flat, locally symmetric manifolds form a special subclass of the class of pseudo-Riemannain manifolds of recurrent curvature (we have (4) with $\varphi = 0$). For symmetric para-Kählerian manifolds, one should see [6]; locally symmetric para-Kählerian manifolds were studied in [8]. The local classification of pseudo-Riemannian spaces of recurrent curvature which are not locally symmetric can be found in [13], [11].

We shall list certain examples of para-Kählerian manifolds of recurrent curvature. They are not locally symmetric in general.

Example 1. Any 2-dimensional para-Kählerian manifold with nonvanishing Gauss curvature everywhere is of recurrent curvature since so is any 2-dimensional pseudo-Riemannian manifold with nonvanishing Gauss curvature everywhere.

Example 2. Let M be an open connected subset of $\mathbf{R}^4$ and denote by (x^1, x^2, x^3, x^4) the Cartesian coordinates in $\mathbf{R}^4$. Let P be a function on M depending on x^1, x^3 only and such that $\frac{\partial^2 P}{\partial (x^3)^2} \neq 0$ everywhere on M. The metric g defined on M by

$$[g_{ij}] = \begin{bmatrix} P & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

is of recurrent curvature (cf. [13],p. 42-43; [11]). Define also the $(1,1)$-tensor field J on M by

$$[J^i_j] = \begin{bmatrix} a & 0 & b & 0 \\ -aP & -a & -\frac{1}{2}bP & -c \\ c & 0 & -a & 0 \\ -\frac{1}{2}bP & -b & 0 & a \end{bmatrix}$$

(the upper index indicates the raw number of the matrix), where a, b, c are constants such that $a^2 + bc = 1$. It is a straighforward verification that J and g fulfil conditions $(1a)$, $(1b)$, so that (J, g) is an almost para-Hermitian structure on M. Moreover, since the only non-zero Christoffel symbols of the metric g are as follows

$$\Gamma^2_{11} = \frac{1}{2}\frac{\partial P}{\partial x^1}, \quad \Gamma^1_{11} = -\frac{1}{2}\frac{\partial P}{\partial x^3}, \quad \Gamma^2_{13} = \frac{1}{2}\frac{\partial P}{\partial x^3},$$

one can immediately check that $(1c)$ is also fulifilled. Thus, (J, g) becomes a para-Kählerian structure on M.

Example 3. Let (M_1, J_1, g_1) be a 2-dimensional para-Kählerian manifold with nonvanishing Gauss curvature everywhere or the para-Kählerian manifold from Example 2. Let $(\mathbf{R}^{2m}_m, J_0, g_0)$ be the pseudo-Euclidean space of signature (m, m) endowed with the standard flat para-Käherian structure. Then the product para-Kählerian manifold $(M = M_1 \times \mathbf{R}^m, J_1 \times J_0, g_1 \times g_0)$ is also of recurrent curvature.

3. The recurrence of the paraholomorhic projective curvature

For an $(0,k)$-tensor field T on a pseudo-Riemannian manifold, define the $(0,k+2)$-tensor field $R\cdot T$ by the condition

$$
\begin{aligned}
(5)\qquad (R\cdot T)_{lmi_1i_2\ldots i_k} &= \nabla_l\nabla_m T_{i_1i_2\ldots i_k} - \nabla_m\nabla_l T_{i_1i_2\ldots i_k}\\
&= -R_{lmi_1}{}^{a}T_{ai_2\ldots i_k} - \cdots - R_{lmi_k}{}^{a}T_{i_1i_2\ldots i_{m-1}a}.
\end{aligned}
$$

In the above, the second equality is in fact the famous Ricci identity.

A pseudo-Riemanian manifold is said to be semi-symmetric if its curvature tensor satisfies the condition (see [12], [1])

$$
(6)\qquad R\cdot R = 0.
$$

Clearly, any (locally) symmetric para-Kählerian manifold is semi-symmetric. Note also that recurrence of the curvature always implies semi-symmetry [13], [11].

In the sequel, we rather use the paraholomorphic projective curvature $(0,4)$-tensor (with local components $P_{hijk} = P_{hij}{}^{s}g_{sk}$). Thus by (3), we have

$$
\begin{aligned}
(7)\qquad P_{hijk} = R_{hijk} - \frac{1}{2(n+1)}\Big(&S_{ij}g_{hk} - S_{hj}g_{ik}\\
&- S_{ia}J_j^a J_{hk} + S_{ha}J_j^a J_{ik} + 2S_{ha}J_i^a J_{jk}\Big),
\end{aligned}
$$

where $J_{ij} = g_{ia}J_j^a (= -J_{ji})$.

Generalizing condition (6), let us call a para-Kählerian manifold to be paraholomorphic projective semi-symmetric if

$$
(8)\qquad R\cdot P = 0.
$$

Using (5) and (7), for the paraholomorphic projective curvature tensor P, we obtain

$$
\begin{aligned}
(9)\qquad (R\cdot P)_{lmhijk} = (R\cdot R)_{lmhijk} - \frac{1}{2(n+1)}\Big(&(R\cdot S)_{lmij}g_{hk}\\
&- (R\cdot S)_{lmhj}g_{ik} - (R\cdot S)_{lmia}J_j^a J_{hk}\\
&+ (R\cdot S)_{lmha}J_j^a J_{ik} + 2(R\cdot S)_{lmha}J_i^a J_{jk}\Big).
\end{aligned}
$$

Let Q be the contracted tensor P, i.e., $Q_{hk} = g^{rs}P_{hrsk}$. Contracting (7) with g^{ij}, we have

$$
(10)\qquad Q_{hk} = \frac{1}{2(n+1)}(2nS_{hk} - rg_{hk}).
$$

Using definition (5) and (10), we find

$$(R \cdot Q)_{lmhk} = \frac{n}{n+1}(R \cdot S)_{lmhk}. \tag{11}$$

Theorem 1. *A para-Kählerian manifold (M, J, g) is paraholomorphic projective semi-symmetric if and only if it is semi-symmetric.*

Proof. It is obvious, that if $R \cdot R = 0$, then $R \cdot S = 0$, and consequently $R \cdot P = 0$ by (9). That is, the semi-symmetry always implies the paraholomorphic projective semi-symmetry. To prove the converse, note that we have in general

$$g^{ij}(R \cdot P)_{lmhijk} = (R \cdot Q)_{lmhk}.$$

Thus, assuming (8), one gets $R \cdot Q = 0$. This, by virtue of (11), gives also $R \cdot S = 0$. Finally, from (9) it follows (6). ■

Theorem 2. *Let (M, J, g) be a para-Kählerian manifold. If the paraholomorphic projective curvature tensor P of (M, J, g) does not vanish at each point of M and satisfies the condition*

$$(R \cdot P)_{lmhijk} = \theta_{lm} P_{hijk}, \tag{12}$$

$\theta = (\theta_{ij})$ being an antisymmetric $(0,2)$-tensor field, then $\theta = 0$. Moreover, the manifold is semi-symetric.

Proof. Contracting (12) with g^{ij}, we obtain

$$(R \cdot Q)_{lmhk} = \theta_{lm} Q_{hk}. \tag{13}$$

Moreover, from (11) and (13), we derive

$$(R \cdot S)_{lmhk} = \frac{n+1}{n} \theta_{lm} Q_{hk}. \tag{14}$$

Relations (12) and (14) applied to (9) enable us to find the following

$$(R \cdot R)_{lmhijk} = \theta_{lm} T_{hijk}, \tag{15}$$

where we have used the tensor $T = (T_{hijk})$ defined by the relation

$$\begin{aligned} T_{hijk} = P_{hijk} + \frac{1}{2n}(&Q_{ij} g_{hk} - Q_{hj} g_{ik} \\ &- Q_{ia} J^a_j J_{hk} + Q_{ha} J^a_j J_{ik} + 2Q_{ha} J^a_i J_{jk}). \end{aligned} \tag{16}$$

We shall use the following curvature identity

$$(R \cdot R)_{lmhijk} + (R \cdot R)_{hijklm} + (R \cdot R)_{jklmhi} = 0, \tag{17}$$

which is valid for arbitrary Riemannian as well as pseudo-Riemannian manifolds (cf. [11], p. 153). Applying (15) into (17), we derive

$$\theta_{lm} T_{hijk} + \theta_{hi} T_{jklm} + \theta_{jk} T_{lmhi} = 0. \tag{18}$$

It is a straightforward verification that if we substitute (7) and (10) into (16), then we can rewrite tensor T in the following way

$$T_{hijk} = R_{hijk} - \frac{r}{4n(n+1)}(g_{ij}g_{hk} - g_{hj}g_{ik} - J_{ij}J_{hk} + J_{hj}J_{ik} + 2J_{hi}J_{jk}). \tag{19}$$

Evidently, by (19), the following relation holds good

$$T_{hijk} = T_{jkhi}. \tag{20}$$

Note that under our assumptions, tensor $T \neq 0$ at every point of the manifold (in fact, at any point $p \in M$, $T(p) = 0$ always implies $P(p) = 0$).

Now, we need to use the following Walker's algebraic lemma (see [11], p. 153): If $a_{\alpha\beta}, b_\gamma$ are numbers satisfying

$$a_{\alpha\beta} = a_{\beta\alpha}, \qquad a_{\beta\gamma}b_\alpha + a_{\gamma\alpha}b_\beta + a_{\alpha\beta}b_\gamma = 0 \tag{21}$$

for $\alpha, \beta, \gamma = 1, 2, ..., N$, then either all the $a_{\alpha\beta}$ are zero or all the b_γ are zero.

Equations (20) and (18) are of the form (21) when suffixes α, β, γ are replaced by pairs hi, jk, lm. By the above Walker's lemma, $\theta_{lm} = 0$. Thus, $R \cdot P = 0$, which in view of the previous theorem gives the semi-symmetry of our manifold. ■

A para-Kählerian manifold will be said of recurrent paraholomorphic projective curvature if its paraholomorphic projective curvature tensor P is non identically zero and satisfies the condition

$$\nabla_s P_{hijk} = \psi_s P_{hijk} \tag{22}$$

for a certain 1-form $\psi = (\psi_i)$ called the recurrence form. Note that by the previously cited Wong's theorem, for a para-Kählerian manifold of recurrent paraholomorphic projective curvature, the tensor P does not vanish everywhere on the manifold.

Corollary. *The recurrence form of any para-Kählerian manifold of recurrent paraholomorphic projective curvature is closed. Moreover, the manifold is semi-symmetric.*

Proof. Note that condition (22) implies (12) with θ given by

$$\theta_{ij} = \nabla_i\psi_j - \nabla_j\psi_i.$$

By Theorem 2, the manifold is semi-symmetric and $\theta = 0$. Consequently $\nabla_i\psi_j - \nabla_j\psi_i = 0$, which means the recurrence form ψ is closed. ∎

Theorem 3. *Let (M, J, g) be a para-Kählerian manifold of dimension $2n \geq 4$.*

(i) If (M, J, g) is of recurrent curvature, then it is projectively flat or of recurrent paraholomorphic projective curvature with the same recurrence form.

(ii) If (M, J, g) is of recurrent paraholomorphic projective curvature, then it is of recurrent curvature with the same recurrence form.

Proof. (i) Assume that R is non-zero and satisfies (4). Then also $\nabla_s S_{ij} = \varphi_s S_{ij}$. Covariant differentiation of tensor P and using these two relations and (7), we find (22) with φ as the recurrence form. Now, we have two possibilities: P is identically zero or non-zero. This completes the proof of the first part of our theorem.

(ii) Assume that P is non-zero and satisfies (22). At first, rewrite (22) with the help (7) in the following manner

$$\begin{aligned}(23)\quad \nabla_s R_{hijk} = \psi_s R_{hijk} + \frac{1}{2(n+1)}\Big((\nabla_s S_{ij} - \psi_s S_{ij})g_{hk} \\ - (\nabla_s S_{hj} - \psi_s S_{hj})g_{ik} - (\nabla_s S_{ia} - \psi_s S_{ia})J^a_j J_{hk} \\ + (\nabla_s S_{ha} - \psi_s S_{ha})J^a_j J_{ik} + 2(\nabla_s S_{ha} - \psi_s S_{ha})J^a_i J_{jk}\Big).\end{aligned}$$

Next, contracting (23) with g^{ij}, we find the relation

$$\nabla_s S_{hk} - \psi_s S_{hk} = \frac{1}{2n}(\nabla_s r - \psi_s r)g_{hk},$$

which applied in (23) gives

$$(24)\quad \nabla_s R_{hijk} = \psi_s R_{hijk} + \xi_s(g_{hk}g_{ij} - g_{ik}g_{hj} - J_{ij}J_{hk} + J_{hj}J_{ik} + 2J_{hi}J_{jk}),$$

with

$$\xi_s = \frac{1}{4n(n+1)}(\nabla_s r - \psi_s r).$$

Because of (24), we have

$$\begin{aligned}(25)\quad (\nabla_p R_{rijk})R^r_{hsm} = \psi_p R_{rijk}R^r_{hsm} + \xi_p(R_{khsm}g_{ij} - R_{jhsm}g_{ik} \\ - R_{rhsm}J^r_k J_{ij} + R_{rhsm}J^r_j J_{ik} + 2R_{rhsm}J^r_i J_{jk}).\end{aligned}$$

As we already know, by Corollary 3, our manifold is semi-symmetric. Thus, by the famous Ricci's identity (see (5)), we have

$$R_{rijk}R^r_{hsm} + R_{hrjk}R^r_{ism} + R_{hirk}R^r_{jsm} + R_{hijr}R^r_{ksm} = 0. \tag{26}$$

Covariant differentiation of (26) yields

$$\begin{aligned}(\nabla_p R_{rijk})R^r_{hsm} + (\nabla_p R_{rhsm})R^r_{ijk} + (\nabla_p R_{rhkj})R^r_{ism} \\ + (\nabla_p R_{rism})R^r_{hkj} + (\nabla_p R_{rkhi})R^r_{jsm} + (\nabla_p R_{rjsm})R^r_{khi} \\ + (\nabla_p R_{rjih})R^r_{ksm} + (\nabla_p R_{rksm})R^r_{jih} = 0.\end{aligned} \tag{27}$$

By applying (25) into (27) and using again (26), we get

$$\begin{aligned}&\xi_p(R_{mijk}g_{hs} - R_{sijk}g_{hm} + R_{mhkj}g_{is} - R_{shkj}g_{im} + R_{mkhi}g_{js} \\ &-R_{skhi}g_{jm} + R_{mjih}g_{ks} - R_{sjih}g_{km} - R_{rijk}J^r_m J_{hs} - R_{rhkj}J^r_m J_{is} \\ &-R_{rkhi}J^r_m J_{js} - R_{rjih}J^r_m J_{ks} + R_{rijk}J^r_s J_{hm} + R_{rhkj}J^r_s J_{im} \\ &+R_{rkhi}J^r_s J_{jm} + R_{rjih}J^r_s J_{km} + 2R_{rijk}J^r_h J_{sm} + 2R_{rhkj}J^r_i J_{sm} \\ &+2R_{rkhi}J^r_j J_{sm} + 2R_{rjih}J^r_k J_{sm}) = 0.\end{aligned}$$

Contracting the last relation with g^{hs}, we find

$$\begin{aligned}&\xi_p(2(n+1)R_{mijk} + R_{mikj} + R_{mkji} + R_{mjik} + S_{ki}g_{jm} - S_{ji}g_{km} \\ &+ R_{rakj}J^r_m J^a_i + R_{rakj}J^{ra}J_{im} - R_{krai}J^r_m J^a_j - R_{krai}J^{ra}J_{jm} + R_{jrai}J^r_m J^a_k \\ &+ R_{jrai}J^{ra}J_{km} + 2R_{rakj}J^r_i J^a_m - 2R_{krai}J^r_j J^a_m + 2R_{jrai}J^r_k J^a_m) = 0.\end{aligned}$$

This, with the help of (2) and the first Bianchi identity, after certain long but standard computations, can be simplified to the form

$$\begin{aligned}&\xi_p(2(n+1)R_{kjim} - S_{ji}g_{km} + S_{ki}g_{jm} + S_{jr}J^r_i J_{km} - S_{kr}J^r_i J_{jm} \\ &-2S_{kr}J^r_j J_{im}) = 0,\end{aligned}$$

which in view of (7) can be written as

$$2(n+1)\xi_p P_{kjim} = 0.$$

Hence, since P is non-zero everywhere, it follows $\xi_p = 0$. Finally, using (24) and $\xi_p = 0$, we get (4) with ψ as the recurrence form. To be the assertion complete, it is sufficient to note that R is non-zero since so is P. ∎

R e m a r k. In Theorem 3, part (i), the projectively flatness can hapen only if the para-Kählerian manifold is locally symmetric. Indeed, paraholomorphic projectively flat para-Kählerian manifolds of dimension ≥ 4 are always locally symmetric, cf. [5].

As an immediate consequence of Theorem 3, we obtain:

Theorem 4. *A para-Kählerian manifold of dimension $2n \geq 4$ has parallel paraholomorphic projective curvature if and only if it is locally symmetric.*

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