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Interaction of Singularities to the Solutions of Some Classes of Semilinear Hyperbolic Equations and Systems with One and Two Space Variables

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1. It is well known that the singularities of the solutions of one space dimensional linear hyperbolic systems generated by the singularities of their initial /Cauchy/ data propagate along the corresponding characteristics. The paper [1] deals with a new, nonlinear effect for a special class of semilinear hyperbolic equations in which anomalous, i.e. new created singularities in comparison with the linear case appear. The new singularities are produced by the interaction of already existing singularities generated by the Cauchy data and propagating along the corresponding characteristics. It is rather interesting that the new born singularities are weaker in appropriate sense than the ones that create them. Moreover, they appear in the cross points of the characteristics carrying out the initial singularities and they propagate along (third) characteristics starting from the above mentioned cross points.

An important example of the appearence of anomalous singularities for one dimensional weakly hyperbolic systems was proposed in [2] and studied in more general frames in [3], [4]. It concerns the nonlinear interaction between two singularities - the first one created by a finite jump type discontinuity in the initial data and the second one generated by the weak hyperbolicity. As far as we know, several papers [3], [4], [8] deal with the optimal order of the anomalous singularities. More precisely, some of the components of the solution with C^∞ initial data possess finite jump type discontinuities which are created at the cross point A of two tangential at A characteristics $L_{1,2}$ carrying out the initial finite jump type discontinuites and moreover, these new singularities propagate along outgoing transversal with respect to $L_{1,2}$ characteristics L_j, $j \geq 3$ starting from A /see Fig. 1/. The larger the order of contact between L_1 and L_2 at the point A the smoother the solution is. We want to point out that in some cases a regularizing effect could aappear, i.e. finite jump type discontinuity of the Cauchy data can produce a C^∞ smooth component of solution near the transversal characteristics starting from A, while the infinite order of tangency between L_1 and L_2 at the point A inplies the C^∞ smoothness of the components of the solution near L_j, $j \geq 3$ /see [8]/.

2. And now we going to discuss the interaction of 3 conormal waves to the semilinear wave equation with 2 space variables. As it was proved by Bony developing the method of the second microlocalization and by Melrose–Ritter applying an other complicated approach in this situation new singularities in comparison with the linear case appear. A modification of the considerations of Keller–Ting [5] gives us an explicit form of the solution u in the case of very special nonlinear right-hand side. The solution u inside and up to the light cone of the future inscribed in the characteristic pyramide with vertex at the origin is written in the form of pointwise differentiable series. Moreover, u has a singularity from inside on the cone $\{|x| = t\}$ of the type $(t - |x|)^{5/2}$, $t > 0$, $|x| < t$, $|x| \to t$. The solution u is C^∞ smooth inside the light cone.

The above formulated results are proved in the following papers: [4], [8], [9].

3. The tools of nonlinear microlocal analysis enable us to prove regularity results and propagation of singularities theorems to the solutions of different classes of nonlinear partial differential equations in the scale of Sobolev spaces. We shall mention some investigations concerning the famous Monge-Ampère equation with two variables, i.e. the solution $u = u(x,y)$. We shall assume that $u \in H^s_{loc}$, $s > 4$ and that the Gaussian curvature K has a simple zero at (x_0, y_0), i.e. $K(x_0, y_0) = 0$, $\nabla K(x_0, y_0) \neq 0$. Put $\gamma = \{(x,y) : K(x,y) = 0\}$, $\Omega_- = \{(x,y) : K(x,y) < 0\}$. Hong-Zuily studied in [6] the case when γ is non-characteristic at (x_0, y_0) to the linearization (first variation) of the operator of Monge-Ampère at $u(x,y)$ and $u \in C^\infty(\Omega_-)$. Then they proved that $u \in C^\infty$ in a full neighbourhood of (x_0, y_0). In contrast with them we assume in [7] that the punctured smooth curve $\gamma \setminus \{(x_0, y_0)\}$ is non-characteristic to the first variation of the operator of Monge-Ampère at $u(x,y)$, γ is characteristic to the first variation at $u(x_0, y_0)$ and that $u \in C^\infty(\Omega_-)$. We show in this case that $u \in C^\infty$ in a full neighbourhood of (x_0, y_0). This way we conclude that there are no isolated singularities of the solutions to the Monge-Ampère type equation. A geometrical explanation of this effect can be given by the following observation. The curve γ turns out be an envelope of the characteristics of the linearized Monge-Ampère equation which are located in Ω_-. Moreover, the characteristics possess cusp point type singularities at $\gamma \setminus \{(x_0, y_0)\}$.

The corresponding proof can be found in [7], [9].

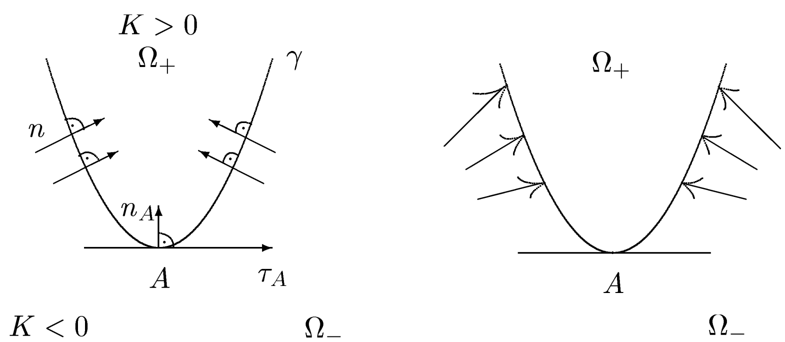

Fig. 1

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