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A Modal Logic Framework for Dynamic Conflict Detection

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This work suggests a set of concepts and procedures for detecting dynamic conflicts in conceptual schemata. Standard modal logic is used for expressing properties of schemata or specifications. The considered model is enriched with correspondence assertions expressing relationships between different properties described in the formal specifications.

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1. Introduction

In [9], a formal model for the analysis of conflicts in sets of autonomous agents described in a first-order language and by a transaction mechanism was presented. The approach took static as well as dynamic aspects of interaction into account. Some technical details of such a model was also considered in [5], [7], [8], where an approach using integration assertions [11], [12] was chosen to demonstrate how different aspects of integration of the models formulated in first-order logic can be analysed with respect to freeness of conflicts. Further, [3], [6], [13] discuss different varieties of temporal logic and BDI logic for the target language. However, due to complexity considerations in particular domains, the above approaches introduced an unnecessarily complicated framework that can be reduced by the introduction of modal operators.

In this work, we express properties of specifications or schemata as formulae containing modal operators. Thus, due to considering together static and dynamic features of specifications, namely as systems of modal logic, we define various procedures for dynamic conflict detection. The advantage here is that many different systems of modal logic can be handled in a uniform way, since all these systems can be given a clear interpretation in terms of common Kripke's semantics of possible worlds. Furthermore, a quite weak language is sufficient for the purpose of modelling and analysing important aspects of schema dynamics.

2. Preliminaries

In this section, we give a short overview of the basic concept used in the sequel. We recall a modal first-order logic framework for conceptual schemata, developed in [2].

2.1. Modal Logic

Modal logic has been developed for formalizing arguments involving the notions of possibility and necessity [4]. The predicate calculus can be extended with these modalities, as well. As was stressed in [10] modal logic (propositional and predicate) can be seen as a sub-logic of predicate logic, possibly with many sorts and generalized quantifiers.

The language of modal logic consists of a set of atomic formulae, logical connectives, e.g., $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$, as well as modal operators of *possibility* $\Diamond$ and *necessity* $\Box$. The formulae of the language are of the following form: *(i)* atomic formulae; *(ii)* if p and q are formulae, so are $\neg p$, $p \wedge q$, $p \vee q$, $p \rightarrow q$, $p \leftrightarrow q$, $\Box p$, $\Diamond p$. Any system of modal logic contains the axiom

$$Df\Diamond. \qquad \Diamond p \leftrightarrow \neg\Box\neg p,$$

and the various different systems of modal logic can be obtained by imposing other additional axioms.

The most important fact in this context is that the different systems have a clear interpretation in terms of Kripke's semantics of possible worlds. Thus, the semantic analysis of a system of modal logic is performed using the notion of a *model of modal logic*, which is usually viewed as a structure of the form [4], $M = \langle W, R, V \rangle$, where W denotes a *set of possible worlds (or states)*, R is a binary relation on W called *accessibility relation* and V is a multivalued mapping from the set of atomic formulae into W called *value assignment function*.

The interpretation of the accessibility relation R in a model of modal logic can vary significantly, but in general it may be thought as expressing the fact that some things may be possible from the standpoint of one world and impossible from the standpoint of another. Thus, $v \in R(w)$ means that world v is an alternative to w or v is possible to world w. Imposing various conditions on the accessibility relation, we obtain different classes of models of modal logic that determine different systems of modal logic. For instance, a system containing the axiom [4]:

$$T. \qquad \Box p \to p$$

corresponds to a class of reflexive models. The latter system is known as the normal system of modal logic, KT.[1]

The value assignment function V associates to each atomic formula p the set $V(p)$ of those possible worlds in which p is true. We use $\|p\|^M$ to denote the *truth set* of a formula p, i.e. the set of all worlds in which p is true. We say that a formula p is *true* (*valid*) in a model M if and only if $\|p\|^M = W$, i.e. in other words M is a model for p. The value assignment function is inductively extended to all non-modal formulae (formulae that do not contain $\Diamond$ and $\Box$) in the standard way. The truth conditions of modal formulae are defined by the accessibility relation R, i.e. for any formula p and any world $w \in W$, we have:

$$w \in \|\Diamond p\|^M \Leftrightarrow (\exists v \in W)(v \in R(w) \wedge v \in \|p\|^M)$$
$$w \in \|\Box p\|^M \Leftrightarrow (\forall v \in W)(v \in R(w) \Rightarrow v \in \|p\|^M).$$

2.2. Schema Representation

In this subsection, we briefly consider translations of the sets of concepts, introduced in [8], [9], in terms of modal logic.

Definition 2.1. Let L be a finite first-order language extended with the modal operators of possibility and necessity.

(1) A *schema* S is a normal system of modal logic KT consisting of a finite set of closed first-order formulae in a language L.

(2) The *transitions* between different states of a schema are represented by formulae $P(z) \to \Diamond Q(z)$, where $P(z)$ and $Q(z)$ are first-order formulae in L, and z is a vector of variables in the alphabet of L. [2]

(3) $L(S)$ is the *restriction* of L to S, i.e. $L(S)$ is the set $\{p \mid p \in L$, but p does not contain any predicate symbol, that is not a formula in $S\}$.

(4) An *integration assertion* expressing S_2 in S_1 is a closed first-order formula: $\forall(p(x) \leftrightarrow F(x))$, where p is a predicate symbol in $L(S_2)$ and $F(x)$ [3] is a formula in $L(S_1)$.

[1] An extended consideration of the *normal systems* of modal logic can be found in [4].
[2] The notation $A(x)$ means that x is free in $A(x)$.
[3] F(x) does not contain modal operators of possibility and necessity.

In [2], the description of a schema is interpreted in terms of standard model of modal logic. This is based on the fact that a normal system of modal logic is determined by each of its *canonical standard models* [4]. A model M is a canonical model for a system of modal logic S iff it verifies just those formulae that are the theorems of the system.

Definition 2.2. The *description of a schema S* is a standard model of modal logic $M = \langle W, R, V\rangle$, i.e. M is characterized by

(1) $W = \{w \mid w$ is $S-$maximal set of formulae$\}$ [4];

(2) $(\forall w \in W)((\Diamond F(x) \in w) \Leftrightarrow ((\exists v \in W)(v \in R(w) \wedge F(x) \in v)))$;

(3) For every atomic formula $F(x) \in L(S), V(F(x))$ is the proof set [5] of the formula $F(x)$.

Thus, W is the set of $S-$maximal sets of formulae, and in M just those atomic formulae are true at a world as are contained by it. Moreover, R is defined so that a world collects all the possibilitations of formulae occurring in its alternatives [4].

Example 2.1. Let us consider a schema $S_1 = \{\neg r(a) \vee r(b), r(c) \leftrightarrow r(a), r(a) \rightarrow \Diamond r(b), \neg r(a) \rightarrow \Diamond(\neg r(b) \wedge \neg r(c))\}$. The description of schema S_1 is a model $M_1 = \langle W_1, R_1, V_1\rangle$, where $W_1 = \{w_1, w_2, w_3\}$ and

$$w_1 = \{r(a), r(b), r(c), \Diamond r(b), \ldots\}$$
$$w_2 = \{\neg r(a), \neg r(b), \neg r(c), \Diamond(\neg r(b) \wedge \neg r(c)), \ldots\}$$
$$w_3 = \{\neg r(a), r(b), \neg r(c), \Diamond(\neg r(b) \wedge \neg r(c)), \ldots\}.$$

Then, due to Def. 2.2, the accessibility relation R_1 is given by the pairs $\{(w_1, w_1), (w_1, w_3), (w_2, w_2), (w_3, w_2), (w_3, w_3)\}$, and the value assignment function V_1 by:

$$V_1(r(a)) = \{w_1\} \qquad V_1(r(b)) = \{w_1, w_3\} \qquad V_1(r(c)) = \{w_1\}$$
$$V_1(\neg r(a)) = \{w_2, w_3\} \qquad V_1(\neg r(b)) = \{w_2\} \qquad V_1(\neg r(c)) = \{w_2, w_3\}.$$

3. Dynamic Conflict Detection

Intuitively, two schemata are in conflict with respect to a set of static integration assertions if one of them together with the integration assertions (IA) restrict the set of worlds (states) for the other one. Thus in [2], we have shown

[4] w is an $S-$maximal set of formulae when it is $S-$consistent and has only $S-$inconsistent proper extensions [4].

[5] A proof set of a formula $F(x)$, is the set of $S-$maximal sets of formulae containing $F(x)$.

that two schemata are free of conflicts w.r.t. IA when the model representing one of them together with the set of integration assertions is, in fact, a model for the other schema. In this work, we apply a different approach to conflict detection. Namely, two schemata are analysed for freeness of conflicts by considering the standard model of modal logic obtained by a disjunctive combination of the models representing the schemata. Such a consideration, as further is shown, is useful for analysing different aspects of schema dynamic.

Consider schemata S_1 and S_2, and assume that $M_1 = \langle W_1, R_1, V_1 \rangle$ and $M_2 = \langle W_2, R_2, V_2 \rangle$ are the corresponding standard models of modal logic that determine them (see subsection 2.2). Let IA be a set of integration assertions expressing S_2 in S_1. Moreover, assume that $L(S_1) \cap L(S_2) = \emptyset$. Further, we regard a model $M = \langle W_1 \times W_2, R, V \rangle$, where R and V^- are defined by : [6]

$$(\forall (w_1, w_2) \in W_1 \times W_2)\, (R\,((w_1, w_2)) = R_1(w_1) \times R_2(w_2))$$

$$(\forall (w_1, w_2) \in W_1 \times W_2)\, \left(V^-\,((w_1, w_2)) = {V_1}^-(w_1) \cup {V_2}^-(w_2)\right).$$

Clearly, M is a model for the system $S_1 \cup S_2$, i.e. each formula $F(x) \in S_1 \cup S_2$ is valid in the built model M.

Now, we consider all those worlds $(w_1, w_2) \in M$, in which the formula $F_{IA} = \bigwedge_{F(x) \in IA} F(x)$ holds, i.e. the worlds that are models for IA. Henceforth, these worlds, constituting the set $\|F_{IA}\|^M$, will be called *secure states*. Note that, in [1], we also regard worlds that are $S_1 \cup S_2 \cup IA-$consistent sets of formulae but in the model determining the system $S_1 \cup IA$. [7] In the framework herein, we define the concept of *freeness of conflicts* in terms of secure states.

Definition 3.1. The schemata S_2 and S_1 are *free of conflicts* w.r.t. IA iff $(\forall w_1 \in W_1)\left((\exists w_2 \in W_2)((w_1, w_2) \in \|F_{IA}\|^M)\right)$.

Thus, two schemata are free of conflicts w.r.t. a set of integration assertions (IA) iff for each world in the first model, there exists a world in the second one such that their union is a model for IA. Moreover, when the cardinality of $\|F_{IA}\|^M$ is less than the cardinality of W_1 then the schemata are in conflicts.

Example 3.1. Consider the schema S_1 from Example 2.1. and a schema $S_2 = \{p(a) \rightarrow p(b), p(b) \vee \neg p(c), (p(a) \wedge p(b)) \rightarrow \Diamond \neg p(b)\}$. Further, let $\forall x (p(x) \leftrightarrow r(x))$ be a possible integration assertion for the schemata. The description of schema S_2 is a model $M_2 = \langle W_2, R_2, V_2 \rangle$, where $W_2 = \{v_1, v_2, v_3, v_4, v_5\}$, and the following atomic formulae are contained by the worlds :

$v_1 \supseteq \{p(a), p(b), p(c)\}$ $\quad v_2 \supseteq \{\neg p(a), \neg p(b), \neg p(c)\}$ $\quad v_3 \supseteq \{\neg p(a), p(b), \neg p(c)\}$
$v_4 \supseteq \{p(a), p(b), \neg p(c)\}$ $\quad v_5 \supseteq \{\neg p(a), p(b), p(c)\}$.

[6] V^- is the inverse of the value assignment function V.

[7] In [1], these worlds are called *conflictfree states*.

Since $\|F_{IA}\|^M = \{(w_1, v_1), (w_2, v_2), (w_3, v_3)\}$, obviously the conditions for conflictfreeness, given in Def. 3.1, are fulfilled.

Observe that the model M contains information about the dynamic compatibility of states, as well. For example, a transition from a world (w_1, w_2) to a secure state is possible if only if $R((w_1, w_2)) \cap \|F_{IA}\|^M \neq \emptyset$. In view of this, we can introduce a definition weakening the conditions for freeness of conflicts. Intuitively, the schemata S_2 and S_1 can be seen as *weakly free of conflicts* with respect to a set of integration assertions if for every world w_1 in the model M_1 there exists a world w_2 in M_2 such that a transition starting from (w_1, w_2) to a secure state (world) in the model M is always possible. Since $\|\Diamond F_{IA}\|^M = R^-(\|F_{IA}\|^M)$ [8] [14], the next definition is given in terms of the set $\|\Diamond F_{IA}\|^M$. Moreover, due to the reflexivity of the model M, we have that $\|F_{IA}\|^M \subseteq \|\Diamond F_{IA}\|^M$, i.e. obviously, this definition states weaker conditions for conflictfreeness than Def. 3.1.

Definition 3.2. The schemata S_2 and S_1 are *weakly free of conflicts* w.r.t. IA iff $(\forall w_1 \in W_1)\Big((\exists w_2 \in W_2)((w_1, w_2) \in \|\Diamond F_{IA}\|^M)\Big)$.

Evidently, the freeness of conflicts according to the above definition ensures that for every world w_1 in the model M_1 there exists a world w_2 in M_2 such that (w_1, w_2) is either a secure state or there is a transition from it to some secure state. Thus, similarly reasoning the schemata S_2 and S_1 can be regarded as *strongly free of conflicts* w.r.t. IA if for every world w_1 in M_1 there exists a world w_2 in M_2 such that (w_1, w_2) is a secure state and moreover, the only possible transitions from it are those to secure states, i.e. $R((w_1, w_2)) \subseteq \|F_{IA}\|^M$.

Definition 3.3. The schemata S_2 and S_1 are *strongly free of conflicts* w.r.t. IA iff $(\forall w_1 \in W_1)\Big((\exists w_2 \in W_2)((w_1, w_2) \in \|\Box F_{IA}\|^M)\Big)$.

Since, R is reflexive we have that $\|\Box F_{IA}\|^M = R^+(\|F_{IA}\|^M)$ [9] [14], and moreover $\|\Box F_{IA}\|^M \subseteq \|F_{IA}\|^M$, i.e. the above definition is stronger than Def. 3.1 and 3.2.

Now, let us recall the concept of dynamic freeness of conflict, considered in [9]. It captures the following idea: if the schemata S_1 and S_2 are dynamically free of conflicts, then for every transition in S_2 from state v to a state w, there is a corresponding sequence of transitions in S_1. This sequence starts from a state of S_1 not in conflict with v and terminates in a state not in conflict with w. In [2], we have introduced conditions for dynamic freeness of conflicts in this sense. However, since this is a very demanding task from a computational viewpoint, in [2], [9], the amount of combined instances that should be investigated are

[8] $R^-(\|F_{IA}\|^M) = \{(w_1, w_2) \mid (w_1, w_2) \in W_1 \times W_2 \wedge R((w_1, w_2)) \cap \|F_{IA}\|^M \neq \emptyset\}$

[9] $R^+(\|F_{IA}\|^M) = \{(w_1, w_2) \mid (w_1, w_2) \in \mathrm{dom}(R) \wedge R((w_1, w_2)) \subseteq \|F_{IA}\|^M\}$

reduced by explicitly expressing which parts of a specification should have a similar behaviour as parts of another specification.

To determine dynamic freeness of conflict, we extend the set of integration assertions by adding formulae expressing correspondence between different transitions in the schemata [2]. The extended set of integration assertions, denoted by IE, contains formulae from the following form :

$$(P_2(z) \to \Diamond Q_2(z)) \leftrightarrow (P_1(z) \to \Diamond^n Q_1(z)), \tag{3.1}$$

where $P_2(z) \to \Diamond Q_2(z)$ and $P_1(z) \to \Diamond^n Q_1(z)$ are formulae, representing transitions between the different states of schemata S_2 and S_1, respectively. [10] Then modifying Def. 3.1 we introduce the concept of dynamic freeness of conflict w.r.t. IE. This concept is also defined in terms of secure states. Note that $\|F_{IE}\|^M \subseteq \|F_{IA}\|^M$ and moreover, the conditions given in the next definition require a correspondence between a transition in S_2 and a sequential combination of transitions in S_1 in addition to static compatibility of the schemata.

Definition 3.4. The schemata S_2 and S_1 are *dynamically free of conflicts* w.r.t. IE iff

(1) $(\forall w_1 \in W_1)\Big((\exists w_2 \in W_2)((w_1, w_2) \in \|F_{IA}\|^M)\Big)$;

(2) $(\forall w_2 \in W_2)\Big((\exists w_1 \in W_1)((w_1, w_2) \in \|F_{IE}\|^M)\Big)$.

Example 3.2. Consider the schemata S_1 and S_2 from Example 3.1 and let $IE = \{\forall x(p(x) \leftrightarrow r(x)), ((p(a) \wedge p(b)) \to \Diamond\neg p(b)) \leftrightarrow (r(a) \to (\Diamond^2(\neg r(b) \wedge \neg r(c))))\}$ be a set of extended integration assertions expressing S_2 in S_1. It can easily be checked that S_1 and S_2 are not dynamically free of conflicts w.r.t. IE, since the cardinality of $\|F_{IA}\|^M$ (see Ex. 3.1) is less than the cardinality of W_2.

In conclusion, note that the check for freeness of conflicts, according to Def. 3.2 and 3.3, takes into account the compatibility of states with respect to the possible transitions, as well, i.e. these definitions also state conditions for dynamic conflict detection. For example, Def. 3.2 can further be elaborated by considering instead the set $\|\bigvee_{i=1}^{m-1} \Diamond^i F_{IA}\|^M$ (m is the cardinality of W_1). Thus, a weaker condition is formulated by checking whether for every world in M_1 there exists a world in M_2 such that the corresponding world in M is either a secure state or is connected with a secure state by some sequence of transitions. Similarly, substituting the right hand side in (3.1) by the formula $(P_1(z) \to \bigvee_{i=1}^{n} \Diamond^i Q_1(z))$, we can weaken the requirements in Def. 3.4.

[10] $\Diamond^n F(x)$ is true in a world w iff for some $v \in W$ such that $v \in R^n(w)$, $F(x)$ is true in v, where R^n is the nth relative product of R with itself.

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