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Objective Systolic Arrays Functions

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Presented by P. Boyvalenkov

A big number of 2D SA exists suitable for implementation of given regular p-nested loop algorithm. Those SA-s could be significantly different, declared with theirs parameters, regardless of their having or not having, same topological structure. If we want to chose the best suitable SA, it is good to know their characteristics in advance, before its design and synthesis. In literature (see [5] and [7]), we can find a definition of big number of space-time characteristics (objective functions) SA and their determination procedure. The main interest of this paper are some of this space and time characteristics.
The importance of this characteristics depends of boundary nested loop algorithms and matrix transformation which enables synthesis of SA. We can't exert influence of boundary nested loop algorithm but it is possible to exert influence on matrix transformation. This privilege is enabled by the fact that the set of good matrix transformation correspond to the one of projection directions. Regardless if this set is finite we try to make it smoller. Therefore, it is important to intensify criteria of determining this subset so that the good matrix transformation which synthesizes SA with bad characteristics will be automatical excluded.

Key Words:systolic arrays,objective functions,adaptable algorithms,matrixs

1. Systolic arrays characteristics

Each regular 3-nested loop algorithm can be characterized(in space presented) by a pair (D;P_{int}), where $D = [\vec{e}_1{}^3\vec{e}_2{}^3\vec{e}_3{}^3]$ is a dependency matrix of dates, $P_{int} = \{(i,j,k) \mid 1 \le i \le N_1,\ 1 \le j \le N_2,\ 1 \le k \le N_3\}$ is set of points where the datas are calculated and N_1, N_2 and N_3 are the points of boundaries.
The SA implementation can be obtained by a linear transformation

$$T = \begin{bmatrix} \vec{\Pi} \\ S \end{bmatrix} = \begin{bmatrix} \vec{\Pi} \\ \vec{S}_1 \\ \vec{S}_2 \end{bmatrix} = \begin{bmatrix} t_{11}t_{12}t_{13} \\ t_{21}t_{22}t_{23} \\ t_{31}t_{32}t_{33} \end{bmatrix}. \tag{1.1}$$

$\vec{\Pi}$ is the time component of T
$\pm\vec{\Pi} = \left((\vec{e}_2^{\,3})^T - (\vec{e}_1\ 3)^T\right) \times \left((\vec{e}_3^{\,3})^T - (\vec{e}_1^{\,3})^T\right)$, $\vec{\Pi}\vec{e}_2^{\,3} > 0(< 0)$ for each i=1,2,3.
S is the space component of T which practical mapps P_{int} into 2D SA. Both are determined,(see [1],[3],[4],[6] and [7]), from following conditions :
matrix T must be nonsingular, $detT \neq 0$, the corresponding allowed projection direction $\vec{\mu} = [\mu_1\mu_2\mu_3]^T$ is orthogonal to the projection plane, $\vec{S}_1\vec{\mu} = 0$ and $\vec{S}_2\vec{\mu} = 0$, the connection between the PE's in synthesized 2D SA must be near-neighbor type, this requirement that elements of matrix $\triangle_S = S \cdot D$ have to be from the set $\{-1,0,1\}$, $t_{ij} \in \{-1,0,1\}$, $2 \leq i \leq 3$, $1 \leq j \leq 3$ and two optional position vectors $\vec{P}_1$ and $\vec{P}_2$ from P_{int} must not satisfy both equality $\vec{\Pi} \cdot \vec{P}_1 = \vec{\Pi} \cdot \vec{P}_2$ and $S \cdot \vec{P}_1 = S \cdot \vec{P}_2$.
Unfortunately the noticed conditions for the given projection direction $\vec{\mu} = [\mu_1\mu_2\mu_3]^T$ do not determine uniform matrix S and valid transformation T. All these matrix generates same SA but they can be significantly different in characteristics. Because of that it is important to create the new conditions:

- Condition that the projection direction $\vec{\mu} = [\mu_1\mu_2\mu_3]^T$ is orthogonal to the projection plane, can be exchanged with stronger

(1.2) $$\vec{\mu} = \vec{S}_1 \times \vec{S}_2 .$$

- In the case of planar 2D SA synthesis, for case $\mu_i \in \{-1,0,1\}$, $i = 1,2,3$, we put on elements of matrix S one of two alternate conditions (respectively for $\mu_1 = 1$ and $\mu_2 = \pm 1$) :

(1.3) $$t_{22}t_{32} + t_{23}t_{33} = 0 ,$$

(1.4) $$t_{21}t_{31} + t_{23}t_{33} = 0 .$$

(1.5) $$(D,P_{int}) \overset{T}{\circ\!\longrightarrow\!\circ} (\triangle, \bar{P}_{int})$$

By standard projection procedure, after selection the valid transformation T for given projection direction $\vec{\mu} = [\mu_1\mu_2\mu_3]^T$, synthesis of corresponding SA is on the basis of mapping T : $(D, P_{int}) -> (\triangle, \bar{P}_{int})$

Now, we give the procedures for determining basic space and time characteristics, using results from papers [5],[6] and [7].

Theorem 1.1.

$$\Omega_p = \begin{cases} N_1 N_2 N_3 & \text{if } a_i > N_i \text{ for some } i \le i \le 3 \\ N_1 N_2 N_3 - (N_1 - a_1)(N_2 - a_2)(N_3 - a_3) & \text{otherwise} \end{cases}, \quad (1.6)$$

$$a_i = \left| \frac{T_{1i}}{gcd(T_{11}, T_{12}, T_{13})} \right|, \quad (1.7)$$

T_{1i} is the (1,i)-cofactor of matrix T, $1 \le i \le 3$. With gcd(T_{11}, T_{12}, T_{13}) is notated the bigest common divisor of numbers T_{11}, T_{12} and T_{13}.
For determining array of 2D SA, in notation g_a, following result was used:

Theorem 1.2.

$$g_a = (N_1 - 1)(N_2 - 1)|T_{13}| + (N_1 - 1)(N_3 - 1)|T_{12}| + (N_2 - 1)(N_3 - 1)|T_{11}|. \quad (1.8)$$

The lengths of SA in direction x and y, in notation l_x and l_y, are given with:

$$\begin{cases} l_x = \max\limits_{(t,x,y) \in P^*_{int}} x - \min\limits_{(t,x,y) \in P^*_{int}} x + 1, \\ l_y = \max\limits_{(t,x,y) \in P^*_{int}} y - \min\limits_{(t,x,y) \in P^*_{int}} y + 1. \end{cases}$$

For determining l_x and l_y , following result was used:

Theorem 1.3.

$$\begin{cases} l_x = 1 + \sum\limits_{j=1}^{3} | t_{2j} | (N_j - 1), \\ l_y = 1 + \sum\limits_{j=1}^{3} | t_{3j} | (N_j - 1). \end{cases} \quad (1.9)$$

The chip area of SA, in notation area, is defined with:

$$area = l_x \cdot l_y . \quad (1.10)$$

Definition 1.1. The number of input/output elements once SA, in notation I/O is defined like the number of connection SA with encirclement. Also, it is well known that the summary time like time necessary for realization of given algorithm on synthetized SA is calculated on the base next:

Theorem 1.4.

$$T_{tot} = T_{in} + T_{exe} + T_{out}; \tag{1.11}$$

$$T_{exe} = \max_{(t,x,y)\in P^*_{int}} t - \min_{(t,x,y)\in P^*_{int}} t + 1. \tag{1.12}$$

t_p , called flow period of processor can be calculated on the base next

Theorem 1.5

$$t_p = |\frac{det(T)}{gcd(T_{11}T_{12}T_{13})}| . \tag{1.13}$$

Definition 1.2. Very important objective function for evaluation the realization once given algorithm on obtained SA is space-time complexity

$$AT = \Omega_p \cdot T \quad i.e. \quad AT^2 = \Omega_p \cdot T^2 . \tag{1.14}$$

Block flow period once SA, in notation ω ,is objective function important in the case when we use same SA for consecutive solving multi number of problems. For example, for M different problem identical volume we have

$$T_M = T + (M-1)\omega . \tag{1.15}$$

2.Main result

Let α be a regular 3-nested loop algorithm with index space $P_{int} = \{(i,j,k) \mid 1 \le i \le N_1, 1 \le j \le N_2, 1 \le k \le N_3\}$. We define the following subclasses of α.

Definition 2.1. If the computations in algorithm α, for some fixed j, can be executed with the permutations of variables for indexes i and k, we say that α is $\alpha(i,k)$ adaptable and if the computations in algorithm α, for some fixed i, can be executed with permutations of variables for indexes j and k, we say that α is $\alpha(j,k)$ adaptable.
If a given algorithm α is $\alpha(i,k)$ and $\alpha(j,k)$ adaptable, we say that α is adaptable.

If the given algorithm is from some of defined classes, its adaptation to the given projection direction $\vec{\mu} = [\mu_1\mu_2\mu_3]^T$ can be given with linear mapping H=(F,G), where F is 3×3 matrix which elements are in function of elements of vector $\vec{\mu}$ and G is 3×1 vector with constant elements which provides that after adaptation mapping

$$\text{H} : P_{int} \longrightarrow \bar{P}_{int} \tag{2.1}$$

the space $\bar{P}_{int}$ is again in coordinate systems first octant. We will now define the mapping H.

Definition 2.2. Suppose that a given algorithm is of type $\alpha(j,k)$. If allowable projection direction is in the form $\vec{\mu} = [1\mu_2\mu_3]^T$ the mapping H=(F,G) is defined by

$$F = \begin{bmatrix} 1 & 0 & 0 \\ \mu_2 & 1 & 0 \\ \mu_3 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 \\ g_2 \\ g_3 \end{bmatrix} \tag{2.2}$$

where g_2 and g_3 are smallest integers determined so that for each $[i,j,k]^T \in P_{int}$ the following equations are valid : $v = \mu_2 i + g_2 + j > 0$ and $w = \mu_3 i + k + g_3 > 0$. The elements v and W are defining on the base of next equation

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = F \cdot \begin{bmatrix} i \\ j \\ k \end{bmatrix} + G = \begin{bmatrix} i \\ \mu_2 i + j + g_2 \\ \mu_3 i + k + g_3 \end{bmatrix}. \tag{2.3}$$

Definition 2.3. Suppose that a given algorithm is $\alpha(i,k)$ type. If allowed projection direction is in the form $\vec{\mu} = [\mu_1 \pm 1 \mu_3]^T$ than mapping H=(F,G) is defined with

$$F = \begin{bmatrix} 1 & \pm\mu_1 & 0 \\ 0 & 1 & 0 \\ 0 & \pm\mu_3 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} g_1 \\ 0 \\ g_2 \end{bmatrix}, \tag{2.4}$$

where g_1 and g_3 are the smallest integers determined so that for each $[i,j,k]^T \in P_{int}$ the following equations are valid : $u = i \pm \mu_1 j + g_1 > 0$ and $w = k \pm \mu_3 j + g_3 > 0$.
The elements u and w are defined on the base of next equation

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = F \cdot \begin{bmatrix} i \\ j \\ k \end{bmatrix} + G = \begin{bmatrix} i \pm \mu_1 j + g_1 \\ j \\ k \pm \mu_3 j + g_3 \end{bmatrix}. \tag{2.5}$$

Now, instead of mapping given with (2.1), we are using two mappings :

(2.6)
$$P_{int} \xrightarrow{\ H\ } \hat{P}_{int} \qquad (D, \hat{P}_{int}) \xrightarrow{\ T\ } (\Delta, \bar{P}_{int})$$

.

where elements of space $\hat{P}_{int}$ are defined with (2.3) or (2.5), in dependence of projection direction $\vec{\mu}$. Let's now determine characteristics of these synthesized SA.

Theorem 2.1 *Suppose that a given algorithm α is $\alpha(j,k)$ (or $(\alpha(i,k))$ adaptable. The number of PEs, Ω_p, and geometric area, g_a, in the 2D SA obtained by the projection direction $\vec{\mu} = [1\mu_2\mu_3^T$ (or $\vec{\mu} = [\mu_1 \pm 1\mu_3]^T$) is*

(2.7) $$\Omega_p = N_2N_3, (\text{ or } \Omega_p = N_1N_3) \ ,$$

and

(2.8) $$g_a = (N_2-1)(N_3-1), (\text{ or } g_a = (N_1-1)(N_3-1)) \ .$$

Proof. Let $\vec{\mu} = [1\mu_2\mu_3]^T$. According to (2.6) and the form of matrix H, see (2.2), we can see that the magnitude of parameter Ω_p and g_a depends of elements of matrix's M
$M \ = \ T \circ F \ =$

(2.9) $$= \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ \mu_2 & 1 & 0 \\ \mu_3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} t_{11}+\mu_2t_{12}+\mu_3t_{13} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & t_{32} & t_{33} \end{bmatrix} .$$

From (2.9) we have $M_{12} = M_{13} = 0$ and $M_{11} = t_{22}t_{33} - t_{23}t_{32}$. According to equality (1.3) we have $M_11 = \mu_1 = 1$. With substitution $T_{1j} = M_{1j}, j = 1,2,3$ in Theorem 1.1 and 1.2 we could obtain needed result.

■

Corollary 2.1. *Suppose that a given algorithm α is adaptable. The number of PEs and geometric area in the 2D SA obtained by the projection direction $\vec{\mu} = [1 \pm 1\mu_3]^T$ is*

(2.10) $$\Omega_p = N_3 min\{N_1, N_2\} \ ,$$

and

(2.11) $$g_a = (N_3-1) min\{N_1-1, N_2-1\} \ .$$

Theorem 2.2 *Suppose that a given algorithm* α *is* $\alpha(j,k)$ (*or* $(\alpha(i,k))$ *adaptable. The chip area, area, in the 2D SA obtained by the projection direction* $\vec{\mu} = [\pm 1\mu_2\mu_3^T$ (*or* $\vec{\mu} = [\mu_1 \pm 1\mu_3]^T$) *is*

$$area \;=\; N_2 \cdot N_3 \;(\; or \; area \;=\; N_1 \cdot N_3 \;) \;. \tag{2.12}$$

Proof. To prove this Theorem we use procedure like in Theorem 2.1. ■

Corollary 2.2. *Suppose that a given algorithm* α *is adaptable. The chip area in the 2D SA obtained by the projection direction* $\vec{\mu} = [1 \pm 1\mu_3]^T$ *is*

$$area \;=\; N_3 \cdot min\{N_1, N_2\} \;. \tag{2.13}$$

Theorem 2.3 *Suppose that a given algorithm* α *is* $\alpha(j,k)$ (*or* $(\alpha(i,k)))$ *adaptable.* T_{exe} *in the 2D SA obtained by the projection direction* $\vec{\mu} = [1\mu_2\mu_3^T$ (*or* $\vec{\mu} = [\mu_1 \pm 1\mu_3]^T$) *is identical for SA obtained with non adaptable algorithm given in chapter 1 this paper i.e.*

$$T_{exe} \;=\; 1 \;+\; \sum_{j=1}^{3} |t_{1j}| \cdot (N_j - 1). \tag{2.14}$$

Proof. According to (1.1) and 2.2 (2.4), we can see that

$$t \;=\; (t_{11} + \mu_2 t_{12} + \mu_3 t_{13})i \;+\; t_{12}j \;+\; t_{13}k \;+\; g_2 t_{12} \;+\; g_3 t_{13},$$

$$T_{exe} = 1 + \mid t_{11} + \mu_2 t_{12} + \mu_3 t_{13} \mid (N_1 - 1) + \mid t_{12} \mid (N_2 - 1) + \mid t_{13} \mid (N_3 - 1) \quad \text{i.e.}$$

$$T_{exe} = \; 1 + \mid \vec{\Pi} \cdot \vec{\mu} \mid (N_1 - 1) + \mid t_{12} \mid (N_2 - 1) + \mid t_{13} \mid (N_3 - 1) \;. \tag{2.15}$$

On other side, according to (1.1) and 2.2 (2.4), we can see that

$$M \;=\; T \circ F \;=\; \begin{bmatrix} \vec{\Pi} \cdot \vec{\mu} & t_{12} & t_{13} \\ 0 & t_{22} & t_{23} \\ 0 & t_{32} & t_{33} \end{bmatrix} \tag{2.16}$$

and on the basis (1.13) for $t_p \;= \mid \vec{\Pi} \cdot \vec{\mu} \mid = \; 1$, we have

$$T_{exe} \;=\; 1 \;+\; \sum_{j=1}^{3} |t_{1j}| \cdot (N_j - 1).$$

■

Corollary 2.3. *Suppose that a given algorithm α is adaptable,$t_p > 1$ and obtained SA for projection direction $\vec{\mu} = [1\mu_2\mu_3^T$ (or $\vec{\mu} = [\mu_1 \pm 1\mu_3]^T$). In two consecutive solvings in some process element SA exists $t_p - 1$ idle speeds.*

3. 2D systolic arrays for matrix-matrix multiplication

Advantage of results obtained with using Theorem 2.1,2.2 and 2.3 and suitable consequences, in relationship with known results from literature, we can see on two rectangular matrix multiplication, A=(a_{ik}) order $N_1 \times N_3$ and B=(b_{kj}) order $N_3 \times N_2$. We determine projection direction $\vec{\mu} = [110]^T, \vec{\mu} = [111]^T$ and $\vec{\mu} = [211]^T$. We have chose them like representatives for synthesis three topological different SA. We meet with those kinds of SA very often in the literature (see [5] and [7]), and they are suitable for comparing with results from this paper. Case $N_1 = N_2 = N_3 = N$ will be given in parenthesis.

For direction $\vec{\mu} = [110]^T$ we obtain orthogonal 2D SA (we mark this SA with SV1). With standard procedure i.e. with procedure from chapter 1., the synthesized array have following characteristics:

$$\Omega_p = N_3(N_1 + N_2 - 1) \ , \ (\Omega_p = N(2N - 1)) \ ,$$

$$g_a = (N_3 - 1)(N_1 + N_2 - 1) \ , \ (g_a = 2(N - 1)^2) \ ,$$

$$area = N_3(N_1 + N_2 - 1) \ , \ (area = N(2N - 1)),$$

$$I/O = 2(N_1 + N_2 + 2N_3 - 1), \ (8N - 2),$$

$$t_p = 2,$$

$$T_{in} = N_2 - 1, \ (N - 1) \ ; \ T_{exe} = N_1 + N_2 + N_3 - 2, \ (3N - 2) \ ; \ T_{out} = N_2 - 1, \ (N - 1).$$

For projection direction $\vec{\mu} = [111]^T$ we obtain hexagonal 2D SA marked SV2, synthesized with standard procedure and $N_1 \geq N_2$, with characteristics:

$$\Omega_p = N_1(N_2 - 1) + N_2(N_3 - 1) + N_3(N_2 - 1) + 1, (3N^2 - 3N + 1),$$

$$g_a = (N_1 - 1)(N_2 - 1) + (N_1 - 1)(N_3 - 1) + (N_2 - 1)(N_3 - 1), \ \left(3(N - 1)^2\right) \ ,$$

$$area = (N_1 + N_2 - 1)(N_1 + N_3 - 1), \ \left((2N - 1)^2\right),$$

$$I/O = 12N,$$

$$t_p = 3,$$

$$T_{in} = N_2 - 1, \ (N - 1) \ ; \ T_{exe} = N_1 + N_2 + N_3 - 2, \ (3N - 2) \ ; \ T_{out} = N_2 - 1, \ (N - 1).$$

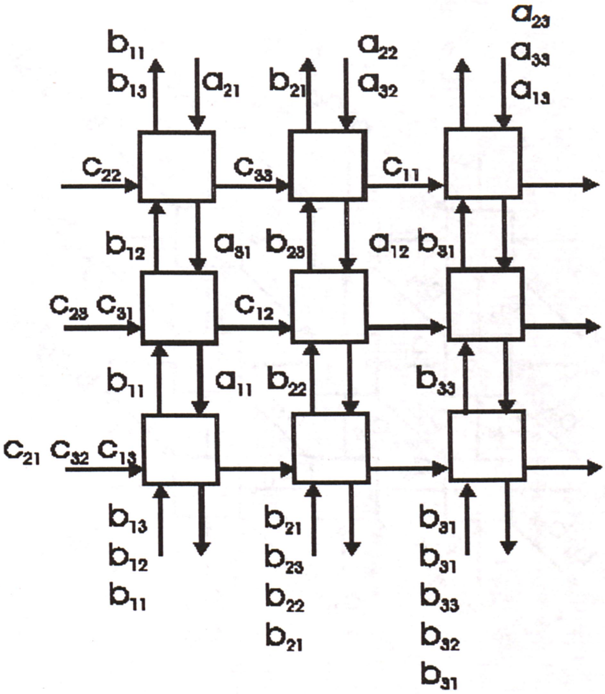

Figure 1.

And also, for projection direction $\vec{\mu} = [211]^T$ we obtain SA with crossing (we mark this SA with SV3). Synthesized with standard procedure from chapter 1. this SA have following characteristics:

$$\Omega_p = 2N_2N_3 + (N_1 - 2)(N_2 + N_3 - 1) \ , \ (\Omega_p = 4N^2 - 5N + 2) \ ,$$

$$g_a = (N_1-1)(N_2-1)+(N_1-1)(N_3-1)+2(N_2-1)(N_3-1) \ , \ (g_a = 4(N-1)^2) \ ,$$

$$area = (N_2 + N_3 - 1)(N_1 + N_2 + N_3 - 2) \ , \ (area = (2N - 1)(3N - 2)) \ ,$$

$$I/O = 2(2N_1 + 3N_2 + 3N_3 - 5), \ (16N - 10),$$

$$t_p = 4 \ ,$$

$$T_{in} = [3N/2] - 1, T_{exe} = N_1 + N_2 + N_3 - 2, \ (3N - 2), \ T_{out} = N - 1.$$

Using the results of Theorems 2.1,2.2 and 2.3 i.e. when we are using the procedure from chapter 2. based on 2.6, situation is completely different.

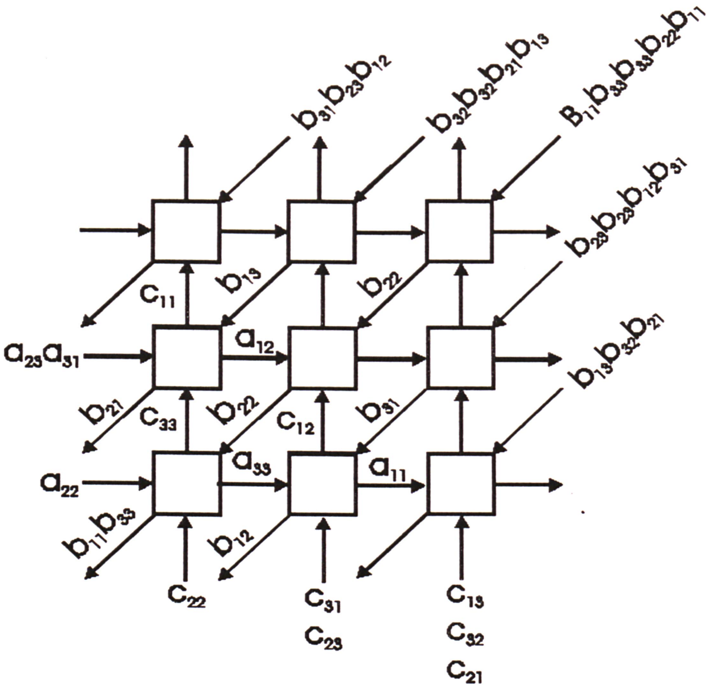

Figure 2.

For projection direction $\vec{\mu} = [110]^T$ and $\vec{\mu} = [111]^T$-($N_1 \geq N_2$), we have SA-s:

$$\Omega_p = N_3 min\,\{N_1, N_2\}\ ,\ (\Omega_p = N^2)\ ,$$

$$g_a = (N_3 - 1)(min\,\{N_1 - 1, N_2 - 1\}\ ,\ (g_a = (N-1)^2)\ ,$$

$$area = N_3 min\,\{N_1, N_2\}\ ,\ (area = N^2)\ ,$$

$$\text{for } \vec{\mu} = [110]^T\ :\ I/O = 2(2N_2 + N_3), (6N),$$

$$\text{for } \vec{\mu} = [111]^T\ :\ I/O = 2(2N_2 + 2N_3 - 1), (8N - 2),$$

$$t_p = 1,$$

$$T_{in} = max\,\{N_2, N_3\} - 1,\ (N-1)\ ;\ T_{exe} = \bar{N}_2 + N_3 - 1,\ (2N-1)\ ;\ T_{out} = 0,$$

$$\hat{N}_2 = \begin{cases} N_2, & N_2 \neq t_p \cdot m \\ N_2 + 1, & N_2 = t_p \cdot m\ . \end{cases}$$

For projection direction $\vec{\mu} = [211]^T$ we obtain SA, in existing notation SV3 :

$$\Omega_p = N_1 N_3\ ,\ (\Omega_p = N^2)\ ,$$

$$g_a = (N_1 - 1)(N_3 - 1)\ ,\ (g_a = (N-1)^2)\ ,$$

$$area = N_3(N_1 + N_3 - 1)\ ,\ (area = N(2N-1))\ ,$$

$$I/O = 2(3N_1 + 2N_3 - 2),\ (10N - 4),$$

$$t_p = 1\ ,$$

$$T_{in} = [3N/2] - 1, T_{exe} = N_1 + N_2 + N_3 - 2,\ (3N-2),\ T_{out} = 0.$$

For ilustration, figures 1, 2, 3 show analysed SA-s for projection direction $\vec{\mu} = [110]^T, \vec{\mu} = [111]^T$ and $\vec{\mu} = [211]^T$ respectively for case $N_1 = N_2 = N_3 = 3$ and when the parameters are defined with Theorems 2.1, 2.2 and 2.3.

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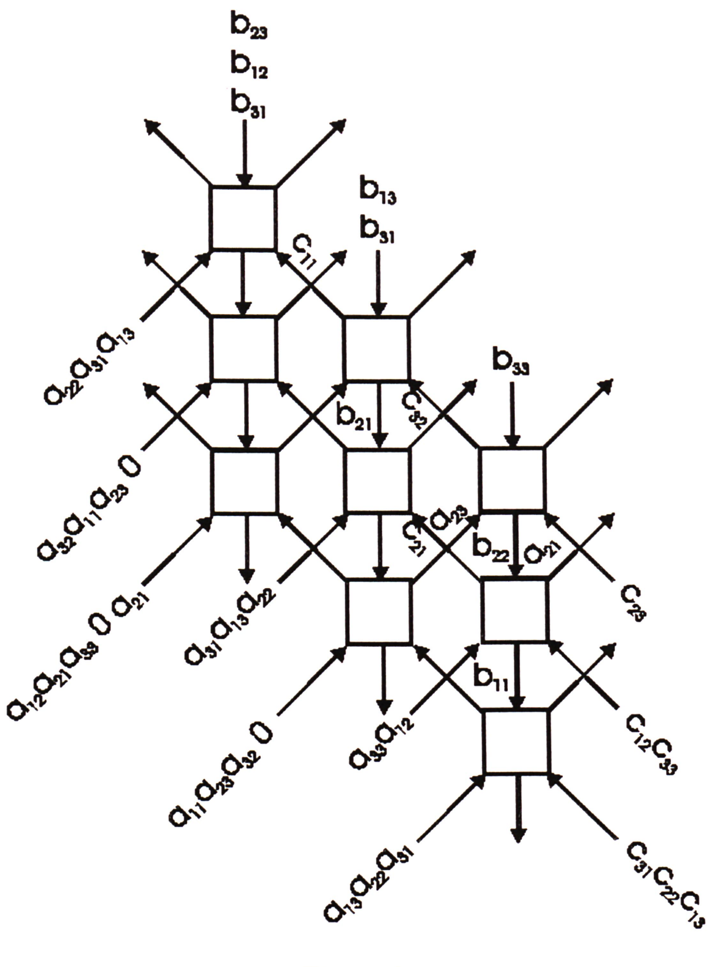

Figure 3.