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Simultaneous Approximation by Bivariate Schurer-Stancu Type Operators

Dan Bărbosu

Presented by P. Kenderov

Let p, q be positive integers and let $\alpha_1, \beta_1, \alpha_2, \beta_2$ be real parameters satisfying the conditions $0 \leq \alpha_1 \leq \beta_1$, $0 \leq \alpha_2 \leq \beta_2$. The bivariate Schurer-Stancu type operator $\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}$: $C\big([0,1+p]\times\times[0,1+q]\big) \to C\left([0,1]\times[0,1]\right)$ is defined at (1.3). Some approximation properties of this operator were investigated in [5]. The deal of the present paper to present properties of simultaneous approximation for the operator $\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q}$. As particular cases follow similar properties for Schurer, Stancu and Bernstein bivariate operators.

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1. Preliminaries

Let α, β be real parameters satysfying the conditions $0 \leq \alpha \leq \beta$ and let p be a non-negative integer. In [4] was considered the Schurer-Stancu operator $\widetilde{S}^{(\alpha,\beta)}_{m,p} : C\big([0,1+p]\big) \to C([0,1])$, defined for any $f \in C([0,1+p])$ and any $m \in \mathbb{N}$ by:

$$\left(\widetilde{S}^{(\alpha,\beta)}_{m,p} f\right)(x) = \sum_{k=0}^{m+p} \widetilde{p}_{m,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right) \tag{1.1}$$

where

$$\widetilde{p}_{m,k}(x) = \binom{m+p}{k} x^k (1-x)^{m+p-k} \tag{1.2}$$

are fundamental Schurer's polynomials [11].

In [4] were established some approximation properties of the operators (1.1), concerning a convergence theorem for the sequence $\left\{\widetilde{S}^{(\alpha,\beta)}_{m,p} f\right\}_{m\in\mathbb{N}}$ and an evaluation of the approximation order of $f \in C([0,1+p])$ by $\widetilde{S}_{m,p}^{(\alpha,\beta)} f$ on $[0,1]$ using the first order modulus of smoothness.

Supposing that $j \in \mathbb{N}$ is given, $f \in C^j([0,1+p])$ and denoting by D^j the j-th order differential operator, in [6] we proved

Theorem 1.1. *For any $f \in C^j([0,1+p])$ the sequence $\left\{D^j \widetilde{S}_{m,p}^{(\alpha,\beta)} f\right\}_{m\in\mathbb{N}}$ converges to $D^j f$, uniformly on $[0,1]$ $(0 \le j \le m+p)$.*

Let p, q be given non-negative integers and let α_1, β_1, α_2, β_2 be real parameters satisfying the conditions $0 \le \alpha_1 \le \beta_1$, $0 \le \alpha_2 \le \beta_2$.

Using the method of parametric extensions [8] in [6] was constructed the bivariate Schurer-Stancu type operator $\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} : C\big([0,1+p] \times [0,1+q]\big) \to C\big([0,1]\times[0,1]\big)$, defined for any $f \in C\big([0,1+p]\times[0,1+q]\big)$ and any $(m,n) \in \mathbb{N}\times\mathbb{N}$ by

$$\left(\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} f\right)(x,y) = \sum_{k=0}^{m+p}\sum_{j=0}^{n+q} \widetilde{p}_{m,k}(x)\widetilde{p}_{n,j}(y) f\left(\frac{k+\alpha_1}{m+\beta_1}, \frac{j+\alpha_2}{n+\beta_2}\right) \tag{1.3}$$

where $\widetilde{p}_{m,k}(x)$, $\widetilde{p}_{n,j}(y)$ are the fundamental Schurer polynomials defined at (1.2).

Some of them approximation properties were investigated in [6].

In what follows, we suppose that i and l are given positive integers and the approximated function f belong to the space $C^{i,j}\big([0,1+p]\times[0,1+q]\big)$.

2. Main results

Let $\widetilde{p}_{m,k}(x)$, $\widetilde{p}_{n,j}(y)$ be the fundamental Schurer polynomials defined at (1.2). By direct computation follows:

Lemma 2.1. *The fundamental Schurer polynomials $\widetilde{p}_{m,k}(x)$, $\widetilde{p}_{n,j}(y)$ verify the following identities:*

$$\begin{aligned} D^1\widetilde{p}_{m,k}(x) &= (m+p)\left\{\widetilde{p}_{m-1,k-1}(x) - \widetilde{p}_{m-1,k}(x)\right\} = \\ &= \frac{k-(m+p)}{x(1-x)}\,\widetilde{p}_{m,k}(x) \end{aligned} \tag{2.1}$$

$$\begin{aligned} D^1\widetilde{p}_{n,j}(y) &= (n+q)\left\{\widetilde{p}_{n-1,j-1}(y) - \widetilde{p}_{n-1,j}(y)\right\} = \\ &= \frac{j-(n+q)}{y(1-y)}\,\widetilde{p}_{n,j}(y) \end{aligned} \tag{2.2}$$

where D^1 denotes the first order differential operator and $\widetilde{p}_{0,0}(x) := 0$, $\widetilde{p}_{0,0}(y) := 0$, $\widetilde{p}_{k,-1}(x) := 0$, $\widetilde{p}_{j,-1}(y) = 0$, $\widetilde{p}_{m-1,m+p-1}(x) := 0$, $\widetilde{p}_{n-1,n+q-1}(y) := 0$.

In what follows the brackets denote devided differences

Lemma 2.2. *The following equalities*

$$D^{1,1}\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}f(x,y) =$$
$$= (m+p)(n+q)\sum_{k=0}^{m+p-1}\sum_{j=0}^{n+q-1}\widetilde{p}_{m-1,k}(x)\widetilde{p}_{n-1,j}(y)\cdot$$
$$\cdot\,\Delta_{\frac{1}{m+p},\frac{1}{n+q}}f\left(\frac{k+\alpha_1}{m+\beta_2},\frac{j+\alpha_2}{n+\beta_2}\right) =$$
$$= \frac{(m+p)(n+q)}{(m+\beta_2)(n+\beta_2)}\sum_{k=0}^{m+p-1}\sum_{j=0}^{n+q-1}\widetilde{p}_{m-1,k}(x)\widetilde{p}_{n-1,j}(y)\cdot$$
$$\cdot\left[\begin{matrix}\frac{k+\alpha_1}{m+\beta_1},\frac{k+\alpha_1+1}{m+\beta_1+1} \\ \frac{j+\alpha_2}{n+\beta_1},\frac{j+\alpha_2+1}{n+\beta_2+1}\end{matrix};f\right] \tag{2.3}$$

hold, where $D^{i,j}$ *denotes the* (i,j)*-th order partial differentiation operator, and:*

$$\Delta_{\frac{1}{m+\beta_1},\frac{1}{n+\beta_2}}f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right) =$$
$$= \Delta_{\frac{1}{m+\beta_1}}\left(\Delta_{\frac{1}{n+\beta_2}}f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)\right) \tag{2.4}$$

$$\left[\begin{matrix}\frac{k+\alpha_1}{m+\beta_1},\frac{k+\alpha_1+1}{m+\beta_1} \\ \frac{j+\alpha_2}{n+\beta_1},\frac{j+\alpha_2+1}{n+\beta_2}\end{matrix};f\right] =$$
$$= \left[\frac{k+\alpha_1}{m+\beta_1},\frac{k+\alpha_1+1}{m+\beta_1};\left[\frac{j+\alpha_2}{n+\beta_2},\frac{j+\alpha_2+1}{n+\beta_2};f\right]\right]. \tag{2.5}$$

P r o o f. In [6], Lemma 2.2 we proved the equalities:

$$D^1\widetilde{S}_{m,p}^{(\alpha,\beta)}f(x) = (m+p)\sum_{k=0}^{m+p-1}\widetilde{p}_{m-1,k}(x)\Delta_{\frac{1}{m+\beta}}f\left(\frac{k+\alpha}{m+\beta}\right) =$$
$$= \frac{m+p}{m+\beta}\sum_{k=0}^{m+p-1}\widetilde{p}_{m-1,k}(x)\left[\frac{k+\alpha}{m+\beta},\frac{k+\alpha+1}{m+\beta+1};f\right].$$

Next we apply the method of parametric extensions [8]. ∎

Theorem 2.1. *For any non-negative integers i and l satisfying the conditions $i \le m+p$, $l \le n+q$, the operator* (1.3) *verifies:*

$$D^{i,l}\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f(x,y) =$$

$$= (m+p)^{[i]}(n+q)^{[l]} \sum_{k=0}^{m+p-i}\sum_{j=0}^{n+q-l} \widetilde{p}_{m-i,k}(x)\widetilde{p}_{n-l,j}(y)\cdot$$

$$\cdot \Delta_{\frac{1}{m+\beta_1},\frac{1}{n+\beta_2}} f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right) \tag{2.6}$$

where

$$\begin{aligned}(m+p)^{[i]} &= (m+p)(m+p-1)\dots(m+p-i+1)\\ (n+q)^{[l]} &= (n+q)(n+q-1)\dots(n+q-l+1).\end{aligned}$$

P r o o f. In [6] Theorem 2.1 we established the equality:

$$D^i\widetilde{S}^{(\alpha,\beta)}_{m,p} f(x) = (m+p)^{[i]} \sum_{k=0}^{m+p-i} \widetilde{p}_{m-i,k}(x) f\left(\frac{k+\alpha}{m+\beta}\right)$$

Taking the above identity into account and applying the method of parametric extensions [8] yields the desired equality (2.6). ∎

Corollary 2.1. *For any integers i, l satisfying the conditions $1 \le i \le m+p$, $1 \le l \le n+q$ the operator* (1.3) *satisfies:*

$$D^{i,l}\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f(x,y) = \frac{(m+p)^{[i]}}{(m+\beta_1)^i}\,\frac{(n+q)^{[l]}}{(n+\beta_2)^l}\, i!\, l! \sum_{k=0}^{m+p-i}\sum_{j=0}^{n+q-l}\cdot$$

$$\cdot\, \widetilde{p}_{m-i,k}(x)\widetilde{p}_{n-j,l}(y) \begin{bmatrix} \frac{k+\alpha_1}{m+\beta_1},\dots,\frac{k+i+\alpha_1}{m+\beta_1} & \\ & ;f \\ \frac{j+\alpha_2}{n+\beta_2},\dots,\frac{j+l+\alpha_2}{n+\beta_2} & \end{bmatrix}. \tag{2.7}$$

P r o o f. One applies Theorem 2.1 taking the following identity

$$\begin{bmatrix} \frac{k+\alpha_1}{m+\beta_1},\dots\frac{k+\alpha_1+i}{m+\beta_1} & \\ & ;f \\ \frac{j+\alpha_2}{n+\beta_2},\dots,\frac{j+\alpha_2+l}{n+\beta_2} & \end{bmatrix} =$$

$$= \frac{(m+\beta_1)^i}{i!}\,\frac{(n+\beta_2)^l}{l!}\,\Delta^i_{\frac{1}{m+\beta_1},\frac{1}{n+\beta_2}} f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)$$

into account. ■

We are now ready to prove the main result of the paper, which is the following

Theorem 2.2. *Let i, l be two given integers satisfying $0 \le i \le m+p$, $0 \le l \le n+q$. For any $f \in C^{i,l}\left([0,1+p]\times[0,1+q]\right)$ the sequence*

$$\left\{D^{i,l}\widetilde{S}_{m,n,p,q}^{(\alpha_1,\beta_2,\alpha_2,\beta_2)}f\right\}_{m,n\in\mathbb{N}}$$

converges to $D^{i,l}f$, uniformly on $[0,1]\times[0,1]$.

P r o o f. For $i = l = 0$ the assertion was proved ([6], Theorem 3.2).

For $i = 0$, $l \ge 1$ or $l = 0$, $i \ge 1$ the assertion follows from Theorem 3.1 [5], applied to the partial function $f(x,*)$ and respectively $f(\circ, y)$.

If $i \ge 1$, $l \ge 1$, using the symbol of Landau (see for example [1]) the relations $\lim\limits_{m\to\infty}\mu\left(\dfrac{1}{m}\right) = 0$, $\lim\limits_{n\to\infty}\gamma\left(\dfrac{1}{n}\right) = 0$, can be expressed under the form $\mu\left(\dfrac{1}{m}\right) = o(1)$ and respectively $\gamma\left(\dfrac{1}{n}\right) = o(1)$.

Taking the above remarks into account, we can write

$$(m+p)^{[i]}(n+q)^{[l]}\Delta^{i,l}_{\frac{1}{m+\beta_1},\frac{1}{n+\beta_2}} f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right) =$$

$$= (m+p)^{i}(n+q)^{l}\{1+o(1)\}^2 i!\,l!\begin{bmatrix}\dfrac{k+\alpha_1}{m+\beta_1},\dots,\dfrac{k+i+\alpha_1}{m+\beta_1} & \\ & ;f \\ \dfrac{j+\alpha_2}{n+\beta_2},\dots,\dfrac{j+l+\alpha_2}{n+\beta_2} & \end{bmatrix} \tag{2.8}$$

Next, using the mean theorem for bivariate devided differences, from (2.8) follows that there exists

$$(\xi_k,\eta_j) \in \left]\frac{k+\alpha_1}{m+\beta_1},\frac{k+\alpha_1+i}{m+\beta_1}\right[\times\left]\frac{j+\alpha_2}{n+\beta_2},\frac{j+\alpha_2+l}{n+\beta_2}\right[$$

such that

$$(m+p)^{[i]}(n+q)^{[l]}\Delta^{i,l}_{\frac{1}{m+\beta_1},\frac{1}{n+\beta_2}} f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right) =$$

$$= (1+o(1))^2\, D^{i,l}f\left(\xi_k,\eta_j\right)\,. \tag{2.9}$$

On the other hand, we have

$$D^{i,l}f\left(\xi_k,\eta_j\right) = \left(D^{i,l}f\left(\xi_k,\eta_j\right) - D^{i,l}f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)\right) +$$

$$+ D^{i,l}f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right) = o(1) + D^{i,l}f\left(\frac{k+\alpha_1}{m+\beta_1},\frac{j+\alpha_2}{n+\beta_2}\right)$$

because $$\left|\xi_k - \frac{k+\alpha_1}{m+\beta_1}\right| < \frac{i}{m+\beta_1}, \ \left|\eta_j - \frac{j+\alpha_2}{n+\beta_2}\right| < \frac{l}{n+\beta_2}$$

and $f \in C^{i,l}\left([0,1+p]\times[0,1+q]\right)$.
This way follows the identity:

$$(1+o(1))^2\, D^{i,l} f\left(\xi_k,\eta_j\right) = D^{i,l} f\left(\frac{k+\alpha_1}{m+\beta_1}, \frac{j+\alpha_2}{n+\beta_2}\right) + o(1)\,. \tag{2.10}$$

Applying next Theorem 2.1 and taking into account that

$$(\xi_k,\eta_j) \in \left]\frac{k+\alpha_1}{m+\beta_1}, \frac{k+\alpha_1+i}{m+\beta_1}\right[\times \left]\frac{j+\alpha_2}{n+\beta_2}, \frac{j+\alpha_2+l}{n+\beta_2}\right[,$$

we get

$$D^{i,l}\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n,p,q} f(x,y) = \widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m+p-i,n+q-l} D^{i,l} f(x,y) + o(1). \tag{2.11}$$

Because the sequence $\left\{\widetilde{S}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m+p-i,n+q-l} g\right\}_{m+p\geq i,n+q\geq l}$ converges to g, uniformly on $[0,1]\times[0,1]$ for any $g \in C\left([0,1+p]\times[0,1+p]\right)$, from (2.11) we arrive to the desired result. ■

Corollary 2.2. *For any $f \in C^{i,l}\left([0,1+p]\times[0,1+q]\right)$, the sequence $\left\{D^{i,l}\widetilde{B}_{m,n,p,q} f\right\}_{m,n\in\mathbb{N}}$ converges to $D^{i,l}f$ uniformly on $[0,1]\times[0,1]$, where $\widetilde{B}_{m,n,p,q}$ is the bivariate Schurer operator, defined by*

$$\left(\widetilde{B}_{m,n,p,q} f\right)(x,y) = \sum_{k=0}^{m+p}\sum_{j=0}^{n+q} \widetilde{p}_{m,k}(x)\widetilde{p}_{n,j}(y) f\left(\frac{k}{m}, \frac{j}{n}\right). \tag{2.12}$$

Proof. Because $\widetilde{B}_{m,n,p,q} = \widetilde{S}^{(0,0,0,0)}_{m,n,p,q}$, the assertion follows from Theorem 2.2. ■

Corollary 2.3. *For any $f \in C^{i,l}\left([0,1]\times[0,1]\right)$ the sequence*

$$\left\{D^{i,l} P^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n} f\right\}_{m,n\in\mathbb{N}}$$

converges to $D^{i,l}f$ uniformly on $[0,1]\times[0,1]$, where $P^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n}$ denotes the bivariate Stancu operator, defined by

$$\left(P^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}_{m,n} f\right)(x,y) = \sum_{k=0}^{m}\sum_{j=0}^{n} p_{m,k}(x)p_{n,j}(y) f\left(\frac{k+\alpha_1}{m+\beta_1}, \frac{j+\alpha_2}{n+\beta_2}\right). \tag{2.13}$$

Proof. Clearly that $P_{m,n}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)} = \widetilde{S}_{m,n,0,0}^{(\alpha_1,\beta_1,\alpha_2,\beta_2)}$ and one applies Theorem 2.2. ∎

Corollary 2.4. *For any $f \in C^{i,l}([0,1]\times[0,1])$ the sequence $\left\{D^{i,l}B_{m,n}f\right\}_{m,n\in\mathbb{N}}$ converges to $D^{i,l}f$, uniformly on $[0,1]\times[0,1]$, where $B_{m,n}$ is the bivariate Bernstein operator, defined by*

$$(B_{m,n}f)(x,y) = \sum_{k=0}^{m}\sum_{j=0}^{n} p_{m,k}(x)p_{n,j}(y)f\left(\frac{k}{m},\frac{j}{n}\right). \tag{2.14}$$

Proof. Because $B_{m,n} = \widetilde{S}_{m,n,0,0}^{(0,0,0,0)}$, the assertion follows from Theorem 2.2. ∎

Remark 2.1. In Corollary 2.1 the integers i,l satisfy the conditions $0 \le i \le m+p$, $0 \le l \le n+q$ while in Corollary 2.2 and Corollary 2.3 the integers i and l satisfy the conditions $0 \le i \le m$, $0 \le l \le n$.

The polynomials $p_{m,k}(x)$, $p_{n,j}(y)$ from (2.13), and respectively (2.14) are the fundamental Bernstein polynomials defined respectively by:

$$p_{m,k}(x) = \binom{m}{k}x^k(1-x)^{m-k}, \quad p_{n,j}(y) = \binom{n}{j}y^j(1-y)^{n-j}. \tag{2.15}$$

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North University of Baia Mare
Department of Mathematics and Computer Science
Victoriei 76, 430122 Baia Mare, ROMÂNIA
E-mail: danbarbosu@yahoo.com

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