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On a Type of P-Sasakian Manifolds

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We consider P-Sasakian manifolds satisfying the conditions $R \cdot P = 0$, $P \cdot R = 0$, $C \cdot P = 0$, $P \cdot C = 0$ and $R \cdot L = 0$, where R is the Riemannian curvature tensor, P is the Weyl projective curvature tensor, C is the Weyl conformal curvature tensor and L is the contact Ricci tensor.

Key Words: para-Sasakian manifold, projective curvature tensor, Weyl tensor, Ricci tensor, Einstein manifold and η-Einstein manifold.

1. Introduction

In [4], T. Adati and K. Matsumoto introduced the notion of para-Sasakian manifold or briefly P-Sasakian manifold which are considered as special cases of an almost para-contact manifold (see [11]).

A Riemannian manifold (M, g) is called *locally symmetric* if its curvature tensor R is parallel, i.e., $\nabla R = 0$. As a proper generalization of locally symmetric manifolds, the notion of semisymmetric manifolds was defined by $R(X,Y) \cdot R = 0$, where $R(X,Y)$ is considered as a derivation of tensor algebra at each point of the manifold. A complete intrinsic classification of these manifolds was given by Z.I.Szabo in [13]. In [7], U. C. De and D. Tarafdar studied semisymmetric P-Sasakian manifolds, that is P-Sasakian manifolds satisfying the condition $R(X,Y) \cdot R = 0$. In [6], U. C. De and N. Guha studied Weyl semisymmetric

P-Sasakian manifolds, that is P-Sasakian manifolds satisfying the condition $R(X,Y)\cdot C=0$ and they prove that such a manifold is an SP-Sasakian manifold, where C denotes the Weyl conformal curvature tensor defined by

$$\begin{aligned}C(X,Y)Z &= R(X,Y)Z-\tfrac{1}{n-1}\{S(Y,Z)X-S(X,Z)Y\\ &+g(Y,Z)QX-g(X,Z)QY)\}+\tfrac{r}{(n-1)(n-2)}\{g(Y,Z)X-g(X,Z)Y\},\end{aligned} \tag{1}$$

and

$$g(QX,Y)=S(X,Y),$$

r is the scalar curvature of M. In [8], U. C. De and J. C. Ghosh studied contact manifold satisfying the condition $R(\xi,X)\cdot R=0$, where ξ belongs to the k-nullity distribution. In [10], C. Özgür studied Weyl pseudosymmetric P-Sasakian manifolds. In this study we consider a P-Sasakian manifolds satisfying the condition $R(X,Y)\cdot P=0$, where P denotes the Weyl projective curvature tensor (see [15]) defined by

$$P(X,Y)Z=R(X,Y)Z-\frac{1}{n-1}[g(Y,Z)QX-g(X,Z)QY]. \tag{2}$$

Here we take dimension $n>2$, because for $n=2$, the projective curvature tensor identically vanishes. $R(X,Y)\cdot P=0$ means the manifold is Weyl projective semi-symmetric manifold.

At first we obtain a necessary and sufficient condition for a P-Sasakian manifold to beWeyl projective semisymmetric. Next in Section 4 , it is shown that if a P-Sasakian manifold satisisfies $P\cdot R=0$ then the scalar curvature is constant. In the fifth section we study the conditions $P\cdot C=0$ and $C\cdot P=0$ on a P-Sasakian manifold and we prove that if one of these conditions is satisfied on a P-Sasakian manifold then the square S^2 of the Ricci tensor S is the linear combination of the Ricci tensor S and the metric tensor g. Here the (0,2)-tensor S^2 is defined by

$$S^2(X,Y)=S(QX,Y).$$

Finally we prove that a P-Sasakian manifold satisfies $R\cdot L=0$ if and only if the manifold is an η-Einstein manifold.

2. P-Sasakian Manifolds

Let M be an n-dimensional contact manifold with contact form η, i.e., $\eta \wedge (d\eta)^n \neq 0$. It is well known that a contact manifold admits a vector field ξ, called the *characteristic vector field*, such that $\eta(\xi) = 1$ and $d\eta(\xi, X) = 0$ for every $X \in \chi(M)$. Moreover, M admits a Riemannian metric g and a tensor field ϕ of type (1,1) such that

$$\phi^2 = I - \eta \otimes \xi, \quad g(X, \xi) = \eta(X), \quad g(X, \phi Y) = d\eta(X, Y).$$

We then say that (ϕ, ξ, η, g) is a contact metric structure. A contact metric manifold is said to be a *Sasakian* if

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X,$$

in which case

$$\nabla_X \xi = -\phi X, \quad R(X, Y)\xi = \eta(Y)X - \eta(X)Y.$$

Now we give a structure similar to Sasakian but not contact.

An n-dimensional differentiable manifold M is said to admit an almost paracontact Riemannian structure (ϕ, ξ, η, g), where ϕ is a (1,1)-tensor field, ξ is a vector field, η is a 1-form and g is a Riemannian metric on M such that

$$\phi\xi = 0, \quad \eta\phi = 0, \quad \eta(\xi) = 1, \quad g(\xi, X) = \eta(X), \tag{3}$$

$$\phi^2 X = X - \eta(X)\xi, \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for all vector fields X and Y on M. The equation $\eta(\xi) = 1$ is equivalent to $|\eta| \equiv 1$, and then ξ is just the metric dual of η, where g is the Riemannian metric on M. If (ϕ, ξ, η, g) satisfy the following equations:

$$d\eta = 0, \qquad \nabla_X \xi = \phi X,$$

$$(\nabla_X \phi)Y = -g(X, Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

then M is called a *Para-Sasakian* manifold or briefly a *P-Sasakian manifold*. Especially, a P-Sasakian manifold M is called a *special para-Sasakian manifold* or briefly a *SP-Sasakian manifold* if M admits a 1-form η satisfying

$$(\nabla_X \eta)(Y) = -g(X, Y) + \eta(X)\eta(Y).$$

It is known that in a P-Sasakian manifold the following relations hold:

$$S(X,\xi) = (1-n)\eta(X), \tag{4}$$

$$\eta(\mathcal{R}(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{5}$$

for any vector fields $X, Y, Z \in \chi(M)$, (see [3], [11] and [12]).

A para-Sasakian manifold M is said to be η-*Einstein* if

$$\mathcal{S} = aI_d + b\eta \otimes \xi, \tag{6}$$

where $\mathcal{S}$ is the Ricci operator and a, b are smooth functions on M [3].

3. P-Sasakian Manifolds satisfying the condition $R \cdot P = 0$

In this section we consider Weyl projective semisymmetric P-Sasakian manifolds, that is; P-Sasakian manifolds satisfying the condition $R \cdot P = 0$.

Now

$$\begin{aligned}(R(X,Y)\cdot P)(U,V)W = R(X,Y)P(U,V)W - P(R(X,Y)U,V)W \\ -P(U,R(X,Y)V)W - P(U,V)R(X,Y)W.\end{aligned} \tag{7}$$

Putting $X = \xi$ in (7) and using $R \cdot P = 0$ we get

$$\begin{aligned} 0 &= g(R(\xi,Y)P(U,V)W,\xi) - g(P(R(\xi,Y)U,V)W,\xi) \\ &\quad -g(P(U,R(\xi,Y)V)W) - g(P(U,V)R(\xi,Y)W). \end{aligned} \tag{8}$$

Using (4), (5) and (2) we obtain

$$\eta(P(X,Y)Z) = 0. \tag{9}$$

By (3) and (9) we get

$$g(R(\xi,Y)P(U,V)W,\xi) = -g(P(U,V)W,Y). \tag{10}$$

From (8) and (9) it follows that the left hand side of (10) is zero and consequently we obtain $g(P(U,V)W,Y) = 0$ for all U, V, W, Y. Hence $P = 0$. This leads to the following theorem:

Theorem 3.1. *A Weyl projective semisymmetric P-Sasakian manifold is projectively flat.*

But it is known that [14], a projectively flat Riemannian manifold is of constant curvature. Also it can be easily seen that a manifold of constant curvature is projectively flat. Hence we conclude that the following result:

Theorem 3.2. *A P-Sasakian manifold is Weyl projective semisymmetric if and only if the manifold is of constant curvature.*

Also it is known that (see [4]) if a P-Sasakian manifold is of constant curvature then the manifold is an SP-Sasakian manifold.

So we have the following result:

Theorem 3.3. *A Weyl projective semisymmetric P-Sasakian manifold is a manifold of constant curvature and hence an SP-Sasakian manifold.*

It is easy to see thatin a projective symmetric Riemannian manifold the condition $R(X,Y)\cdot P=0$ is satisfied trivially.

4. P-Sasakian Manifolds satisfying the condition $P\cdot R=0$

In this section we investigate P-Sasakian manifolds satisfying the condition $P\cdot R=0$. The condition $R\cdot P=0$ does not imply $P\cdot R=0$. Then we have

$$\begin{aligned}(P(X,Y)\cdot R)(U,V)W &= P(X,Y)R(U,V)W - R(P(X,Y)U,V)W \\ &\quad -R(U,P(X,Y)V)W - R(U,V)P(X,Y)W.\end{aligned} \tag{11}$$

Putting $X=\xi$ in (11), by using $P(X,Y)\cdot R=0$, we get

$$\begin{aligned} 0 &= g(P(\xi,Y)R(U,V)W,\xi) - g(R(P(\xi,Y)U,V)W,\xi) \\ &\quad -g(R(U,P(\xi,Y)V)W,\xi) - g(R(U,V)P(\xi,Y)W,\xi).\end{aligned} \tag{12}$$

Putting $X=\xi$, replacing Z by U in (2) and by virtue of (4) and (5) we have

$$\eta(R(P(\xi,Y)U,V)W = \eta(U)\left[\eta(R(Y,V)W) + \frac{1}{n-1}\eta(R(QY,V)W)\right]. \tag{13}$$

Putting $X=\xi$, replacing Z by V in (2) and by virtue of (4) and (5) we also have

$$\eta(R(U,P(\xi,Y)V)W=\eta(V)\left[\eta(R(U,V)W)+\frac{1}{n-1}\eta(R(U,QY)W)\right]. \tag{14}$$

Again putting $X=\xi$, replacing Z by V in (2) and by virtue of (4) and (5) we obtain

$$\eta(R(U,V)P(\xi,Y)W)=\eta(W)\left[\eta(R(U,V)Y)+\frac{1}{n-1}\eta(R(U,V)QY)\right]. \tag{15}$$

Using (9), (13) and (15) it follows from (12) that

$$\begin{aligned} 0 &= \eta(U)[\eta(R(Y,V)W)+\frac{1}{n-1}\eta(R(QY,V)W)] \\ &\quad +\eta(V)[\eta(R(U,Y)W)+\frac{1}{n-1}\eta(R(U,QY)W)] \\ &\quad +\eta(W)[\eta(R(U,V)Y)+\frac{1}{n-1}\eta(R(U,V)QY)]. \end{aligned} \tag{16}$$

Now putting $Y=U$ in (16) we get

$$\begin{aligned} 0 &= \eta(U)[\eta(R(U,V)W)+\frac{1}{n-1}\eta(R(QU,V)W)] \\ &\quad +\eta(V)[\eta(R(U,U)W)+\frac{1}{n-1}\eta(R(U,QU)W)] \\ &\quad +\eta(W)[\eta(R(U,V)U)+\frac{1}{n-1}\eta(R(U,V)QU)]. \end{aligned} \tag{17}$$

So using (4) and (5) we obtain

$$\eta(W)[g(U,V)\eta(U)-g(U,U)\eta(V)+\frac{1}{n-1}(\eta(U)S(U,V)-\eta(V)S(U,U))]=0.$$

Now putting $U=e_i$, where $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i, $1\leq i\leq n$, we get $r=n(1-n)$, since $\eta(V)\neq 0$. Hence we have the following:

Theorem 4.1. *If a P-Sasakian manifold satisfies the condition $P\cdot R=0$ then its scalar curvature is constant.*

5. P-Sasakian manifolds satisfying $P \cdot C = 0$ and $C \cdot P = 0$

Theorem 5.1. *Let M ba a P-Sasakian manifold satisfying the condition $P(\xi, X) \cdot C = 0$ then the condition*

$$S^2(X,Y) = \left(2 - n + \frac{r}{n-1}\right) S(X,Y) + [n - 1 + r]g(X,Y) \tag{18}$$

holds on M.

Proof. Assume that the condition $P(\xi, X) \cdot C = 0$ holds on M. Then

$$\begin{gathered} 0 = (P(\xi,X) \cdot C)(Y,Z)W = P(\xi,X)C(Y,Z)W - C(P(\xi,X)Y,Z)W \\ -C(Y,P(\xi,X)Z)W - C(Y,Z)P(\xi,X)W. \end{gathered} \tag{19}$$

So taking the inner product of (19) with ξ we have

$$\begin{aligned} 0 \ &= \ g(P(\xi,X)C(Y,Z)W,\xi) - g(C(P(\xi,X)Y,Z)W,\xi) \\ &\quad -g(C(Y,P(\xi,X)Z)W,\xi) - g(C(Y,Z)P(\xi,X)W,\xi). \end{aligned} \tag{20}$$

On the other hand using (1), (2), (4) and (5) we get

$$g(P(\xi,X)C(Y,Z)W,\xi) = 0, \tag{21}$$

$$\begin{gathered} g(C(P(\xi,X)Y,Z)W,\xi) = g(P(\xi,X)Y,W)\eta(Z) \\ +\tfrac{1}{n-2}[S(P(\xi,X)Y,W)\eta(Z) + (1-n)g(P(\xi,X)Y,W)\eta(Z)] \\ -\tfrac{r}{(n-1)(n-2)}g(P(\xi,X)Y,W)\eta(Z), \end{gathered} \tag{22}$$

$$\begin{gathered} g(C(Y,P(\xi,X)Z)W,\xi) = -g(P(\xi,X)Z,W)\eta(Y) \\ -\tfrac{1}{n-2}[S(P(\xi,X)Z,W)\eta(Y) + (1-n)g(P(\xi,X)Z,W)\eta(Y)] \\ +\tfrac{r}{(n-1)(n-2)}g(P(\xi,X)Z,W)\eta(Y), \end{gathered} \tag{23}$$

$$g(C(Y,Z)P(\xi,X)W,\xi) = 0, \tag{24}$$

$$g(C(Y,Z)P(\xi,X)W,\xi) = g(Y,P(\xi,X)W)\eta(Z) - g(Z,P(\xi,X)W)\eta(Y) \tag{25}$$

$$\begin{aligned}0 = -\tfrac{1}{n-2}[S(Z,P(\xi,X)W)\eta(Y) - S(Y,P(\xi,X)W)\eta(Z)\\ +(1-n)g(Z,P(\xi,X)W)\eta(Y) - (1-n)g(Y,P(\xi,X)W)\eta(Z)]\\ +\tfrac{r}{(n-1)(n-2)}[g(Z,P(\xi,X)W)\eta(Y) - g(Y,P(\xi,X)W)\eta(Z)],\end{aligned} \tag{26}$$

$$g(Y,P(\xi,X)W) = g(X,Y)\eta(W) + \frac{1}{n-1}S(X,Y)\eta(W) \tag{27}$$

and

$$S(Y,P(\xi,X)W) = S(X,Y)\eta(W) + \frac{1}{n-1}S^2(X,Y)\eta(W). \tag{28}$$

Now putting (21)-(28) into (20) and using (1), (2), (4) and (5) it follows that

$$\begin{aligned}0 = [1 - \tfrac{r}{(n-1)(n-2)}][g(X,Y)\eta(W)\eta(Z) + -\tfrac{1}{n-1}S(X,Z)\eta(W)\eta(Y)]\\ \tfrac{1}{n-1}S(X,Y)\eta(W)\eta(Z) - g(X,Z)\eta(W)\eta(Y)\\ -\tfrac{1}{n-2}[\tfrac{1}{n-1}S^2(X,Z)\eta(W)\eta(Y) - \tfrac{1}{n-1}S^2(X,Y)\eta(W)\eta(Z)]\\ -\tfrac{1}{n-2}[(1-n)g(X,Z)\eta(W)\eta(Y) - (1-n)g(X,Y)\eta(W)\eta(Z)].\end{aligned} \tag{29}$$

So by a suitable contraction of (29) we get (18). ∎

Theorem 5.2. *Let M ba a P-Sasakian manifold satisfying the condition $C(\xi,X)\cdot P = 0$ then the condition (18) holds on M.*

Proof. Assume that the condition $C(\xi,X)\cdot P = 0$ holds on a P-Sasakian manifold M. Then we can write

$$\begin{aligned}0 &= C(\xi,X)P(Y,Z)W - P(C(\xi,X)Y,Z)W\\ &\quad -P(Y,C(\xi,X)Z)W - P(Y,Z)C(\xi,X)W.\end{aligned} \tag{30}$$

Taking the inner product of (30) with ξ we get

$$\begin{aligned}0 &= g(C(\xi,X)P(Y,Z)W,\xi) - g(P(C(\xi,X)Y,Z)W,\xi)\\ &\quad -g(P(Y,C(\xi,X)Z)W,\xi) - g(P(Y,Z)C(\xi,X)W,\xi).\end{aligned} \tag{31}$$

On the other hand using (1) and (2) we have

$$\begin{aligned}C(\xi,X)P(Y,Z)W = R(\xi,X)P(Y,Z)W\\ -\tfrac{1}{n-2}[S(X,P(Y,Z)W)\xi + (1-n)g(X,P(Y,Z)W)\xi]\\ +\tfrac{r}{(n-1)(n-2)}g(X,P(Y,Z)W)\xi,\end{aligned} \tag{32}$$

(33)
$$P(C(\xi,X)Y,Z)W = R(C(\xi,X)Y,Z)W - \frac{1}{(n-1)}[g(Z,W)QC(\xi,X)Y - g(C(\xi,X)Y,W)QZ],$$

(34)
$$P(Y,C(\xi,X)Z)W = R(Y,C(\xi,X)Z)W - \frac{1}{(n-1)}[g(C(\xi,X)Z,W)QY - g(Y,W)QC(\xi,X)Z],$$

and

(35)
$$P(Y,Z)C(\xi,X)W = R(Y,Z)C(\xi,X)W - \frac{1}{(n-1)}[g(Z,C(\xi,X)W)QY - g(Y,C(\xi,X)W)QZ].$$

Hence using (32), (33), (34) and (35) into (31) and using (4) and (5) we obtain

$$\begin{aligned} 0 &= g(X,P(Y,Z)W) + \frac{1}{n-2}S(X,P(Y,Z)W) + \frac{n-1}{n-2}g(X,P(Y,Z)W) \\ &\quad + \frac{r}{(n-1)(n-2)}g(X,P(Y,Z)W). \end{aligned}$$

So in view of (2) by a suitable contraction of last equation we find (18). ■

For symmetric $(0,2)-$tensor fields A and B on M we define Kulkarni-Nomizu product $A \barwedge B$ of A and B by ([5],p.47)

(36)
$$\begin{aligned} A \barwedge B(X_1,...,X_4) &= A(X_1,X_4)B(X_2,X_3) - A(X_1,X_3)B(X_2,X_4) \\ &\quad + A(X_2,X_3)B(X_1,X_4) - A(X_2,X_4)B(X_1,X_3) \end{aligned}$$

Lemma 5.3. *[9]. Let A be a symmetric (0,2)-tensor at point x of a semi-Riemannian manifold (M,g), $dimM \geq 3$, and let $T = g \barwedge A$ be the Kulkarni- Nomizu product of g and A. Then the relation*

(37)
$$T \cdot T = \alpha Q(g,T),\ \alpha \in \mathsf{R}$$

is satisfied at x if and only if the condition

(38)
$$A^2 = \alpha A + \lambda g,\ \alpha \in \mathsf{R}$$

holds at x.

Hence we have the following corollary:

Corollary 5.4. *Let M ba a P-Sasakian manifold satisfying the condition $C(\xi, X) \cdot P = 0$ or $P(\xi, X) \cdot C = 0$ then $T \cdot T = \alpha Q(g, T)$, where $T = g \barwedge S$ and $\alpha = \left(2 - n + \frac{r}{n-1}\right)$*

6. P-Sasakian manifolds satisfying $R \cdot L = 0$

Contact Ricci tensor L (see [15]) is defined by

$$L(X, Y) = S(X, Y) + 2g(X, Y) - 2(n + 1)\eta(X)\eta(Y), \tag{39}$$

where S is the Ricci tensor.

Now consider a P-Sasakian manifold satisfying the condition $R \cdot L = 0$. So we have

$$(R(X, Y) \cdot L)(U, V) = -L(R(X, Y)U, V) - L(U, R(X, Y)V) = 0. \tag{40}$$

Putting $X = \xi$ in (40) we can write

$$L(R(\xi, Y)U, V) + L(U, R(\xi, Y)V) = 0. \tag{41}$$

Using (5) and putting $U = V$ we get from (41)

$$\eta(U)L(Y, U) - g(Y, U)L(\xi, U) = 0. \tag{42}$$

Putting $X = \xi$ in (39) and replacing Y by U and using (4) we get

$$L(\xi, U) = -(3n - 1)\eta(U). \tag{43}$$

Now using (43) in (42) we get

$$\eta(U)[S(Y, U) + (3n + 1)g(Y, U) - 2(n + 1)\eta(Y)\eta(U)] = 0. \tag{44}$$

From (44)it follows that

$$S(Y, U) = -(3n + 1)g(Y, U) + 2(n + 1)\eta(Y)\eta(U). \tag{45}$$

Hence the manifold is an η-Einstein manifold.

Conversely if the manifold is an η-Einstein manifold of the form (45), then it can be easily seen that in a P-Sasakian manifold $R(X,Y)\cdot L = 0$ holds. Thus we can state the following:

Theorem 6.1. *A P-Sasakian manifold satisfies* $R(X,Y)\cdot L = 0$ *if and only if the manifold is an* η*-Einstein manifold of type (45).*

By a contraction of the equation (45) we have the following result:

Corollary 6.2. *A P-Sasakian manifold satisfies* $R(X,Y)\cdot L = 0$ *then it has constant scalar curvature* $r = -3n^2 + n + 1$.

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