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Mathematical Optimization for the Train Timetabling Problem

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Rail transportation is very rich in terms of problems that can be modelled and solved using mathematical optimization techniques. The train scheduling problem as the most important part of a rail operating policy has a very significant impact on a rail company profit considering the fact that from the quality of a train timetable depends a flow of three most important resources on rail network: cars, locomotives and crews. The train timetabling problem aims at determining a periodic timetable for a set of trains that does not violate track capacities and satisfies some operational constraints. In this paper, we developed an integer programming approach for determining an optimal train schedule for a single, one-way track linking two major stations, with a number of intermediate stations between. The application has been tested on a realistic example suggested by the PE "Serbian Railways". Obtained results show a potential for a practical application of proposed approach.

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1. Introduction

In most countries the railway traffic system represents a very important part of the backbone transport system. Traffic and transport policies are striving towards decreasing road traffic pollution by increasing railway usage when appropriate. At the same time, the available railway systems are partly oversaturated creating bottlenecks on major links. An important issue is thus how to best use the existing capacity while ensuring sustainability and attractiveness. The train scheduling problem arises in several contexts [1],[2]. In real-time scheduling [3], it helps the dispatchers who are managing the traffic to make optimal decision about which trains to stop and where, as updated data about train positions becomes available [6],[8]. In tactical planning, it consists of finding the best master schedule or timetable on a regular basis (weekly, monthly, yearly)

[5], [7]. In strategic planning, it relates to investment decisions in rail infrastructure such as building new stations, extending current station track lengths, and upgrading single-line segments [9], [4] to double lines. The timetable planning problem aims at determining a timetable for a set of trains which does not violate track capacities and satisfies some operational constraints. The timetable is a schedule of trains on a given rail infrastructure. It contains the arrival and departure times of the trains at each intermediate point of their route. Due to its complex nature, the construction of the actual timetable as it is operated in practice, is still mainly a human planning process. Traditionally, planners use two types of graphs as the main tools for constructing a timetable. The so-called time-space diagram graphically represents the train movements that take place in between stations, on the tracks. One axis displays time, and the other axis depicts space. Each line in the time-space diagram depicts a train, which is indicated by the number next to the line. Lines with a positive inclination correspond to trains from Station E to Station D, and lines with a negative inclination to trains in opposite direction. Flat lines indicate fast trains, because those trains cover a large distance in a short time, and steep lines indicate slow trains. When a train dwells for some time at a station, this gives a vertical line at that station, because time moves on while the location of the train remains unchanged. Intersecting lines indicate that the two corresponding trains meet at the point of intersection. This is clearly only allowed at stations, or if the trains are using different tracks. The advantage of this diagram is that it makes it much more simple and intuitive to read the timetable and to detect conflicts. In Fig.1 is an example of a train diagram with 21 trains and 19 stations.

The X axis represents time of day, the Y axis the sequence of stations in distance scale. Lines indicate the movement of trains, with the slope indicating direction and speed, horizontal meaning stand-still. Usually, the outbound direction is defined upwards. Fig 2. displays a possible movement of four trains on a single track line connecting the stations A-E. Intermediate meet-points are located at all stations from B to D. The horizontal and vertical axes represent time and space, respectively.

Even in this simple example there are many possible combinations of stations and times for trains to be pulled over to allow meets and passes. Therefore, the train meet-pass problem is a very large-scale combinatorial optimization problem. When several trains in both directions are scheduled, many conflicts may arise. Each conflict involves trains moving either in opposite directions or in the same direction. Depending on the chosen solution for a conflict involving two trains, the location and the time of later conflicts may change, new conflicts at different locations and times may arise, and existing conflicts may

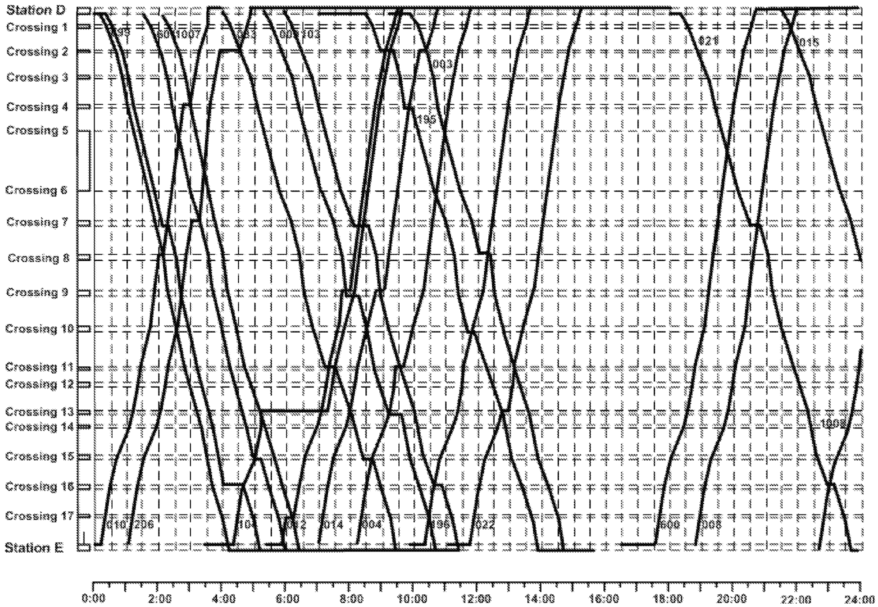

FIGURE 1. Train diagram example

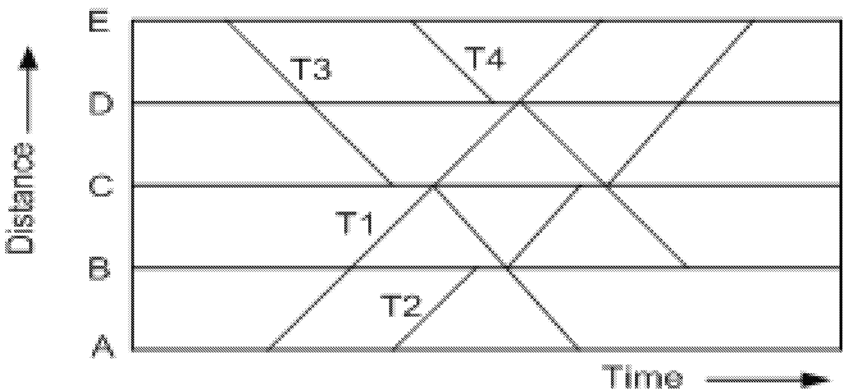

FIGURE 2. Graph of train movements

disappeared. Thus, the number of feasible solutions to train conflicts can be very large. Also, it is very important to notice that train scheduling decision may have different economic effects. Sequences that reduce travel times may decrease investments in cars and engines as well as crew and fuel costs. Regularity and transportation velocity, which also depend on the schedule, are important factors to the satisfaction of customers needs and thus are related to railway company revenues.

In practice, a timetable is constructed by specifying, a time-space path for each train through a railway network, which is drawn in the time-space diagrams, and by specifying the platform tracks that the train occupies, which are drawn in the platform-occupation charts. Generally, a timetable is not constructed from scratch. Rather, adjustments are made to an existing timetable, typically, to the timetable of the previous year. Trains are added to the existing timetable, deleted from it, or the schedule of an already existing train is adjusted. During the process of adding, deleting, and adjusting the schedules of trains, one usually runs into problems at a certain moment in time. It may not be possible to schedule a new train as desired, because there is no capacity on the track or in the station, or because capacity is only available when a connection can not be realized. In such a case, some of the already scheduled trains have to be rescheduled. This can be achieved by shifting an already scheduled train to some earlier or later point in time, by adjusting the planned trip time of a train, or by relocating a train to a different platform. Cycling through this process of scheduling trains, and backtracking on previously made choices in case of a 'dead end', one may eventually arrive at a complete timetable. The timetable construction process is quite complex, and it may take a team of planners several months to create a complete timetable. Modification of the last year timetable implies that the new timetable inherits properties that may be unnecessary and costly. The construction process is very time consuming. Thus, for time reasons, there is no possibility to optimize the timetable, and the planner is often satisfied to find a feasible timetable.

In this paper we present an optimization model for the problem of timetable optimization. This algorithm treats some kind of trade-off between departure times for each of a given set of trains and the total waiting time needed for train conflict resolution. Considering the fact that an optimal timetable gives a good base for making the optimal decision for routing of locomotives, cars and crews, this strategy of making timetables based on a flexible train departures can be a very efficient strategy for improving the timetable construction. We test this algorithm on a realistic example. The computation times are moderate considering the sizes of the optimization problems. Of course, there are many aspects we do not consider, but these may be added later. We hope that the suggested procedure will be a first step towards intelligent computer assisted profit maximizing allocation of track capacity in timetabling.

2. **Model formulation**

We consider a single, one-way track linking two major stations with a number of intermediate stations. This case is particularly interesting because railway networks are mostly contained from single track sections. On these sections made of a track carrying traffic in opposite directions, track resource is limited by great traffic densities.

The following data represent problem parameters:

T - set of trains
S - set of stations
Q - set of discrete time intervals
d^t - departure station of train t
a^t - arrival station of train t
$Path(t)$ - set of stations between d^t and a^t
ed^t - estimated departure of train t
md^t - maximum allowed delay of train t
$ld^t = ed^t + md^t$
$f(t, s)$ - travel time of train t alongside the track segment s
$Prev(t, s)$ - previous station in the path of train t
$Next(t, s)$ - next station in the path of train t
$ea_{t,s}$ - earliest arrival of train t to station s (calculated from ed_t and $f(t, l)$)
$ea_{t,s} = ed^t + \sum_{l=d^t}^{s} f(t, l)$
mw_s - maximum number of trains waiting simultaneously at station s
mi_s - maximum number of trains simultaneously waiting at, or exiting from station s
$x^{dep}_{t,d^t,k}$ - an arc which denotes train t departing at time k
$x^{arr}_{t,a^t,k}$ - an arc which denotes train t arriving at time k
$x^{wait}_{t,s,k}$ - an arc denoting train t waiting in station s at time k
$x^{out}_{t,s,k}$ - an arc denoting that train t is leaving station s at time k
Arcs - set of all arcs $x^{dep}_{t,d^t,k}, x^{arr}_{t,a^t,k}, x^{wait}_{t,s,k} and x^{out}_{t,s,k}, \forall t, \forall k, \forall s$
$Arcs = \{x^{dep}_{t,d^t,k} | \forall t, \forall k\} \cup \{x^{arr}_{t,a^t,k} | \forall t, \forall k\} \cup \{x^{wait}_{t,s,k} | \forall t, \forall k, \forall s\} \cup \{x^{out}_{t,s,k} | \forall t, \forall k, \forall s\}$
$Exit(t, s, k)$ - set of arcs denoting train t leaving node (s,k) except for departure arcs, because there is no enter arc for them
$Exit(t, s, k) = \left\{x^{out}_{t,s,k}, x^{wait}_{t,s,k,}\right\} \cup \left\{x^{arr}_{t,a^t,k} | s = a^t\right\}$

$Enter(t,s,k)$ - set of arcs denoting train t entering node (s,k) except for arrival arcs, because there is no exit arc for them

$Enter(t,s,k) = \left\{ x^{out}_{t,\Pr ev(s),k-f(t,s)}, x^{wait}_{t,s,k-1,} \right\} \cup \left\{ x^{dep}_{t,a^t,k} | s = d^t \right\}$

The problem is formulated as an integer programming model with decision variables $x \in Arcs$ representing the flow of a train on an arc. The formulation is as follows:

$$minimize\ z = \sum_{t \in T} \left(\sum_{k=ed_t}^{ld_t} (k - ed_t) x^{dep}_{t,d^t,k} + \sum_{s \in Path(t)} \sum_{k=ea_{t,s}}^{ea_{t,s}+md_t} x^{wait}_{t,s,k} \right) \quad (1)$$

Subject to:

$$\sum_{k=ed_t}^{ld_t} x^{dep}_{t,d^t,k} = 1, \forall t \in T \quad (2)$$

$$\sum_{k=sa_t}^{la_t} x^{arr}_{t,a^t,k} = 1, \forall t \in T \quad (3)$$

$$\sum_{x \in Enter(t,s,k)} x - \sum_{x \in Exit(t,s,k)} x = 0,$$

$$\forall t \in T, \forall s \in Path(t), \forall k \in Q, ea(t,s) \le k \le ea(t,s) + md(t) \quad (4)$$

$$\sum_{t \in T} x^{wait}_{t,s,k} \le mw_s, \forall s \in S, \forall k \in Q \quad (5)$$

$$\sum_{t \in T} (x^{wait}_{t,s,k} + x^{out}_{t,s,k}) \le mi_s, \forall s \in S, \forall k \in Q \quad (6)$$

$$\sum_{t \in T^{out}} \sum_{\substack{l \le k-1, \\ l+f(t,s) \ge k}} x^{out}_{t,s,l} + \sum_{t \in T^{in}} \sum_{\substack{l \le k-1, \\ l+f(t,s+1) \ge k}} x^{out}_{t,s+1,l} \le 1, \forall s \in S, \forall k \in Q \quad (7)$$

$$x \in \{0,1\}, \forall x \in Arcs$$

The objective function (1) minimizes the sum of train delays since the arc costs represent the delays in discretized time units. Equation (2) ensures that for each train there is a unit outflow from its departure node to one of the nodes that is the copy of its origin station at a time within its possible departure window. The constraint of equation (3) ensures that for each train there is a unit flow from one of the nodes that is the copy of its destination node to its arrival node. Constraint set (4) provides the flow conservation constraints. Constraint set (5) ensures that the number of trains waiting simultaneously in station be equal or less than the maximum number of tracks serving for train operations. Constraint set (6) ensures that the total number of trains staying in station and trains which at the same time pass through the station be less than the maximum number of available tracks. Constraint (7) is dedicated to capacity of sections between the stations. Therefore, the total number of trains can not be greater than one, if we are considering inter-station system of traffic regulation.

Train No.	Departure station	Arrival station	Departure time	Arrival time
1	Novi Sad	Subotica	22:52	24:50
2	Subotica	Novi Sad	02:40	03:37
3	Novi Sad	Subotica	09:04	11:00
4	Subotica	Novi Sad	17:00	18:54
5	Novi Sad	Subotica	05:09	07:16
6	Subotica	Novi Sad	18:40	20:45
7	Novi Sad	Subotica	11:35	13:32
8	Subotica	Novi Sad	05:45	07:42
9	Novi Sad	Subotica	20:19	22:18
10	Subotica	Novi Sad	14:22	16:19
11	Novi Sad	Subotica	10:10	12:16
12	Subotica	Novi Sad	11:17	13:17
13	Novi Sad	Subotica	13:05	15:17
14	Subotica	Novi Sad	07:20	09:30
15	Novi Sad	Subotica	15:25	17:55
16	Subotica	Novi Sad	04:33	06:54
17	Novi Sad	Subotica	04:33	06:42
18	Subotica	Novi Sad	10:29	12:53
19	Novi Sad	Subotica	07:38	09:49
20	Subotica	Novi Sad	15:48	18:04
21	Novi Sad	Subotica	19:15	21:26
22	Subotica	Novi Sad	13:00	15:18
23	Novi Sad	Subotica	22:20	24:25
24	Subotica	Novi Sad	19:21	21:38
25	Novi Sad	Subotica	22:22	24:34

TABLE 1. Timetable for a given set of trains

3. Computational results

In this section, we present computational results for a real world example of the train dispatching problem. The example is based on actual data for a 24-hour planning horizon on a part of the major line of Serbian railways. Within this time interval we have 25 trains of opposite directions and 12 potential meet/pass points for each of them. According to data from the graphical train timetable, for solving all conflicts raising on this set of trains a total waiting time of 42 minutes is planned. Next table contains all necessary data.

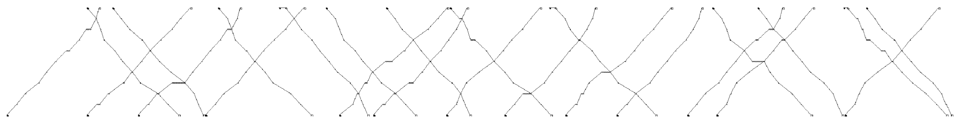

FIGURE 3. Optimal solution for a train timetabling problem

Of course, we also considered the possibility that is not for any train allowed to wait in any station due to incompatibility of its length and the length of station tracks or for some other reasons. Fig. 3 contains an optimal schedule with minimal train delays for given example. Table 2. presents the computational results for a given set of trains.

The minimum total waiting time, as a sum of the time spent in departure stations and the time needed for meeting/overtaking operations for this set of trains on a part of Serbian rail network is 185 minutes. We implemented our algorithm in C programming language and CPLEX 10.0 solver. All computational tests were conducted on a 3.0-GHz Intel Pentium 4 processor with 2Gb RAM. For this example CPLEX loading time was 8 minutes long and optimal solution has been found for 25 seconds of CPU time.

4. **Conclusions**

Due to the complexity of rail operations, the expected growth of traffic and the limited possibilities of enhancing the infrastructure, effective timetable design strategy play a key role in improving the level of railway service. In this paper we presented a model for tactical train scheduling on a single line track. Unlike most models, we tried here to make a trade-off between the time of train departing and the train schedule. For each train, depending on its importance and future train connections we defined an interval of departure as a rolling horizon within which we can depart our train. Obtained results are very promising. Considering the fact that time for deriving the optimal solution is the crucial parameter if we want to use this model as a part of decision support system for on-line train dispatching, this model can be used as a tool for real-time train dispatching.

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Train No.	Optimal departure time	Time deviation in minutes respect to earliest departure time
1	22:54	2
2	02:49	9
3	09:05	1
4	17:01	1
5	05:09	0
6	18:50	10
7	11:43	9
8	06:12	27
9	20:30	11
10	14:34	12
11	10:10	0
12	11:27	10
13	13:15	10
14	07:27	7
15	15:25	0
16	04:43	10
17	04:33	0
18	10:31	2
19	07:39	1
20	15:48	0
21	19:32	17
22	13:04	4
23	22:39	19
24	19:21	0
25	22:23	1

TABLE 2. Optimal solution for a given train timetabling problem

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