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New Vacuum Solutions for Quadratic Metric-Affine Gravity - a Metric Affine Model for the Massless Neutrino?

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In this paper we present an overview of our research that was presented at the MASSEE International Congress on Mathematics MICOM 2009 in Ohrid, Macedonia. We deal with *quadratic metric–affine gravity*, which is an alternative theory of gravity. We present *new vacuum solutions* for this theory and an attempt to give their *physical interpretation* on the basis of comparison with existing classical models. These new explicit vacuum solutions of quadratic metric–affine gravity are constructed using *generalised pp-waves*. A classical pp-wave is a 4-dimensional Lorentzian spacetime which admits a non–vanishing parallel spinor field. We *generalise* this definition to metric compatible spacetimes with torsion, describe basic properties of such spacetimes and eventually use them to construct new solutions to the field equations of quadratic metric–affine gravity. The physical interpretation of these solutions we propose is that these new solutions represent a *conformally invariant metric–affine model for the massless neutrino*. We give a comparison with a classical model describing the interaction of gravitational and massless neutrino fields, namely *Einstein-Weyl theory*. Future research topics are briefly discussed.

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1. Introduction

There are a number of different alternative theories of gravity that try to further the completion of Einstein's theory of gravity. One such theory, propagated by Einstein himself for some time, is the *metric–affine gravity*.

A number of developments in physics in the last several decades have evoked the possibility that the treatment of spacetime might involve more than just the Riemannian spacetime of Einstein's general relativity. The smallest departure from a Riemannian spacetime of Einstein's general relativity would consist of admitting *torsion*, arriving thereby at a Riemann–Cartan spacetime, and, furthermore, a possible nonmetricity, resulting in a '*metric–affine*' spacetime.

The metric–affine gravity is a natural generalisation of Einstein's general relativity, which is based on a spacetime with a Riemannian metric g of a Lorentzian signature. Similarly, in the metric–affine gravity we consider spacetime to be a connected real 4–manifold M equipped with a Lorentzian metric g and an affine connection Γ. Note that the characterisation of the spacetime manifold by an *independent* linear connection Γ initially distinguishes metric–affine gravity from general relativity. The connection incorporates the inertial properties of spacetime and it can be viewed, according to Hermann Weyl [28], as the guidance field of spacetime. The metric describes the structure of spacetime with respect to its spacio-temporal distance relations.

The 10 independent components of the (symmetric) metric tensor $g_{\mu\nu}$ and the 64 connection coefficients $\Gamma^\lambda{}_{\mu\nu}$ are the unknowns of metric–affine gravity.

We mostly deal with *quadratic* metric-affine gravity. In the quadratic metric-affine gravity, we define our action as

$$S := \int q(R) \tag{1}$$

where q is a quadratic form on curvature R. The coefficients of this quadratic form are assumed to depend only on the metric, and the form itself is assumed to be $\mathrm{O}(1,3)$ invariant.

An independent variation of (1) with respect to the metric g and the connection Γ produces the system of Euler–Lagrange equations which we will write symbolically as

$$\partial S/\partial g = 0, \tag{2}$$

$$\partial S/\partial \Gamma = 0. \tag{3}$$

The objective of our work was the study of the combined system of field equations (2), (3). This is a system of $10+64$ real nonlinear partial differential equations with $10+64$ real unknowns. the quadratic curvature Lagrangians were first discussed by Weyl [28], Pauli [17], Eddington [6] and Lanczos [10, 11, 12] in an attempt to include the electromagnetic field in the Riemannian geometry.

Our motivation comes from the Yang–Mills theory. The Yang–Mills action for the affine connection is a special case of (1) with

$$q(R) = q_{\mathrm{YM}}(R) := R^\kappa{}_{\lambda\mu\nu}\, R^\lambda{}_\kappa{}^{\mu\nu}\,. \tag{4}$$

With this choice of $q(R)$, equation (3) is the Yang–Mills equation for the affine connection, which was analysed by Yang [29].

The idea of using a purely quadratic action in the General Relativity goes back to Hermann Weyl, who argued that the most natural gravitational action should be quadratic in curvature and involve all possible invariant quadratic combinations of curvature, like the square of Ricci curvature, the square of scalar curvature, etc. By choosing a purely quadratic curvature Lagrangian we are hoping to describe phenomena whose characteristic wavelength is sufficiently small and curvature sufficiently large.

As presented in [16], we were able to obtain a new class of solutions for quadratic metric–affine gravity.

2. A short introduction to pp-waves

PP-waves are well known spacetimes in the general relativity, first discovered by Brinkmann [2] in 1923, and subsequently rediscovered by several authors, for example Peres [18] in 1959. We used them as the basis for constructing new solutions for quadratic metric–affine gravity. Hence, an introduction to classical pp-waves is required in order to fully understand this construction.

Recall first the well-known notion of a pp-wave.

Definition 1. A (classical) *pp-wave* is a connected 4-manifold M equipped with Lorentzian metric g and Levi-Civita connection Γ which admits a nonvanishing parallel spinor field.

Another way of characterising a pp-wave is by its restricted holonomy group Hol^0. Definition 1 is equivalent to

Definition 2. A (classical) *pp-wave* is a connected 4-manifold M equipped with Lorentzian metric g and Levi-Civita connection Γ whose holonomy Hol^0 is, up to conjugation, a subgroup of the group

$$B^2 := \left\{ \left(\begin{array}{cc} 1 & q \\ 0 & 1 \end{array} \right) \middle| \quad q \in \mathbb{C} \right\}. \tag{5}$$

The group (5) is, up to conjugation, the unique nontrivial Abelian subgroup of $\mathrm{SL}(2, \mathbb{C})$, where "non-trivial" is understood as "weakly irreducible and not 1-dimensional" and dimension understood as real dimension. "Weak irreducibility" means that the only non-degenerate invariant subspaces of the tangent space are $\{0\}$ and the tangent space itself.

Yet another equivalent way of characterising a pp-wave is via an explicit formula for the metric.

Definition 3. A (classical) *pp-wave* is a connected 4-manifold M equipped with Lorentzian metric g and Levi-Civita connection Γ whose metric can be written locally in the form

$$ds^2 = 2\,dx^0\,dx^3 - (dx^1)^2 - (dx^2)^2 + f(x^1, x^2, x^3)\,(dx^3)^2 \tag{6}$$

in some local coordinates (x^0, x^1, x^2, x^3).

PP-waves are well known in general relativity for their beautiful and amazing properties. For example, the curvature tensor R of a pp-wave is linear in f (in special local coordinates (6)) and is given by a simple explicit formula

$$R_{\alpha\beta\gamma\delta} = -\frac{1}{2}(l \wedge \partial)_{\alpha\beta}\,(l \wedge \partial)_{\gamma\delta} f \tag{7}$$

where l is a parallel null light–like vector and $(l \wedge \partial)_{\alpha\beta} := l_\alpha \partial_\beta - \partial_\alpha l_\beta$. See Section 3 in [16] for more details.

The main aim of our research was to extend the classical notion of a pp-wave to metric compatible spacetimes with torsion, i.e. with

$$\Gamma^\lambda{}_{\mu\nu} \neq \frac{1}{2} g^{\lambda\kappa}(\partial_\mu g_{\nu\kappa} + \partial_\nu g_{\mu\kappa} - \partial_\kappa g_{\mu\nu}),$$

and to do it in such a manner that all the nice properties are preserved and they are still easy to work with in practical applications, as presented in [16]. One natural way of generalising the notion of a pp-wave is simply to extend it to general metric–compatible spacetimes. However, this would give us a class of spacetimes which is too wide and difficult to work with. We choose to extend the classical definition in a more special way, and in this section we do it by introducing torsion explicitly.

Let A be a complex vector field defined by

$$A = h(x^3)m + k(x^3)l \tag{8}$$

where l is a parallel null light–like vector and m is a complex isotropic vector field orthogonal to l. We choose the set of local coordinates for which $l^\mu = (1, 0, 0, 0)$ and $m^\mu = (0, 1, \mp i, 0)$. The functions $h, k : \mathbb{R} \to \mathbb{C}$ are arbitrary.

We can then define a *generalised pp-wave* as a metric–compatible spacetime with pp–metric and torsion

$$T := \frac{1}{2}\mathrm{Re}(A \otimes \mathrm{d}A). \tag{9}$$

Torsion can be expressed more explicitly in our local coordinates as

$$T^\alpha{}_{\beta\gamma} = \frac{1}{2}\mathrm{Re}\left[(k(x^3)h'(x^3)l^\alpha + h(x^3)h'(x^3)m^\alpha)\,(l \wedge m)_{\beta\gamma}\right].$$

Torsion is purely tensor and it has 4 non-zero independent components. The formula for curvature in our local coordinates is

$$R_{\alpha\beta\gamma\delta} = -\frac{1}{2}(l \wedge \partial)_{\alpha\beta}(l \wedge \partial)_{\gamma\delta} f + \frac{1}{4}\mathrm{Re}\left((h(x^3)^2)''\,(l \wedge m)_{\alpha\beta}\,(l \wedge m)_{\gamma\delta}\right). \tag{10}$$

Curvature only has two irreducible pieces, namely symmetric trace-free Ricci and Weyl and it can be written down as

$$R_{\kappa\lambda\mu\nu} = \frac{1}{2}(g_{\kappa\mu}Ric_{\lambda\nu} - g_{\lambda\mu}Ric_{\kappa\nu} + g_{\lambda\nu}Ric_{\kappa\mu} - g_{\kappa\nu}Ric_{\lambda\mu}) + \mathcal{W}_{\kappa\lambda\mu\nu}.$$

The Ricci and Weyl curvatures are given by

$$\begin{aligned} Ric_{\mu\nu} &= \frac{1}{2}(f_{11} + f_{22})\, l_\mu l_\nu, \\ \mathcal{W}_{\kappa\lambda\mu\nu} &= \sum_{j,k=1}^{2} w_{jk}(l \wedge m_j) \otimes (l \wedge m_k), \end{aligned}$$

where $m_1 = \mathrm{Re}(m), m_2 = \mathrm{Im}(m)$, $f_{\alpha\beta} := \partial_\alpha \partial_\beta f$ and w_{jk} are real scalars given by

$$w_{11} = \frac{1}{4}[-f_{11} + f_{22} + \mathrm{Re}((h^2)'')], \quad w_{22} = -w_{11},$$
$$w_{12} = \pm\frac{1}{2}f_{12} - \frac{1}{4}\mathrm{Im}((h^2)''), \quad w_{21} = w_{12}.$$

Note that our generalised pp-waves have the same irreducible pieces of curvature as classical pp-waves and that their curvature has all the usual symmetries of curvature in the Riemannian case.

3. The main result

The main result of our research thus far is the following

Theorem 1. *Generalised pp-waves of parallel Ricci curvature are solutions of the system of equations (2), (3).*

Note that when using Theorem 1 it does not really matter whether the condition 'parallel Ricci curvature' is understood in the non-Riemannian sense $\nabla Ric = 0$, the Riemannian sense $\{\nabla\}\{Ric\} = 0$, or any combination of the two ($\{\nabla\}Ric = 0$ or $\nabla\{Ric\} = 0$). Here curly brackets refer to the Levi–Civita connection.

In special local coordinates, the condition that Ricci curvature is parallel is written as $f_{11}+f_{22} = \mathrm{const}$, where $f_{\alpha\beta} := \partial_\alpha\partial_\beta f$. Hence, generalised pp-waves of parallel Ricci curvature admit a simple explicit description.

The proof of the main theorem is done by 'brute force'. We write down our field equations (2), (3) explicitly under certain assumptions on the properties of the spacetime, which generalised pp-waves automatically posses. The proof of the theorem is then quite straightforward, as we explicitly show that the field equations are satisfied by inserting the formulae for the irreducible pieces of curvature and torsion of generalised pp-waves.

For the proof of Theorem 1, see [16].

4. **Physical interpretation of generalised pp-waves**

Our analysis of vacuum solutions of quadratic metric–affine gravity shows, see Theorem 1, that classical pp-spaces of parallel Ricci curvature should not be viewed on their own. They are a particular (degenerate) representative of a wider class of solutions, namely, generalised pp-spaces of parallel Ricci curvature. The latter appear to admit a sensible physical interpretation. Indeed, according to formula (10) the curvature of a generalised pp-space is a sum of two curvatures: the curvature

$$-\frac{1}{2}(l \wedge \{\nabla\}) \otimes (l \wedge \{\nabla\}) f \tag{11}$$

of the underlying classical pp-space and the curvature

$$\frac{1}{4}\mathrm{Re}\left((h^2)''\,(l \wedge m) \otimes (l \wedge m)\right) \tag{12}$$

generated by a torsion wave traveling over this classical pp-space. Our torsion (9), (8) and corresponding curvature (12) are waves traveling at speed of light. The underlying classical pp-space of parallel Ricci curvature can now be viewed as the 'gravitational imprint' created by a wave of some massless matter field. Such a situation occurs in the Einstein–Maxwell theory[1] and the Einstein–Weyl theory[2]. The difference with our model is that Einstein–Maxwell and Einstein–Weyl theories contain the gravitational constant which dictates a particular relationship between the strengths of the fields in question, whereas our model is conformally invariant and the amplitudes of the two curvatures (11) and (12) are totally independent.

The physical interpretation of the solution from Theorem 1 we proposed is that these new solutions represent a *conformally invariant metric–affine*

[1]The Einstein–Maxwell theory is a classical model describing the interaction of gravitational and electromagnetic fields

[2]The Einstein–Weyl theory is a classical model describing the interaction of gravitational and massless neutrino fields

model for a massless elementary particle by comparing them to solutions of the *Einstein-Weyl theory.*

In the Einstein–Weyl theory the action is given by

$$S_{EW} := 2i \int \left(\xi^a \, \sigma^\mu{}_{a\dot{b}} \, (\{\nabla\}_\mu \bar{\xi}^{\dot{b}}) \;-\; (\{\nabla\}_\mu \xi^a) \, \sigma^\mu{}_{a\dot{b}} \, \bar{\xi}^{\dot{b}} \right) + k \int \mathcal{R}, \tag{13}$$

where the constant k can be chosen so that the non-relativistic limit yields the usual form of Newton's gravity law.

In the Einstein–Weyl theory the connection is assumed to be Levi-Civita, so we only vary the action (13) with respect to the metric and the spinor to obtain the well known Einstein–Weyl field equations

$$\frac{\delta S_{EW}}{\delta g} = 0, \tag{14}$$

$$\frac{\delta S_{EW}}{\delta \xi} = 0. \tag{15}$$

We pointed out the fact that the nonlinear system of Einstein–Weyl field equations has solutions in the form of pp-waves. The main difference between the two models is that in the metric–affine model our generalised pp-waves solutions have parallel Ricci curvature, whereas in the Einstein–Weyl model the pp-wave type solutions do not necessarily have parallel Ricci curvature. However, when we look at monochromatic pp-wave type solutions in the Einstein–Weyl model their Ricci curvature also becomes parallel and we conclude that while in the metric–affine case the Laplacian of f can be *any* constant, in the Einstein–Weyl case it is required to be a *particular* constant. This should not be surprising as our metric–affine model is conformally invariant, while the Einstein–Weyl model is not.

We pointed out a very interesting fact that that generalised pp-waves of parallel Ricci curvature are sufficiently similar to pp-type solutions of the Einstein–Weyl model, which is a classical model describing the interaction of massless neutrino and gravitational fields, to suggest that *generalised pp-waves of parallel Ricci curvature represent a metric–affine model for the massless neutrino.*

5. **Planned future research**

The main aims of the research we plan to do in the immediate future would be the following:

(i) *Physical interpretation of previous results.* We would first try to solidify and expand the physical interpretation of the new solutions obtained thus far. As our generalised pp-waves of parallel Ricci curvature clearly have interesting properties, the main objective of this part of research would be to further investigate the possibility that these solutions represent a metric–affine (and thus conformally invariant) model for some massless particle. This would be done in collaboration with several people who are perhaps more involved with the physical aspects of this area. I expect this research to produce publishable results fairly soon, as the majority of the work has already been done.

(ii) *Further comparison with existing solutions.* There are several results from this area that can be compared to our solution in order to see if additional solutions can be obtained or not. The two papers of Singh [21, 22] are an example of this. In [21] Singh presents solutions of the vacuum field equations with *purely axial torsion*, which is a class of solutions unobtainable by the double duality ansatz of [1, 13].

In the second paper [22], Singh also constructs solutions unobtainable by the double duality ansatz, but this time that have *purely trace torsion.* These solutions are similar in many ways, as the metric and the hence the Riemannian pieces of curvature are the same - which leads the author to stipulate that it might be possible to combine these two solutions but he however shows that this is unfortunately not possible.

It should be pointed out that in [21, 22] Singh was not working within the setting of the most general purely quadratic action and the solutions were obtained for the Yang–Mills case (4). It is clear that these solutions differ from the ones presented in our work, as our torsion is purely tensor. It would however be of interest to us to see whether this construction of Singh's can be expanded to our most general $\mathrm{O}(1,3)$-invariant quadratic form q.

One other and more recent non-metric–compatible result comes from Obukhov [15]. The quadratic form on curvature considered is the most general, and identical to the quadratic form used in our work and in [16, 24, 27]. However, unlike the solutions presented in these works, Obukhov

constructs new solutions that have non-zero nonmetricity, which are generalisations of pp-waves. Obukhov presents solutions that have not only torsion waves present but the nonmetricity has a non-trivial wave behaviour as well, which is different from the generalised pp-waves presented in this thesis. Moreover, Obukhov suggests that his solutions provide a minimal generalisation of the pseudoinstanton, see [23] for definition of a pseudoinstanton. However, it should be pointed out that solutions presented in [15] are *not* non-metric–compatible generalisations of solutions presented in this thesis.
It would be of great interest to us to respond to this work of Obukhov's, for example by seeing what the relaxation of our condition on metric-compatibility would produce and to investigate a possible combination of these solutions.

(iii) *Teleparallelism.* The last, but definitely the most interesting part of the research we plan to do in the near future would be in the field of *teleparallelism.*

Teleparallelism is a very interesting alternative theory of gravity and it can be considered as a special case of Cosserat elasticity initially investigated by the Cosserat brothers in [5] and used by Einstein and Cartan to try to unify electromagnetism and gravity, i.e. as a candidate for the *theory of everything.* The subject of teleparallelism has a long history and its origins lie in the pioneering works of Eugène and Francois Cosserat, Élie Cartan, Albert Einstein and Roland Weitzenböck. Modern reviews of the physics of teleparallelism are given in [7, 9, 14, 20].

The basic idea of teleparallelism is to work with a Lorentzian metric, vanishing curvature and *non-vanishing torsion*, so it could be viewed as a special case of metric–affine gravity. However, in practice instead of using the metric as the unknown of this theory, one uses a quartet of covectors (a *coframe*).

An interesting recent result in teleparallel gravity related to our previous result was done by Vassiliev in [25, 26], where a new (teleparallel) representation for the Weyl Lagrangian is given. The advantage of the teleparallel approach is that it does not require the use of spinors, Pauli matrices or covariant differentiation, as we did in our work. The only geometric concepts used are those of a metric, differential form, wedge product and exterior derivative. It would be interesting to see whether this can be applied to our previous research.

Another interesting result that we plan to investigate and further build on can be found in [3] where the authors suggest an alternative mathematical model for the electron using the teleparallel approach and where the electron mass and external electromagnetic field are incorporated into the model by means of a *Kaluza–Klein extension*.

One of the main topics of research interest for us in this field would be the calculation the ground energy state of the hydrogen atom based on the model presented in [3]. The model presented in [4] would be interesting in comparison with our research thus far as [4] deals with a teleparallel model for the massless neutrino, while we dealt with the metric–affine model for the massless neutrino.

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