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Research of the Estimate Calculating Algorithms on One Class of Tables Generated by the Multidimensional Boolean Functions [1]

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The work examines four parametric family of algorithms for calculating the estimates on the number of binary tables (matrices), caused by monotonous Boolean functions. Estimate calculating algorithms (ECA) are based on the classification of the recognition subject in the class depending on the value of quantitative performance - estimates (where comes the name of the method).We prove that under some additional restrictions on the set of admissible tables, algorithms of this family have a high accuracy of recognition.

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1. Description of the model

The set of objects to be recognized by the system is described by n binary features, i.e. $\sigma_i = \{0, 1\}, i = 1, 2, ..., n$. The tables for training and supervision are sets of admissible rows, broken down into two classes - m for the training table and $t$table for control. Sets of sets of admissible rows for the classes K_1 and K_2 are described as follows:

Let $f(x) = f(x_1, x_2, ..., x_n)$ is monotonic Boolean function;

$D_1 = \{x = (x_1, x_2, ..., x_n)|f(x) = 1\}$ is the set of function units;

$D_0 = \{x = (x_1, x_2, ..., x_n)|f(x) = 0\}$ is the set of zeros for function $f(x)$.

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The point $x \in D_1$ is defined as a limit point in the set D_1 such item for which there is such $y \in D_0$ that $\rho(x, y) = 1$. (This $\rho(x, y)$ is distance in terms of Hemming, i.e. a number of different components). Similarly is determined limit point in the set of zeros of monotonic Boolean function: the point $x' \in D_0$ is called the limit, if there is a point $y' \in D_1$ such that $\rho(x', y') = 1$.

Let Γ_1 is the set of limit points of the set D_1 and Γ_0 is set of limit points of D_0. The set $\Gamma = \Gamma_0 \bigcup \Gamma_1$ is formed the set of limit points of the Boolean function $f(x)$.

Let φ is any natural number. We define the following subsets D_1^{φ} - the set of units and D_0^{φ} - the set of zeros of the function:

$$D_1^{\varphi} \equiv \{x | x \in D_1 \text{ and } \rho(x, \Gamma) \geq \varphi\}, \quad D_0^{\varphi} \equiv \{x | x \in D_0 \text{ and } \rho(x, \Gamma) \geq \varphi\},$$

where $\rho(x, \Gamma) = min\, \rho(x, z),\ z \in \Gamma$.

The set of admissible objects for the class K_1 coincides with the set of coordinates of points belonging to the set D_1^{φ} (briefly coincides with the set D_1^{φ}).

Similarly, the set of admissible rows for the class K_2 forms the set D_0^{φ}. In further work with tables T_1, T_2 filled with admissible rows from built thus sets we will denote with T_1^{φ}, T_2^{φ}.

But any arbitrary algorithm belonging to the class algorithms $A(k, \varepsilon, \delta_1, \delta_2)$ described in [1] are supplied sequentially to recognition rows (objects) of a fixed control table, as decision set and use table. We defined the quality of the algorithm by the proportion of correctly recognized control rows:

$$Q(A) = \frac{t'}{2t},$$

where t' is s number of correctly identified rows of the table T_2^{φ} by algorithm A. The extremal algorithm $A^* \in A(k, \varepsilon, \delta_1, \delta_2)$ is determined as:

$$Q(A^*) = \sup_{A \in A(k, \varepsilon, \delta_1, \delta_2)} Q(A).$$

As performance characteristics of the given family (a class) $A(k, \varepsilon, \delta_1, \delta_2)$ of the algorithms on a set of ordered pairs $\{(T_1^{\varphi}, T_2^{\varphi})\}$ of admissible tables is used value, specified by the condition:

$$Q_{\varphi}(A^*) = \min_{(T_1^{\varphi}, T_2^{\varphi})} Q(A^*).$$

The task is to find the minimum value of the parameter (if it even exists for the function f), where the algorithm detects extreme permissible faultless couple tables. We will attempt to formulate a criterion for correctly (faultless) recognition to formulated procedure to reduce the checks in order to solve the problem. We will show that for a sufficiently wide class of monotonic function there is an integer non-negative value of φ, such that for any pair T_1, T_2 of corresponding set, the algorithm is recognized unmistakably.

2. Criterion for correct recognition

Let the rows of K_1 and K_2 be arbitrary and not intersected subsets G_1 and G_2 of the set of tops of unit n-metric cube E_n.

$$R(G_1) = \max_{y \in G_1} [\max_{x_1 \in G_1} \rho(x_1, y) - \min_{x_2 \in G_2} \rho(x_2, y)],$$

$$R(G_2) = \max_{z \in G_2} [\max_{x_2 \in G_2} \rho(x_2, z) - \min_{x_1 \in G_1} \rho(x_1, z)],$$

$$R(G) = \max\{R(G_1), R(G_2)\}.$$

Theorem 1. *The extremal algorithm recognizes correctly each pair of admissible tables if and only if* $R(G) < 0$.

Proof. Sufficiency. Let $R(G) < 0$. Consider algorithm $A \in A(k, \varepsilon, \delta_1, \delta_2)$ with parameters k, ε satisfying the conditions (parameters δ_1, δ_1 are not fixed so far):

$$(1) \qquad \varepsilon + 1 \leq m(G),$$

$$(2) \qquad n - k + \varepsilon \geq M(G),$$

where

$$m(G) = \min_{x \in G_1} \min_{y \in G_2} \rho(x, z), \qquad M(G) = \max\{M(G_1), M(G_2)\}$$

and

$$M(G_1) = \max_{x,y \in G_1} \rho(x, z), \qquad M(G_2) = \max_{x,y \in G_2} \rho(x, z).$$

Inequalities system (1), (2) is compatible, as the sets G_1 and G_2 do not intersect, and $m(G) \geq 1$, but by condition $R(G) < 0$, where $M(G) \leq n - 1$.

Let us $T_1 = (S_1, S_2, \ldots S_{2m})$, $T_2 = (S'_1, S'_2, \ldots S'_{2t})$ have arbitrary admissible tables for training and supervision, ie

$$S_i \in G_1, \; S_{i+m} \in G_2, \; i = 1, 2, \ldots, m,$$

$$S'_j \in G_1, \; S'_{j+t} \in G_2, \; j = 1, 2, \ldots, t.$$

Algorithm with parameters satisfying conditions (1), (2), assigned, according to (1), the following estimates for the row S_i', $(1 \le i \le t)$:

$$\Gamma_1^{S_i} = \sum_{q=1}^{m} \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(S_q,S_i')}{k-\lambda} \binom{\rho(S_q,S_i')}{\lambda},$$

$$\Gamma_2^{S_i} = \sum_{q=m+1}^{2m} \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(S_q,S_i')}{k-\lambda} \binom{\rho(S_q,S_i')}{\lambda}.$$

Therefore the following inequalities have been satisfied:

$$\Gamma_1^{S_i'} - \Gamma_2^{S_i'} \ge m \cdot \min_{S' \in G_1} \left[\min_{S \in G_1} \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(S,S')}{k-\lambda} \binom{\rho(S,S')}{\lambda} - \right.$$

$$\left. - \max_{S \in G_2} \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(S,S')}{k-\lambda} \binom{\rho(S,S')}{\lambda} \right], \ i = 1,2,\ldots,t \tag{3}$$

i.e,

$$\rho(x_1,y) < \rho(x_2,y),$$

for any

$$x_1, y \in G_1, x_2 \in G_2.$$

We receive:

$$\sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_1,y)}{k-\lambda} \binom{\rho(x_1,y)}{\lambda} \ge \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_2,y)}{k-\lambda} \binom{\rho(x_2,y)}{\lambda}.$$

Since $\rho(x_1,y) \le M(G_1) \le M(G) \le n-k+\varepsilon$, $\rho(x_2,y) \ge m(G) \ge \varepsilon+1$ (see (1) and (2)) has been fulfilled, then according to the inequality:

$$\sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_1,y)}{k-\lambda} \binom{\rho(x_1,y)}{\lambda} > \sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_2,y)}{k-\lambda} \binom{\rho(x_2,y)}{\lambda}.$$

for arbitrary and inequality (3) follows:

$$\Gamma_1^{S_i'} - \Gamma_2^{S_i'} \ge m \quad i = 1,2,\ldots,t. \tag{4}$$

Reflecting similarly with respect to any order of the class K_2 of the control table, we get:

$$\Gamma_1^{S_i'} - \Gamma_2^{S_i'} \ge m \quad i = i+1, i+2, \ldots, 2t. \tag{5}$$

Let us consider the value $\frac{\Gamma_1'}{\Gamma_1'+\Gamma_2'}$ of arbitrary row $S' \in G_1$. Since, under (4)

$$\frac{\Gamma_1^{S'}}{\Gamma_1^{S'}+\Gamma_2^{S'}} \geq \frac{1}{1+\max_{S\in G_1}\frac{\Gamma_2^S}{\Gamma_1^S}} \geq \left[1+\frac{\max_{S\in G_1}\Gamma_2^S}{\min_{S'\in G_1}\Gamma_1^{S'}}\right]^{-1} \tag{6}$$

Similarly for arbitrary row $\tilde{S}' \in G_2$ is fulfilled:

$$\frac{\Gamma_1^{S'}}{\Gamma_1^{S'}+\Gamma_2^{S'}} \geq \left[1+\frac{\max_{S\in G_2}\Gamma_2^S}{\min_{S'\in G_2}\Gamma_2^{S'}}\right]^{-1} \tag{7}$$

and using inequalities (6) and (7) we obtain:

$$\min\left\{\min_{S'\in\Gamma_1}\frac{\Gamma_1^{S'}}{\Gamma_1^{S'}+\Gamma_2^{S'}}, \min_{\tilde{S}'\in G_2}\frac{\Gamma_1^{\tilde{S}'}}{\Gamma_1^{\tilde{S}'}+\Gamma_2^{\tilde{S}'}}\right\} \geq$$

$$\left[1+\frac{\sum_{\lambda=0}^{\varepsilon}\binom{n-m(G)}{k-\lambda}\binom{m(G)}{\lambda}}{\sum_{\lambda=0}^{\varepsilon}\binom{n-M(G)}{k-\lambda}\binom{M(G)}{\lambda}}\right]^{-1}. \tag{8}$$

Let the parameters δ_1, δ_2 are such that

$$\delta_1 \leq m, \quad \delta_2 \leq \left[1+\frac{\sum_{\lambda=0}^{\varepsilon}\binom{n-m(G)}{k-\lambda}\binom{m(G)}{\lambda}}{\sum_{\lambda=0}^{\varepsilon}\binom{n-M(G)}{k-\lambda}\binom{M(G)}{\lambda}}\right]^{-1}. \tag{9}$$

From inequalities (4),(5),(8) it follows that the algorithm with parameters $k, \varepsilon, \delta_1, \delta_2$ satisfying the system of conditions (1),(2),(9) will unmistakably recognize in each pair admissible tables. This sufficiency is proven.

II. Necessity. Let an extremal algorithm work faultlessly on any pair of admissible tables. It will be shown that the inequality $R(G) < 0$ is satisfied.Let us assume the contrary, ie $R(G) \geq 0$. Let for the certainty $R(G_1) \geq 0$ has been met, then $x_1, y \in G_1, x_2 \in G_2$ exist, such that $\rho(x_1, y) \geq \rho(x_2, y)$. Let us build a training table in the following way:

$$S_i = x_1, \quad S_{i+m} = x_2, \quad i = 1, 2, \ldots, m.$$

The table for control contains at least one row, equal to y in the K_1 class, then:

$$\Gamma_1^y - \Gamma_2^y = m\left[\sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_1,y)}{k-\lambda}\binom{\rho(x_1,y)}{\lambda} - \right.$$
$$\left.\sum_{\lambda=0}^{\varepsilon} \binom{n-\rho(x_2,y)}{k-\lambda}\binom{\rho(x_2,y)}{\lambda}\right] \leq 0$$

for any value of each parameters k, ε.

For the arbitrary algorithm of the family algorithms $A(k,\varepsilon,\delta_1,\delta_2)$ the decision rule will include the row y in the class K_2 or will refuse to classify it, i.e. the extremal algorithm will not work properly, which contradicts to the assumption that $R(G_1) \geq 0$ assumption leads to contradiction. This need has been demonstrated.

Let us introduce the following:

$$R_1(\varphi) = R(D_1^\varphi), \quad R_0(\varphi) = R(D_0^\varphi), \quad R(\varphi) = \max\{R_1(\varphi), R_1(\varphi)\}.$$

■

Corollary 1. *Let φ is fixed and such that the sets D_1^φ and D_0^φ are not empty. The extremal algorithm works unmistakably for each pair admissible tables $(T_1^\varphi, T_2^\varphi)$ then only when $R(\varphi) < 0$.*

If set $f(x_1, x_2, \ldots, x_n)$ monotonic Boolean function it can be found φ such that D_1^φ and D_0^φ both are not empty, and $R(\varphi) < 0$ then the corresponding model is unmistakably recognized in the class of algorithms. The largest interest causes "the richest"set of admissible rows corresponding to maximum φ satisfying inequality $R(\varphi) < 0$. The problem of finding a value of the parameter φ allows a trivial solution by the method of complete exhaustion, but for large enough n, the volume of work is large.

3. Structure of the sets D_1^φ and D_0^φ

Let $x \in E_n$, $x = (x_1, x_2, \ldots, x_n) andby J_x$ is denoted the set of all unit coordinates of point x, i.e. $J_x = \{j \,|x_j = 1\}$. Obviously, for $x \prec y$, $J_x \subseteq J_y$ and conversely, if $J_x \subseteq J_y$ then point x precedes y, i.e. $x \prec y$ and $J_x = J_y$ if and only if $x = y$.

Lemma 1. *The distance between $\rho(x,y) = |(J_x \cup J_y) \setminus (J_x \cap J_y)|$ (By $|y|$ is denote cardinality of the set).*

Proof. The distance between tops x, y of the unit n-metric cube is equal to the number of non-coincidence coordinates, i.e. if $J_z = j|x_j \neq y_j$,

$j \in \{1, 2, \ldots, n\}$, then $\rho(x, y) = |J_z|$. If $x_j \neq y_j$, then $j \in J_x \bigcup J_y$, $j \notin J_x \cap J_y$ and therefore $j \in (J_x \cup J_y) \backslash (J_x \cap J_y)$. If $x_j = y_j = 1$, then $j \in J_x \cap J_y$. In case $x_j = y_j = 0$ $(1 < j \leq n$) it is obtained that $j \notin (J_x \cup J_y)$. Therefore, we can record $j \notin (J_x \cup J_y) \setminus (J_x \cap J_y)$. Where it follows that $J_z = (J_x \cup J_y) \setminus (J_x \cap J_y)$. The lemma is proven. ■

Corollary 2. *Corollary: If $x \prec y$, then $\rho(x, y) = |J_y \backslash J_x|$.*

Let $f(x_1, x_2, \ldots, x_n)$ is given monotonic Boolean function, setting with the parameter sets and of admissible rows.

Theorem 2. *For any $\varphi\, (\varphi \geq 0,\ \varphi \in \mathbb{N})$ there are monotonous Boolean functions $f_\varphi^1(x), f_\varphi^0(x)$ such that a set of the units of the function $f_\varphi^1(x)$ coincides with the set D_1^φ and the set of zeros of the function $f_\varphi^0(x)$ coincides with D_0^φ.*

4. Structure of the sets R_1^φ and R_0^φ

Let for a function $f(x)$ and arbitrary φ be constructed the corresponding functions $f_\varphi^1(x)$ and $f_\varphi^0(x)$. We denote by $W_1^1(\varphi)$ a set of "lower units"of the function $f_\varphi^1(x)$ and by $W_0^0(\varphi)$ the set of "top zeros"of the function.

Theorem 3.
Let us $D_1^\varphi \neq \varnothing\emptyset$, $D_0^\varphi \neq \varnothing\emptyset$, then to $R_1(\varphi)$ and $R_0(\varphi)$ is fulfilled:

$$(10)\qquad R_1(\varphi) = n - \min_{x \in W_1^1(\varphi)} \left\{ \min_{y \in W_1^1(\varphi)} |J_x \cap J_y| + \min_{z \in W_0^0(\varphi)} |J_x \backslash J_z| \right\}$$

$$(11)\qquad R_0(\varphi) = n - \min_{x \in W_0^0(\varphi)} \left\{ \min_{y \in W_0^0(\varphi)} |J_x \cap J_y| + \min_{z \in W_1^1(\varphi)} |J_x \backslash J_z| \right\}$$

P r o o f. By definition

$$R_1(\varphi) = \max_{x \in D_1^\varphi} \left[\max_{y \in D_1^\varphi} \rho(x, y) - \min_{z \in D_0^\varphi} \rho(x, z) \right].$$

Let us consider the functions:

$$\psi_1 = \max_{y \in D_1^\varphi} \rho(x, y) \quad \text{and} \quad \psi_0 = \max_{z \in D_0^\varphi} \rho(x, z),$$

which are defined in the set D_1^φ. Then $R_1(\varphi)$ can be presented as follows:

$$R_1(\varphi) = \max_{x \in D_1^\varphi} \left[\psi_1(x) - \psi_0(x) \right].$$

■

Lemma 2. *Lemma 2. If $x \prec x'$, $x \in D_1^{\varphi}$ then 1) $\psi_1(x) \geq \psi_1(x')$, 2) $\psi_0(x) \leq \psi_0(x')$.*

P r o o f. First we will prove inequality 1) Let $x = (x_1, x_2, \ldots, x_n)$, $x' = (x'_1, x'_2, \ldots, x'_n)$ and $\tilde{z}' = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be such a point, that condition $\max_{z \in D_1^{\varphi}} \rho(x', z) = \rho(x', \tilde{z}')$ is satisfied.

Let us consider the point $\tilde{z}'' = (\beta_1, \beta_2, \ldots, \beta_n)$, where

$$\beta_i = \begin{cases} 1, & i \in J_{x'} \backslash J_x \\ \alpha_i, & i \in N \backslash (J_{x'} \backslash J_x). \end{cases}$$

It is obvious that $\tilde{z}' \prec \tilde{z}''$ or $\tilde{z}' \equiv \tilde{z}''$, and therefore $\tilde{z}'' \in D_1^{\varphi}$. We will show that $\rho(x', \tilde{z}') \leq \rho(x, \tilde{z}'')$ if $i \in N \backslash ({}_{x'} \backslash J_x)$, than $x_i = x_i'$, $\beta_i = \alpha_i$. If $i \in J_{x'} \backslash J_x$ than $x_i = 0, x_i' = 1, \beta_i = 1, \alpha_i \geq 0$ and therefore the number of non-coincidence values for the coordinates of pair of points $x', \tilde{z}'$ is not superior to the number of non-coincidence values for the coordinates for the pair $x', \tilde{z}''$, i.e. inequality is satisfied: $\rho(x', \tilde{z}') \leq \rho(x, \tilde{z}'')$.

Then for $\psi_1(x)$, which is fulfilled:

$$\psi_1(x) = \max_{z \in D_1^{\varphi} \rho(x,z) \geq \rho(x,\tilde{z}'') \geq \rho(x',\tilde{z}') = \max_{z \in D_1^{\varphi}} \rho(x',z) \rho(x',z) = \psi_1} (x'),$$

i.e. proved assertion 1) of the lemma. And to prove the second statement:

Let $\tilde{z}' = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$, $\tilde{z}' = (\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n)$.

Consider a point $\tilde{z}'' = \left(\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_n\right)$ as

$$\tilde{\beta}_i = \begin{cases} 0, & i \in J_{x'} \backslash J_x \\ \tilde{\alpha}_i, & i \in N \backslash (J_{x'} \backslash J_x). \end{cases}$$

Point $\tilde{z}'' \prec \tilde{z}'$ or $\tilde{z}' \equiv \tilde{z}''$, i.e. $\tilde{z}'' \in D_0^{\varphi}$. We will show that $\rho(x, \tilde{z}'') \leq \rho(x', \tilde{z}')$. Let us assume that $i \in N \backslash (J_{x'} \backslash J_x)$, then $x_i = x_i'$, $\tilde{\beta}_i = \tilde{\alpha}_i$. If $i \in J_{x'} \backslash J_x$ than $x_i = 0, x_i' = 1, \tilde{\beta}_i = 0, \tilde{\alpha}_i \leq 1$ and therefore the number of non-coincidence values for the coordinates for the pair is not superior to the number of non-coincidence values for the coordinates for the pair $x, \tilde{z}''$, i.e. the inequality is satisfied: $\rho(x, \tilde{z}'') \leq \rho(x', \tilde{z}')$.

Then for $\psi_0(x)$, which is fulfilled:

$$\psi_0(x) = \max_{z \in D_0^{\varphi} \rho(x,z) \geq \rho(x,\tilde{z}'') \geq \rho(x',\tilde{z}') = \max_{z \in D_0^{\varphi}} \rho(x',z) \rho(x',z) = \psi_0} (x'),$$

i.e. proved assertion 2) of the lemma. ■

Lemma 2 has been demonstrated and it is a lower unit x of function $f_{\varphi}^{1}(x)$ (i.e. $x \in W_1^1(\varphi)$), such that:

$$R_1(\varphi) = \psi_1(x) - \psi_0(x) \tag{12}$$

. Let

$$\psi_1(x) = \rho(x, y''),\ y \preceq y'',\ y \in W_1^1(\varphi),$$

$$\psi_0(x) = \rho(x, z''),\ z \preceq z'',\ z \in W_0^0(\varphi). \tag{13}$$

We will prove the following two lemmas:

Lemma 3. *The distance between* $\rho(x, y'') = n - |(J_x \cup J_y)|$.

P r o o f. According to lemma 1 has been

$$\rho(x, y'') = \left|\left(J_x \bigcup J_{y''}\right) \setminus \left(J_x \bigcap J_{y''}\right)\right|$$

met but $J_x \cap J_{y''} = N$, the existing index would applier $\alpha \notin J_x, \alpha \notin J_{y''}, \alpha \in \{1, 2, \ldots, n\}$ and then $\rho(x, y''') > \rho(x, y'')$, $J_{y'''} = \{\alpha\} \cup J_{y''}$ will be fulfilled, which contradicts to the condition $\psi_1(x) = \rho(x, y'')$. So we can make the conclusion that

$$\rho(x, y'') = \left|N \setminus \left|J_x \bigcap J_{y''}\right|\right|, \tag{15}$$

moreover

$$J_x \cap J_{y''} = J_x \cap J_y. \tag{16}$$

Actually $J_x \cap J_y \subseteq J_x \cap J_{y''}$, since $J_y \subseteq J_{y''}$, on the other hand $J_x \cap J_{y''} \subseteq J_x \cap J_y$, because otherwise an index $\alpha \in J_x \cap J_y$ would not exist, such that $\alpha \notin J_y$ and could be found point $y''' \in D_1^{\varphi}, J_{y'''} = J_{y''} \setminus \{\alpha\}$, which in turn contradicts to the condition $\psi_1 = \rho(x, y''')$. By permission (15) in (16), we obtain that $\rho(x, y'') = n - |J_x \bigcap J_y|$. ■

Lemma 4. *The distance between*

$$\rho(x, z'') = n - |(J_x \setminus J_z)|. \tag{17}$$

Proof. : According to lemma 1.$\rho(x, z'') = |(J_x \bigcup J_{z''}) \setminus (J_x \bigcap J_{z''})|$ is satisfied, but $J_{z''} \subseteq J_x$. Let us assume otherwise - there is an index $\alpha \in J_{z''}$ and $\alpha \notin J_x$. Consider the point $z''' \in D_0^{\varphi}, J_{z'''} = J_{z''} \setminus \{\alpha\}$. It is condition $\rho(x, z''') < \rho(x, z'')$ is met: contrary to the condition $\psi_0(x) = \rho(x, z'')$. Since it follows that $J_x \bigcup J_{z''} = J_x$ and $\rho(x, z'') = |J_x \setminus (J_x \bigcap J_{z''})|$. We will show that $J_x \cap J_{z''} = J_x \cap J_z$.

1. $J_x \cap J_{z''} \subseteq J_x \cap J_z$ since $z'' \preceq z$.

2. Let $\alpha \in J_x \cap J_z$. Let us assume that $\alpha \notin J_{z''}$. Consider the point z''' satisfies the condition: $J_{z'''} = J_{z''} \cup \{\alpha\}$. Then it follows that $z''' \in D_0^{\varphi}, \rho(x, z''') < \rho(x, z'')$, contrary to the condition $\psi_0(x) = \rho(x, z'')$, i.e. the assumption that $\alpha \notin J_{z''}$ is incorrect and therefore the condition $\alpha \in J_{z''}$. Where $J_x \cap J_{z''} \subseteq J_x \cap J_z$ is implemented. From 1. and 2. it follows that $J_x \cap J_{z''} \subseteq J_x \cap J_z$, $J_x \cap J_{z''} \subseteq J_x \cap J_z$ we receive:

$$\rho(x, z'') = |J_x \setminus (J_x \bigcap J_z)| = |J_x \setminus J_z|.$$

It follows that lemma 4 is proved. ∎

From (12)-(15) it follows, that there are $x, y \in W_1^1(\varphi)$, $z \in W_0^0(\varphi)$ points such that:

$$R_1(\varphi) = n - |J_x \cap J_z| - |J_x \setminus J_z|$$

.

From the above, by determining $R_1(\varphi)(10)$ follows. Similarly the equality (11) for $R_0(\varphi)$ is obtained.You will note that (11) for $R_0(\varphi)$ can be obtained from proven equality (10), which replaces the function by its dual function [3,4,5]:

$$f * (x_1, x_2, \ldots, x_n) = \bar{f}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n),$$

$$f *_{\varphi}^{0} (x_1, x_2, \ldots, x_n) = \bar{f}_{\varphi}^{1}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n).$$

Under construction $f_{\varphi}^1(x)$ and $f_{\varphi}^0(x)$ the functions are fulfilled:

$$f *_{\varphi}^{1} (x_1, x_2, \ldots, x_n) = \bar{f}_{\varphi}^{0}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n),$$

$$f *_{\varphi}^{0} (x_1, x_2, \ldots, x_n) = \bar{f}_{\varphi}^{1}(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n),$$

and therefore $R_{f*}^1(\varphi) = R_f^0(\varphi)$. Then, if we denote by $\tilde{W}_1^1(\varphi)$ the set of the lower units of the function $f *_{\varphi}^{1}(x)$ and with $[\tilde{W}_0^0(\varphi)$ a top set of zeros of the function $f *_{\varphi}^{0}(x)$, if $x \in \tilde{W}_1^1(\varphi)$, $y \in \tilde{W}_0^0(\varphi)$ and only if, $\bar{x} \in W_0^0(\varphi)$ and $\bar{y} \in W_1^1(\varphi)$ as

$$x = (x_1, x_2, \ldots, x_n), \quad \bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$$

$$y = (y_1, y_2, \dots, y_n), \quad \bar{y} = (\bar{y}_1, \bar{y}_2, \dots, \bar{y}_n),$$

i.e. $J_x = \bar{J}_{\bar{x}}$ and $J_y = \bar{J}_{\bar{y}}$.

According to (10)

$$R^1_{f_*}(\varphi) = n - x \in \tilde{W}^1_1(\varphi)\min\{y \in \tilde{W}^1_1(\varphi)\min|J_x \bigcap J_y| + z \in \tilde{W}^0_0(\varphi)\min|J_x \backslash J_z|\}$$

is fulfilled, and theorem 3 is proven. ■

Thus, under the proven theorem finding the parameter φ ensures unmistakably the recognizing and crawl sets D^φ_1 and D^φ_0 is replaced by the crawl set of their end points.

5. Concluding remarks

In the text the existence of the parameter value φ (for a function f) is not mentioned for which extremal faultless recognition algorithm performed on each pair of admissible tables. We will show a class of monotonic functions for which such values of parameters exist. Let $\psi(n,\tau)$ be the set of Boolean functions defined on the collections, whose number of units is in the range from $[\frac{n}{2}] - \tau$ to $[\frac{n}{2}] + \tau$ (in the collections we set the choice of values for the coordinates). We believe that outside this interval with fewer units in the collection, the value function is equal to 0, where a larger number of units is equal to 1. Then $\varphi < [\frac{n}{2}] - \tau$ for each function $f \in \psi(n,\tau)$ the sets D^φ_1 and D^φ_0 are not empty. On the other hand a set of many collections, corresponding to the lower units of the function $f^1_\varphi(x)$ contains not less than $[\frac{n}{2}] - \tau + \varphi$ units, and each set of many sets, corresponding to the zeros of functions $f^0_\varphi(x)$ contain no more than $[\frac{n}{2}] + \tau - \varphi$ units. Where according to the criteria for recognition and faultless formula (10) (11), it follows that $\varphi \geq [\frac{n}{4}] + \tau$ for each function $f \in \psi(n,\tau)$ the extremal algorithm from family of algorithms $A(k, \varepsilon, \delta_1, \delta_2)$ made recognition for each pair admissible tables $(T^\varphi_1, T^\varphi_2)$ unmistakably .
Let $\tau < [\frac{n}{4}]$ and τ be integer, $[\frac{n}{4}] + \tau < [\frac{n}{2}] - \tau$ is satisfied, where we can make a conclusion that there is at least one parameter value φ, such that the extremal algorithm made recognition for each pair admissible tables unmistakably. According to [2]

$$|\psi(n,\tau)| \geq 2^{\binom{n}{\frac{n}{2}}(1+\alpha_n)} \tag{19}$$

where $\alpha_n = ce^{-\frac{n}{4}}$, c is constant. Furthermore, the number of Boolean functions is true assessment [2]:

$$2^{\binom{n}{\frac{n}{2}}(1+\alpha_n)} \leq \psi(n) \leq 2^{\binom{n}{\frac{n}{2}}(1+\beta_n)} \tag{20}$$

where $\alpha_n = c'e^{-\frac{n}{4}}, \beta_n = \frac{c'' \log n}{\sqrt{n}}$, c', c'' are constants.

Let $\bar{\psi}(n)$ be the number of monotonous Boolean functions for which there is a parameter, extremal algorithm from family of algorithms $A(k, \varepsilon, \delta_1, \delta_2)$ made recognition for each pair admissible tables $(T_1^{\varphi}, T_2^{\varphi})$ unmistakably. Then according to (19) and (20) we obtain:

$$\log \tilde{\psi}(n) \sim \log \psi(n) \sim \binom{n}{[\frac{n}{2}]}.$$

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