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Estimation of Minimal Path Vectors of Multi State Two Terminal Networks with Cycles Control

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One of the hardest problems in two terminal networks reliability theory is to obtain minimal path or cut vectors of such a network. Moreover, the bigger problem is appeared when we have a network with cycles. Here we give one algorithm solution for such a problem.

1. Introduction

Two-terminal reliability (2TR) is a well-known problem in the area of network binary reliability. For the binary network it is assume that a whole network and its components can be in two states: working or failed state. However, the binary approach does not completely describe some networks. These networks and its components may operate in any of several intermediate states and better results may be obtained using a multi-state reliability approach.

In this paper we propose an algorithm for obtaining minimal path vectors of multi-state two terminal network. Some algorithms for obtaining minimal path or cut vectors are given in [1], [2], [4] and [5], but these algorithms give candidates for minimal cut or path vectors that are not minimal, so you must do additional calculations to eliminate them. In fact these vectors are eliminated by mutual comparison; when some candidate for minimal path (cut) vector is greater (smaller) than another candidate, then it is not a minimal, and it is eliminate. This procedure is relatively expensive, because the number of minimal path vectors is much greater than the number of nodes and links in the network. Therefore, we want to find a way of determining whether any given path vector is a minimal path vector, without vector comparison. For that reason, in this paper we improve the algorithms given in [1] and [2], so that we analyze the properties of the minimal path vectors, in order to separate them from those vectors that are not minimal path vectors.

Further in this section we give some definitions about multi-state two-terminal networks. More of these definitions are given in [4] and [5]. Let n is a number of link in the network. A **multi-state link** is defined as an arc of a network having a set of states $\{0, s_1, s_2, \ldots, M_i\}$, $0 < s_1 < s_2 \ldots < M_i$, $1 \leq i \leq n$. Let set $\mathbf{M} = (M_1, M_2, \ldots, M_n)$ to be the vector of maximal state of the network.

A vector $\mathbf{x}$ that reflects the state of a component is called a **state space set**. For every multi-state link, its **capacity state set** is obtained as the product of full capacity of the component and its states. For entire system is defined **system capacity state set** S, as the set of all available capacities from source to sink. A vector $\mathbf{X}$ that describes the state of all the system's components is called a **state vector**. The set of all state vectors is denoted by E, $E = S_1 \times, \ldots, \times S_n$ (where S_i is capacity vector of the i-th link).The **structure function** $\phi(\mathbf{x}) : E \to S$, maps the state vector into a system state. In fact, $\phi(\mathbf{x})$ is available capacity from the source to the sink under state vector $\mathbf{x}$. The vector $\mathbf{x}$ is a **minimal path vector to level** d ($\mathbf{MPV}_d$), if $\phi(\mathbf{x}) \geq d$ and for every other $\mathbf{y} < \mathbf{x}$, $\phi(\mathbf{y}) < d$.

2. Some properties about graphs appropriate to minimal path vectors

Suppose that the network works in the state $\mathbf{x}$, where $\mathbf{x}$ is a minimal path vector to level d. Then, in order to deliver d units from the source to the sink, each link is used only in one direction, [3]. Let us regard only the network structure, ignoring the capacity of its links, i.e. the unweighted graph. Suppose that the links are oriented as they are used and the other links are removed from the graph. The obtained subgraph is acyclic oriented graph, [3]. We will denote it by $G_{\mathbf{x}}(V, E_{\mathbf{x}})$. The accessibility matrix corresponding to $G_{\mathbf{x}}$ will be denoted by $A_{\mathbf{x}}$. Now, we may define an ordering of the nodes in respect to $\mathbf{x}$.

Definition 1. Let $\mathbf{x}$ be a minimal path vector to level d. For two nodes u and v we will say that $u <_{\mathbf{x}} v$ if there is a path in G from u to v. Such type of ordering will be called: ordering of the nodes in respect to $\mathbf{x}$.

In fact, the relation $<_{\mathbf{x}}$ is a accessibility relation for $G_{\mathbf{x}}$. Note that this relation is an ordering only in the case when the $\mathbf{x}$ is a minimal path vector for some level d.

E x a m p l e 1 Let us consider the network given in Figure 1 a). One minimal path vector of level 2 for the network given in Figure 1 a) is the vector (1,1,1,2,0,0,0,2). Figure 1 b) shows the ordering of the nodes in respect to (1,1,1,2,0,0,0,2), that actually is the graph $G_{(1,1,1,2,0,0,0,2)}$.

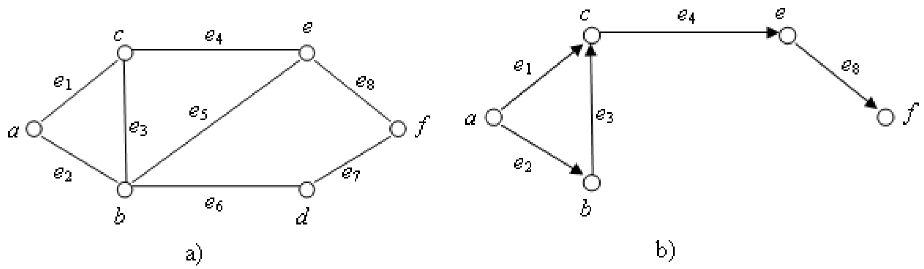

Figure 1

Definition 2. Let $\mathbf{x}$ and $\mathbf{y}$ are paths in G for the levels d and d' respectively. We define a graph $G_{\mathbf{x}+\mathbf{y}} = (V, \widetilde{E}_{\mathbf{x}+\mathbf{y}})$, where $(u,v) \in \widetilde{E}_{\mathbf{x}+\mathbf{y}} \Leftrightarrow (u,v) \in E_{\mathbf{x}}$ or $(u,v) \in E_{\mathbf{y}}$.

Proposition 1. *Let* $\mathbf{x}$ *and* $\mathbf{y}$ *are minimal path vectors in* G *for the levels* d *and* d' *respectively, and* $\mathbf{x}+\mathbf{y} < \mathbf{M}$. *If* $G_{\mathbf{x}+\mathbf{y}}$ *is an acyclic graph, then the vector* $\mathbf{x}+\mathbf{y}$ *is a minimal path vector for level* $d+d'$.

P r o o f. It is clear that $\mathbf{x}+\mathbf{y}$ is a path vector for level $d+d'$. Suppose that it is not a minimal path vector, i.e. there is a minimal path vector $\mathbf{z}$ for level $d+d'$, such that $\mathbf{z} < \mathbf{x}+\mathbf{y}$. Since $\mathbf{z}$ is a minimal path vector for level $d+d'$, from the Kirchhoff's Current law follows that when the network is in the state $\mathbf{z}$, the number of units flowing into the sink node is equal to the number of units flowing out of the source node, and this umber is exactly $d+d'$. Moreover for all other nodes, the number of units flowing into that node is equal to the number of units flowing out of that node.

Let us regard what will happen when the network is in the state $\mathbf{x}+\mathbf{y}-\mathbf{z}$. It is clear that there no units flowing into the sink node and flowing out of the source node. But, there are nodes such that the number of units flowing into these nodes is equal to the number of units flowing out of these nodes and this number is strictly greater then 0. This is possible only when the graph $G_{\mathbf{x}+\mathbf{y}}$ ordered in respect to $\mathbf{x}+\mathbf{y}$ has at last one cycle, which is in contradiction with our assumption that this graph is acyclic. ■

The last Proposition gives that to check whether a sum of d minimal path vectors of level 1 is a minimal path vector of level d, it is sufficient to check whether the corresponding graph is acyclic. This can be done using the accessibility matrix, in which, when the graph is acyclic, all diagonal elements are equal to 0.

It is clear that the accessibility relation is a transitive closure of the adjacency relation. But, the procedure for finding the transitive closure, in general case, has a big complexity, the adjacency matrix should be multiplied by itself at most $|V|$ times. Next we will be prove that it can be obtained by only 3 multiplications. For that reason, firs we will define the relation $\widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}$ by

$$u \ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} \ v \Leftrightarrow u <_{\mathbf{x}} v \ or \ u <_{\mathbf{y}} v$$

and the relation $\alpha_{\mathbf{x}+\mathbf{y}}$ as a transitive closure of the $\widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}$. It is clear that $\alpha_{\mathbf{x}+\mathbf{y}}$ is a accessibility relation for $G_{\mathbf{x}+\mathbf{y}}$.

E x a m p l e 2 The vectors (1,0,0,1,1,1,1,0) and (0,1,1,1,0,0,0,1) are minimal path vectors of level 1 for the network in Figure 1 a).

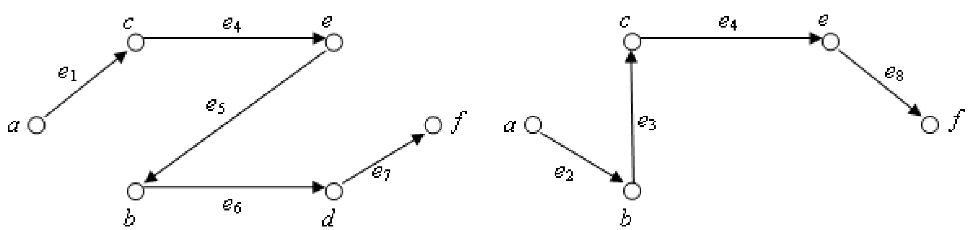

Figure 2

Adding these vectors we get (1,1,1,2,1,1,1,1), which is not a minimal path vector, because it is greater than (1,1,0,1,0,1,1,1). Figure 3 shows that the graph obtained adding these vectors, contains a cycle.

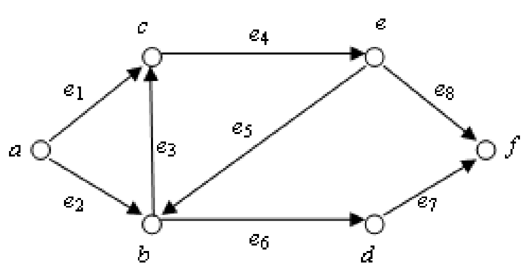

Figure 3

Let $\widetilde{A}_{\mathbf{x}+\mathbf{y}} = A_{\mathbf{x}} \oplus A_{\mathbf{y}}$, where $\oplus$ is the binary or operation ($0 \oplus 0 = 0, 0 \oplus 1 = 1, 1 \oplus 0 = 1, 1 \oplus 1 = 1$). It is clear that $A_{\mathbf{x}+\mathbf{y}} = \sum_{i=1}^{|V|} \widetilde{A}^{i}_{\mathbf{x}+\mathbf{y}}$, where the sum is the binary or operation ($\oplus$) and the multiplication is the binary and operation, $\otimes$ ($0 \otimes 0 = 0, 0 \otimes 1 = 0, 1 \otimes 0 = 0, 1 \otimes 1 = 1$).

Let $G = (V, E)$ is the graph appropriate for a given multi-state network. For each simple path $s = u_0, u_1, u_2, \ldots, u_{k-2}, u_{k-1}, u_k = t$, $(u_i, u_{i+1}) \in E$, we construct minimal binary path vector, putting 1 on the coordinate corresponding to (u_i, u_{i+1}), and 0 otherwise. In fact, a minimal binary path vector for multi-state network is a minimal path vector for a binary network with the same unweight graph. For example, the appropriate binary minimal path vector for the simple path a, c, e, f for the network in the Figure 1 a) is (1,0,0,1,0,0,0,1). It is clear that the nodes lying on the simple path corresponding to $\mathbf{x}$ are linearly ordered by $<_{\mathbf{x}}$. This is directly use in the proof of the next Proposition:

Proposition 2. *Let $\mathbf{x}$ be a binary minimal path vector and $u, v, u_1, v_1 \in V$, then from $u <_{\mathbf{x}} u_1$ (or $u_1 <_{\mathbf{x}} u$) and $v <_{\mathbf{x}} v_1$ (or $v_1 <_{\mathbf{x}} v$) follows that either $u <_{\mathbf{x}} v$ or $v <_{\mathbf{x}} u$.*

P r o o f. From $u <_{\mathbf{x}} u_1$ follows that u lies on the simple path corresponding to $\mathbf{x}$. Similarly, u lies on the simple path corresponding to $\mathbf{x}$. Since all nodes lying on the simple path corresponding to $\mathbf{x}$ are linearly ordered by $<_{\mathbf{x}}$, we have that either $u <_{\mathbf{x}} v$ or $v <_{\mathbf{x}} u$. ■

The results from the next Proposition and its Corollary are helpful for reduction of the number of steps in determining accessibility relation of the graph $G_{\mathbf{x}+\mathbf{y}}$.

Proposition 3. *Let $\mathbf{x}$ is a minimal path vector to level d, $\mathbf{y}$ is a binary minimal path vector and u, v, w and r are nodes such that $u <_{\mathbf{y}} v <_{\mathbf{x}} w <_{\mathbf{y}} r$. Then, either $u <_{\mathbf{y}} r$ or at least one diagonal element in the matrix $\widetilde{A}^2_{\mathbf{x}+\mathbf{y}}$ is equal to 1.*

P r o o f. Let $u <_{\mathbf{y}} v <_{\mathbf{x}} w <_{\mathbf{y}} r$. $\mathbf{y}$ is a binary minimal path vector, so from Proposition 2, we have that $u <_{\mathbf{y}} w$ or $w <_{\mathbf{y}} u$.

In the case $u <_{\mathbf{y}} w$, it is obtain $u <_{\mathbf{y}} w <_{\mathbf{y}} r \Rightarrow u <_{\mathbf{y}} r$.

In the case $w <_{\mathbf{y}} u$, we have $w <_{\mathbf{y}} u <_{\mathbf{y}} v \Rightarrow w <_{\mathbf{y}} v$. Now, $w <_{\mathbf{y}} v$ and $v <_{\mathbf{x}}$, so $w \ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} \ v$ and $v \ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} w$, which implies that $w \ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}} \ w$. ■

Corollary 1 *Let $\mathbf{x}$ is a minimal path vector to level d, $\mathbf{y}$ is a minimal binary path vector and u, v, w, r and m are nodes such that $u <_{\mathbf{x}} v <_{\mathbf{y}} w <_{\mathbf{x}} r <_{\mathbf{y}} m$. Then either $u \ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}} \ m$ or at least one diagonal element in the matrix $\widetilde{A}^2_{\mathbf{x}+\mathbf{y}}$ is equal to 1.*

P r o o f. From the given conditions, $v <_{\mathbf{y}} w <_{\mathbf{x}} r <_{\mathbf{y}} m$. Applying Proposition 3 we have that either at least one diagonal element in the matrix $\widetilde{A}^2_{\mathbf{x+y}}$ is equal to 1 or $v <_{\mathbf{y}} m$. When $v <_{\mathbf{y}} m \Rightarrow u <_{\mathbf{x}} v <_{\mathbf{y}} m \Rightarrow u\ \widetilde{\alpha}^2_{\mathbf{x+y}}\ m$. ■

Next result shows that all cycles in $G_{\mathbf{x+y}}$ can be found by comparing elements from $A_{\mathbf{x}}$ and $A_{\mathbf{y}}$.

Theorem 1 *Let $G_{\mathbf{x+y}}$ have a cycle. then there are nodes u and v such that $u <_{\mathbf{x}} v$ and $u <_{\mathbf{y}} v$.*

P r o o f. Let k be the smallest positive integer such that for some node w from $G_{\mathbf{x+y}}$, $w\widetilde{\alpha}^k_{\mathbf{x+y}}w$. Then there are nodes $w_i, i = \overline{0,k}$ such that

$$w = w_0\widetilde{\alpha}_{\mathbf{x+y}}w_1\widetilde{\alpha}_{\mathbf{x+y}}\ldots\widetilde{\alpha}_{\mathbf{x+y}}w_{k-1}\widetilde{\alpha}_{\mathbf{x+y}}w_k = w. \tag{0.1}$$

Suppose that there is a subarray of 0.1 such that $w_j <_{\mathbf{y}} w_{j+1} <_{\mathbf{x}} w_{j+2} <_{\mathbf{y}} w_{j+3}$. We have two cases, either $w_j <_{\mathbf{y}} w_{j+2}$, or $w_{j+2} <_{\mathbf{y}} w_j$. In the first case $w\widetilde{\alpha}^{k-1}_{\mathbf{x+y}}w$, so the array 0.1 is not the shortest array of that type. In the second case, $w_{j+2} <_{\mathbf{y}} w_j <_{\mathbf{y}} w_{j+1} \Rightarrow w_{j+2} <_{\mathbf{y}} w_{j+1}$, so the Theorem is true for $u = w_{j+1}$ and $v = w_{j+2}$.

We obtain that 0.1 should has one of the following forms: $w <_{\mathbf{y}} w_1 <_{\mathbf{x}} w$, $w <_{\mathbf{x}} w_1 <_{\mathbf{y}} w$ or $w <_{\mathbf{x}} w_1 <_{\mathbf{y}} w_2 <_{\mathbf{x}} w$. It is clear that in the Theorem is true for the first two cases. In the last one we have $w_2 <_{\mathbf{x}} w <_{\mathbf{x}} w_1 \Rightarrow w_2 <_{\mathbf{x}} w_1$, so the Theorem is true for $u = w_2$ and $v = w_1$. ■

According to this Theorem, $G_{\mathbf{x+y}}$ has a cycle if and only if there are i and j such that $A_{\mathbf{x}}[i,j] = A_{\mathbf{y}}[j,i] = 1$. To check this you need $\Theta(|V|^2)$ time.

Now we a ready to prove that the accessibility matrix for the graph obtained by adding new simple path $\mathbf{y}$ to some graph $G_{\mathbf{x}}$ can be determined in 3 steps.

Theorem 2 *Let $\mathbf{x}$ is a minimal path vector to level d, $\mathbf{y}$ is a binary path vector and $G_{\mathbf{x+y}}$ does not have cycles. Then*

$$\widetilde{\alpha}^4_{\mathbf{x+y}} \subseteq \widetilde{\alpha}_{\mathbf{x+y}} \cup \widetilde{\alpha}^2_{\mathbf{x+y}} \cup \widetilde{\alpha}^3_{\mathbf{x+y}}$$

P r o o f. Let $u\ \widetilde{\alpha}^2_{\mathbf{x+y}}\ v$. We have

$$\begin{aligned} u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ v &\Rightarrow (\exists w \in V) u\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ w\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ v \\ &\Rightarrow (\exists w \in V)(u <_{\mathbf{x}} w \vee u <_{\mathbf{y}} w) \wedge (w <_{\mathbf{x}} v \vee w <_{\mathbf{y}} v) \\ &\Rightarrow (\exists w \in V)(u <_{\mathbf{x}} w <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \\ &\quad \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{y}} v). \end{aligned}$$

The relations $<_{\mathbf{x}}$ and $<_{\mathbf{y}}$ are transitive, so

$$(0.2) \qquad (\exists w \in V)\ u <_{\mathbf{x}} v\ \vee\ u <_{\mathbf{y}} w <_{\mathbf{x}} v\ \vee\ u <_{\mathbf{x}} w <_{\mathbf{y}} v\ \vee\ u <_{\mathbf{y}} v$$

Let $u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ v$. From (0.2) we have

$$\begin{aligned} u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ v &\Rightarrow (\exists w_1 \in V_1) u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ w_1\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ v \\ &\Rightarrow (\exists w_1 \in V)\left((\exists w \in V)(u <_{\mathbf{x}} w_1) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1)\right. \\ &\quad \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1) \vee (u <_{\mathbf{y}} w_1))) \wedge ((w_1 <_{\mathbf{x}} v) \vee (w_1 <_{\mathbf{y}} v)) \\ &\Rightarrow (\exists w, w_1 \in V)\left((u <_{\mathbf{x}} w_1 <_{\mathbf{x}} w) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} w)\right. \\ &\quad \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \\ &\quad \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{y}} v) \\ &\quad \left.\vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} w \vee u <_{\mathbf{y}} w_1 <_{\mathbf{y}} w)\right). \end{aligned}$$

Since the relations $<_{\mathbf{x}}$ and $<_{\mathbf{y}}$ are transitive, we have

$$\begin{aligned} (\exists w, w_1 \in V) \quad & (u <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \\ & \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \\ & \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} v). \end{aligned}$$

Having in mind that there are not cycles in $G_{\mathbf{x}+\mathbf{y}}$, using Proposition 3 we may do the following reduction

$$\begin{aligned} (\exists w, w_1 \in V) \quad & (u <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w_1 <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w <_{\mathbf{x}} v) \vee (u <_{\mathbf{y}} v) \\ & \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} v) \vee (u <_{\mathbf{y}} w_1 <_{\mathbf{x}} v). \end{aligned}$$

So,

$$(0.3) \qquad (\exists w, w_1 \in V)(u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v).$$

At the end, let $u\ \widetilde{\alpha}^4_{\mathbf{x}+\mathbf{y}} v$. From (0.2) and (0.3) we have

$$\begin{aligned} u\ \widetilde{\alpha}^4_{\mathbf{x}+\mathbf{y}} v &\Rightarrow (\exists w_2) u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ w_2\ \wedge\ w_2\ \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}}\ v \\ &\Rightarrow (\exists w, w_1, w_2 \in V)((u\ \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}}\ w_2) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} w_2)) \\ &\quad \wedge (w_2 <_{\mathbf{x}} v \vee w_2 <_{\mathbf{y}} v) \\ &\Rightarrow (\exists w, w_1, w_2 \in V)((u\ \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}\ v) \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} v) \\ &\quad \vee (u <_{\mathbf{x}} w <_{\mathbf{y}} w_1 <_{\mathbf{x}} w_2 <_{\mathbf{y}} v). \end{aligned}$$

Using Proposition 3 and the fact that $G_{\mathbf{x}+\mathbf{y}}$ has not cycles, we obtain

$$u\ \widetilde{\alpha}^4_{\mathbf{x}+\mathbf{y}} v \Rightarrow u \widetilde{<}^3_{\mathbf{x}+\mathbf{y}} v \vee u \widetilde{<}^2_{\mathbf{x}+\mathbf{y}} v,$$

which proves that $\widetilde{\alpha}^4_{\mathbf{x}+\mathbf{y}} \subseteq \widetilde{\alpha}_{\mathbf{x}+\mathbf{y}} \cup \widetilde{\alpha}^2_{\mathbf{x}+\mathbf{y}} \cup \widetilde{\alpha}^3_{\mathbf{x}+\mathbf{y}}$. ■

The following result comes directly from the previous theorem

Corollary 3 *Let* $\mathbf{x}$ *is a minimal path vector to level* d*,* $\mathbf{y}$ *is a binary minimal path vector and* $G_{\mathbf{x}+\mathbf{y}}$ *does not have cycles, then* $A_{\mathbf{x}+\mathbf{y}} = \widetilde{A}_{\mathbf{x}+\mathbf{y}} \oplus \widetilde{A}^2_{\mathbf{x}+\mathbf{y}} \oplus \widetilde{A}^3_{\mathbf{x}+\mathbf{y}}$.

3. Algorithm for a networks with component capacity state set {0,1,2, ..., M_i}

In this section we use the results given before for an algorithm for obtaining the set of all minimal path vectors for multi state network with capacity state set of the i-components $\{0,1,2, \ldots, M_i\}$.

Steps of the algorithm:

1. Find all binary minimal path vectors $\mathbf{x}$ and the matrix $A_{\mathbf{x}}$.
2. Set $\mathbf{MPV}^{`}_{d+1} = \emptyset$.
3. For each minimal path vector of level d, $\mathbf{x}$, and each binary minimal path vector $\mathbf{y}$, check whether there are i and j such that $A_{\mathbf{x}}[i,j] = A_{\mathbf{y}}[i,j] = 1$
   - 3.1. If there not such elements, then find $\widetilde{A}_{\mathbf{x}+\mathbf{y}}$ and $A_{\mathbf{x}+\mathbf{y}} = \widetilde{A}_{\mathbf{x}+\mathbf{y}} \oplus \widetilde{A}^2_{\mathbf{x}+\mathbf{y}} \oplus \widetilde{A}^3_{\mathbf{x}+\mathbf{y}}$.
   - 3.2 $\mathbf{MPV}^{`}_{d+1} = \mathbf{MPV}^{`}_{d+1} \cup \{\mathbf{x}+\mathbf{y}\}$
4. The set $\mathbf{MPV}_{d+1}$ is obtain from the set $\mathbf{MPV}^{`}_{d+1}$ by elimination the element that appear more then once.
5. Repeat the steps 2, 3 and 4 for all $d \leq M$.

As an illustration of the algorithm, we give the following example.

Example 4 Regard the network shown in Figure 4.

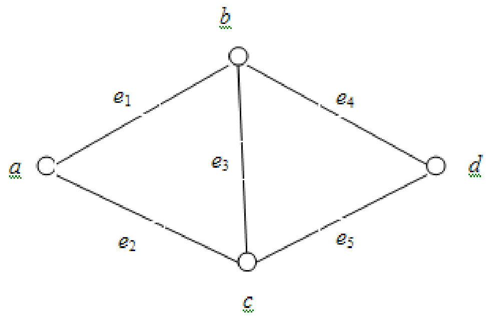

Figure 4

The set of minimal path vectors to level 1 is

$\mathbf{MPV}_1 = \{(1,0,1,0,1),(1,0,0,1,0),(0,1,1,1,0),(0,1,0,0,1)\}$

Table 1 shows all minimal path vectors to level 2. From the table, we can see that the accessibility matrix $A_{\mathbf{x}+\mathbf{y}}$ for the vector $\mathbf{x}+\mathbf{y}$=(1,0,1,0,1) +(0,1,1,1,0) = (1,1,2,1,1) have 1 on the diagonal, so this vector is not a minimal path vector (it is greater than (1,1,0,1,1)).

$\mathbf{MPV}_1$	$\mathbf{MPV}_1$	$A_{\mathbf{x}+\mathbf{y}}$	$\mathbf{MPV}_2$	$\mathbf{MPV}_1$	$\mathbf{MPV}_1$	$A_{\mathbf{x}+\mathbf{y}}$	$\mathbf{MPV}_2$
1 0 1 0 1	1 0 1 0 1	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	2 0 2 0 2	1 0 0 1 0	0 1 1 1 0	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	1 1 1 2 0
1 0 1 0 1	1 0 0 1 0	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	2 0 1 1 1	1 0 0 1 0	0 1 0 0 1	0 1 1 1 0 0 0 1 0 0 0 1 0 0 0 0	1 1 0 1 1
1 0 1 0 1	0 1 1 1 0	0 1 1 1 0 1 1 1 0 1 1 1 0 0 0 0	1 1 2 1 1 not in $\mathbf{MPV}_2$	0 1 1 1 0	0 1 1 1 0	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	0 2 2 2 0
1 0 1 0 1	0 1 0 0 1	0 1 1 1 0 0 1 1 0 0 0 1 0 0 0 0	1 1 1 0 2	0 1 1 1 0	0 1 0 0 1	0 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0	0 2 1 1 1
1 0 0 1 0	1 0 0 1 0	0 1 0 1 0 0 0 1 0 0 0 0 0 0 0 0	2 0 0 2 0	0 1 0 0 1	0 1 0 0 1	0 0 1 1 0 0 0 0 0 0 0 1 0 0 0 0	0 2 0 0 2

Table 1: Minimal path vectors to level 2

4. Conclusion

In this paper we proposed algorithm for obtaining the set of all minimal path vector for multi-state network. This problem belong in the group of very hard and complex problems, since the complexity of the set of minimal path vector is $O(2^{d|V|})$, where V is the set of nodes, and d is level of work of the network. So, it is of great importance to find an procedure that does not gives candidates for minimal path vectors that are not minimal.

By the proposed algorithm, each candidate for minimal path vector is recognized as a minimal path vector by $\Theta(|V|^2)$ binary operations, and when it is not minimal, it is rejected. The advantage is in the fact that this takes less time than the vector comparing operation, which is used for elimination of the non minimal path vectors.

References

[1] M. Mihova, N. Synagina, An algorithm for calculating multi-state network reliability using minimal path vectors, *The 6th international conference for Informatics and In-formation Technology (CIIT 2008)*

[2] M. Mihova, N. Maksimova, Ž. Popeska, An algorithm for calculating multi-state network reliability with arbitrary capacities of the links- *Fourth International Bulgarian-Greek Conference Computer Science'2008* , 170-175.

[3] M. Mihova, N. Maksimova, K. Gorgiev, *Optimal improving of network reliability: Reliability of the improved network.* (in print)

[4] J.E. Ramirez-Marquez and D. Coit, Alternative Approach for Analyzing Multistate Network Reliability, *IERC Conference Proceedings 2003*

[5] J.E. Ramirez-Marquez, D. Coit, and M. Tortorella, *Multi-state Two-terminal Reliability: A Generalized Cut-Set Approach*, Rutgers University IE Working Paper

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