

## **An Innovative Approach in Teaching Mathematics in Elementary and High Schools by Using the Software Package GeoGebra**

*Slaviša Radović*

The constant progress of society, economics and even modern multimedia technology permanently affect the development of systems for improving education. One of the main problems of today's school system is how to maintain focus, concentration and interest of students for learning during classes. An important feature of modern teaching is multimedia. However, the use of multimedia brings a certain transformation in the teaching process. Given the fact that the focus of the teaching process has been shifting from the curriculum and teacher to student, multimedia undoubtedly have a major contribution to the modernization of traditional teaching. It is real to expect that in the near future the multimedia will represent daily routine in teaching. In this work, an innovative approach in teaching mathematics in elementary and high schools by using the software package GeoGebra is introduced. It is demonstrated on the example of 'surface area'. The goal is to increase interactivity between professors and students and to improve the quality of teaching.

*MSC2010: 97C70*

*Key Words:* interactive learning, multimedia, GeoGebra applets, student motivation, teaching methods, surface area, geometric figures.

### **1. Introduction**

The future of the school cannot be understood without understanding its development and history [1]. One of the key needs of the human race is education which is reflected in the maintenance of the species. In order to achieve that, adult community members cared about youth. As the knowledge and skills that were to be transferred to the next generation accumulating, it was clear that the care of young people have to stand apart from everyday life and to be organized differently. Over time, education and teaching the young have become a special social activity which gradually professionalizes and institutionalizes. A concern

about young people in the form of survival and maintaining the species grows in organized education of youth. The school is one of the oldest institutions of society. Old is about seven thousand years, there is from Sumerian empire to China, Greece and the Roman Empire, created in the various religions, during the civil wars, developed during the 19th and 20 century, to today's school. There is even remotely developed community which had its own school. The school is able to survive so long because it is constantly changing, adapting and harmonizing its work with the needs and demands of the community within which it existed. The school has changed and continued to act in a new form, always transforming into a new school - school that meets the needs of the community. For all those concerned with education in any way, no significant issues of what changes should be made in the modern school that she could successfully achieve its social role in the future, in the future society.

Social development, progress of technology and economics certainly entail the development of the current educational system. The problems that we will encounter in the future, the direction of technical advances, the development of economic systems could no longer be predicted using past experience. Currently we cannot even be aware that the world will look like in ten or twenty years, when these students, which we are teaching today, will work and have families. Our goal as teachers is to prepare students properly to face problems that await them in the future. On the contrary to the previous school system, where it was required to be submissive and obedient to reproduce the stored information, we now ask from the students to understand, think, and choose the correct answers. With them we must care about creativity and freedom of thought, which is very often neglected.

Before we start talking about the development of the current education system, we must look at its shortcomings. Perhaps the greatest one is the lack of attention and concentration of students during each lesson. The goal of every teacher is to make lessons fun and interesting enough so that more students were motivated to be attentive to the classes, think and follow the presented material. However, when it comes to math classes, especially with abstract mathematical topics, this is not an easy task. Teachers can use the modern technology and educational software [2] and present abstract mathematical concepts in a virtual environment where students feel very safe. In this way, the place where students are accustomed to playing and having fun turned into a place where students can train and learn [3].

Current classes are designed to fit the needs of average students. This is another drawback of the traditional education system. The use and implementation of multimedia and educational software classes may significantly contribute

to the modernization and individualization of learning [4]. Educational software that is designed with good didactic and methodical preparation of materials is able to correspond to the current knowledge, abilities and skills of each student. Instruction is adjusted to each student and his/her individual needs, since the progress of a student does not have to be conditioned by the progress of the whole class, but his/her own work and opportunities. The flow of the teaching process is shifted from teachers and teaching materials to the student. The student is finally in the center of teaching.

The main task of teachers is to approach a problem to a student, to make it more understandable and to prepare interactive worksheets that enable students to research individually and verify new properties of known objects. In this sense, software package GeoGebra as a tool for modeling and dynamic structure, can develop learning through discovery to the students, the ability to research the problems, and what is the most important, individual learning.

GeoGebra is a mathematical software developed by Markus Hohenwarter for teaching mathematics in schools. It was created as his master work and then became a software package that is used around the world. The author has continued to improve the program through his doctoral dissertation. Dr. Hohenwarter has won many European and international awards in the field of educational software. GeoGebra is a software package that connects algebra, geometry and analysis, and on that basis comes its name. It is free and available in over 50 languages. You can download it from [www.geogebra.org](http://www.geogebra.org) as well as user guide and lots of examples. GeoGebra is written in Java, such that operating system is not a prerequisite for its use. In addition, it is possible to run it from any web browser. At the same address there is a forum designed to gather various experiences while working with the program. GeoGebra is a software package that combines two different approaches of visualizing mathematical objects. More precisely, geometry and algebra are completely equally represented, i.e. it is possible to assign objects to equations, and the change the graphics objects and observe how the equations change when these objects are changing and reverse.

Besides enabling the construction of geometric shapes to record as a picture, we can use GeoGebra to create interactive Web pages, so-called dynamic drawings. Easy implementation and publication on the web is another benefit of GeoGebra. Without knowledge of HTML language and creating Web pages, using GeoGebra "Wizard" after a few clicks and entering the name, title, page description and name of the author, we create web page that is ready to share with other GeoGebra users. This method is suitable to approach the material to the student, attract attention and awaken interest for individual work [5].

Interactivity of applets in web pages is increased if javascript buttons are present on the site, allowing the interaction of text and applets. Having this prepared together with related page, we get a powerful tool of every teacher in bringing abstract mathematical concepts to students of all ages [6].

## 2. Interactive educational material

In this chapter an interactive educational material is presented, related to the term surface area of figure, in the elementary and high schools. The whole content is publicly available at <http://alas.matf.bg.ac.rs/~m106125/index.html>. The material is in the form of web pages with GeoGebra applets. Dynamics and interactivity is achieved by using JavaScript and PHP functions, MatJax function enabled the writing of mathematical text and formulas using standard  $\text{\LaTeX}$  commands.

All educational materials are written primarily for students (students can independently learn, expand their knowledge and enable better individual progress) and teacher (as well as ideas for their multimedia classes). The whole material is divided into classes in which are studied mathematical topics related to surface area of figure, according to educational standards issued by the Ministry of Education of the Republic Serbia. Each class there contains four different sections of interactive materials which are mutually complementary.

The first section concerns the aspect of learning. By using interactive GeoGebra applets and dynamic web pages, geometric figures, objects and their basic properties are represented to students in an interesting way, as well as the motives and different ways to calculate the surface of the figure. The main feature of this presentation of education material is that it allows students to individually explore and discover relationships between observed objects.

By moving objects, points and slider on the applets, the students notice the changes that occur and in this way make conclusions. In every Web page with applets included, there is a mathematical text that introduces the students to an event which takes place on the applet and later explains to the students what they should conclude.

The second part refers to the tasks for exercises. When the students learn the material that concerns the properties of geometric objects and the ways in which compute surface of figures, the next step is the confrontation with the problem assignments.

By opening the link "Interesting tasks" within each class, we get a web page with the problems that need to be solved. The problems are divided into several groups depending on the difficulty or type of tasks. If students have some problems while solving the tasks, they may simply click on the "solution"



Surface area of geometric figures

[Index](#) [Title](#) [Motivation](#) [Registers](#) [Links](#) [Contact](#)

The concept of surface

### Elementary School

- 4. grade
- 6. grade
- 7. grade
- 8. grade

### High School

- 3. grade
- 4. grade
- Literature

"Mathematics and its style of thinking must become an integral part of general culture modern man, ie. man which educate today's schools, whether or not he would perform a job that uses math or not."  
 UNESCO CONFERENCE 1956.

## The concept of surface

Surface figure in the plane of one of the most famous mathematical concepts. Students successfully count surfaces of different figures using known formulas. *But what is that surface area?* On this question only a little number of a student knowing to answer, and it often confuses students of mathematics.

For many centuries in the past people have measured the surface, they do it constantly even today, and need for study of surfaces is not necessary to emphasize, but despite that we need to constantly remind students of that. When measuring the surface people have come up with different surface properties. Naturally distinguish the following four simple properties from which we perform all other properties:

- The surface is always nenegativan number.
- If it a character is made up of parts, then its surface area surface area equal to the sum of these parts.
- The same figures have equal surface areas.
- Square with the side length 1 has an area equal to 1.

It can be no doubt that these facts the students know intuitively. It is necessary only to activate knowledge. That is the easy way, the motivation to introduce the concept of surface polygons that students can easily accept.

**Definition.** Let  $P$  the set of all polygons in the plane, including even empty set. Surface area on a set  $P$  is a mapping  $p : P \rightarrow \mathbb{R}$  which has the following properties

1.  $p(P) \geq 0$ , for every polygon  $P$ .
2. If the polygon  $P$  disjoint union of polygons  $P_1$  and  $P_2$ , then  $p(P_1 + P_2) = p(P_1) + p(P_2)$ .
3. If the polygons  $P_1 \sim P_2$  matching figures, then  $p(P_1) = p(P_2)$ .
4. If  $K$  square with the side length 1, then  $p(K) = 1$ .

Number  $p(P)$  is surface of polygon  $P$ .

Figure 1: Site Surface area of figures,  
<http://alas.matf.bg.ac.rs/~ml06125/index.html>

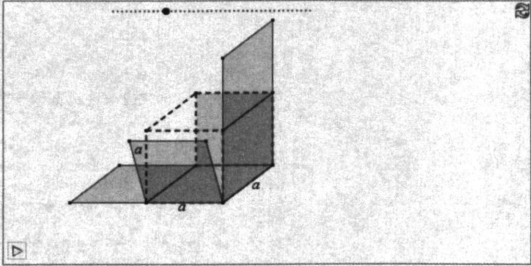
button that appears below each task. This button opens a field in which the steps of the solution are explained in details and exact result are given. This type of initiation of the problem allows students to solve tasks independently and (if needed) simultaneously check if are they working well or if there is another, perhaps more simple, way of solving problems.

The third section is related to the self-test knowledge. Students who have successfully done tasks in the part of site dedicated for the exercise, can further test their knowledge by solving test. By clicking on the "Test" button within each class, we open the web page with the instruction for students and selection of the level of assignments. The tasks are grouped in three tests, arranged by difficulty, as instructed by "Educational standards for the end of compulsory

The concept of surface  
 4. grade  
 Comparison and measurement of surface  
 Calculation of the rectangle area  
 Calculation of the surface of the cuboid and the cube  
 Interesting tasks  
 Test of knowledge  
 Homework  
 6. grade  
 7. grade  
 8. grade  
 High School  
 3. grade  
 4. grade  
 Literature

### Calculation surface of the rectangular and the cube

Objectives and tasks of these classes are introducing students to the figures of cubes and squares, edges, vertices, sides, and to develop in the surface plane of the figure. The following applet shows that the network consists of 6 blocks of matching square with sides equal to the edge of a cube. If  $P_1$  surface of a square, then the surfaces of the cube

$$P = 6P_1 = 6a^2.$$


The network rectangular consists of a six rectangles, of which two nonadjacent congruent. The surfaces of these rectangles are

$$P_1 = a \cdot c, \quad P_2 = a \cdot b, \quad P_3 = b \cdot c,$$

and rectangular area:

$$\begin{aligned}
 P &= 2P_1 + 2P_2 + 2P_3 \\
 &= 2(a \cdot b + b \cdot c + c \cdot a).
 \end{aligned}$$

Figure 2: Surfaces of the cube, 4 grade of elementary school.

education for mathematics,” the Ministry of Education and Institute for the Evaluation of Education of Republic Serbia.

When the test is opened, the measuring of time starts. At the end of each question, the number of points that carries the correct response is written and there is place to enter the correct answer. When a student is done with the tasks, by clicking on the ”Check” button, his/her answers are evaluated and he/she wins certain number of points. The student can win a total 100 points.

The ”Check” button opens a new page where all tasks are reviewed and where the student can read information about the test - which tasks are correct, the number of points he/she has won, which grade got, the time spent solving test and the exact solutions of tasks (in order to realize its mistakes).

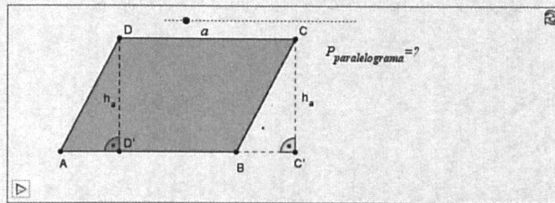
The fourth part refers to homeworks. This part of the site is dedicated to students who work at home and whose knowledge teachers want to check by using an e-mail.

When student opens a homework, before he/she starts to work, it is

- ② The concept of surface
- Elementary School
- ④ 4. grade
- ⑤ 5. grade
  - The need to measure
  - Surface of rectangular
  - surface of parallelogram
  - Surface of triangle
  - Surface of trapezoid
  - Surface of the quadrilateral whose
- diagonal normal
- Interesting tasks
- Test of knowledge
- Homework
- ⑦ 7. grade
- ⑧ 8. grade
- High School

### Surface of parallelogram

let the  $ABCD$  arbitrary parallelogram. And are some  $C'$  and  $D'$  point base perpendiculars from the vertices  $C$  and  $D$  on the site of a parallelogram  $AB$ . Students show that the parallelogram  $ABCD$  and rectangle  $D'C'CD$  have the same surface- with applet, and then we prove that the triangles  $AD'D \cong BC'C$ , than  $ABCD$  and  $D'C'CD$  are equal. Since the surface of the rectangle we have learned to calculate, we can now calculate the area of a parallelogram.



**Theorem.** Area of a parallelogram equal to the product length of its sides and the right height, respectively:  
 $P = a \cdot h_a = b \cdot h_b$ .

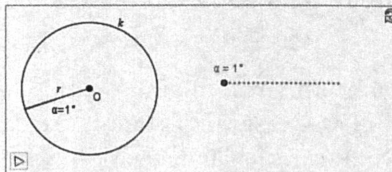
Figure 3: The area of parallelogram, the 6th grade of elementary school.

- ② The concept of surface
- elementary School
- ④ 4. grade
- ⑥ 6. grade
- ⑦ 7. grade
  - Surface of polygons
  - Surface of a circle
  - Surface of the sector and the ring
- Interesting tasks
- ⑧ 8. grade
- High School
- ③ 3. grade
- ④ 4. grade
- ⑤ Literature

### Surface of the sector, slice, and ring

The circular slice is part of the circle limited by radius and arc. Each circuit slice has a corresponding central angle  $\alpha$  and an arc length  $l$ .

Let draw circuit slice of which is the central angle  $1^\circ$ . Its area is equal to 360-part of the circle area  $P_l = \frac{r^2 \cdot \pi}{360}$ .



If a central angle of the sector  $2^\circ$  then its surface is  $P_l = \frac{r^2 \cdot \pi}{360} \cdot 2$ , if a central angle is  $3^\circ$  than is  $P_l = \frac{r^2 \cdot \pi}{360} \cdot 3$ , and so on...

If  $\alpha$  central angle of the sector, then its surface is:

$$P_l = \frac{r^2 \cdot \pi}{360} \cdot \alpha.$$

Figure 4: Surface area of the sector, slice and ring, 7 grade of elementary school.

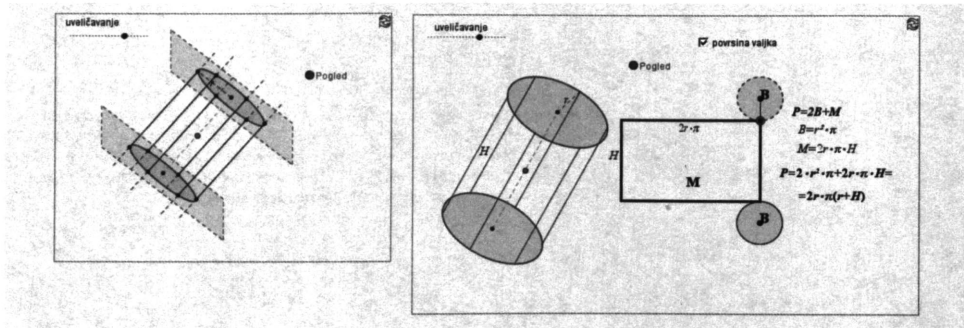


Figure 5: The area of the cylinder, the 8th grade of elementary school.

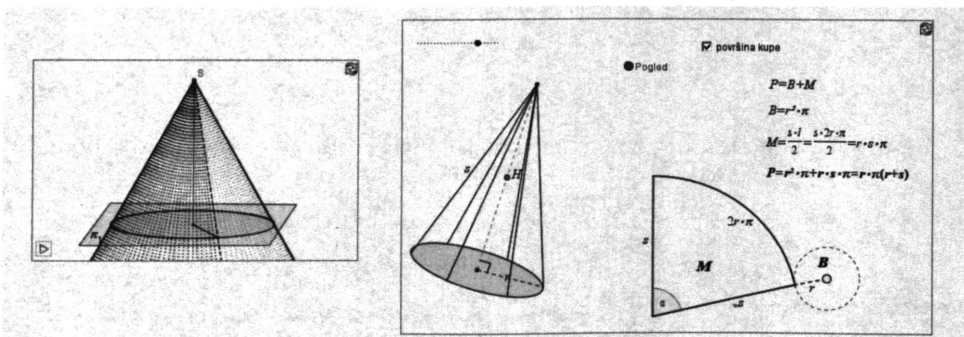


Figure 6: The Surface of cone, the 3th grade of high school.

necessary to enter a name, an e-mail and teacher's e-mail in the corresponding fields. When all the tasks are completed, the student fills the last field that refers to the comments on homework, clicks on "send homework" in order to send the answer to the teacher.

When the student clicks on "send" button, he/she gets a confirmation e-mail that his homework is sent to the teacher and also needs to verify the teacher's mail address. The teacher also gets mail with the homework title, the student's name, the e-mail address, the text of the tasks, the correct answer and the student's answer in a table format. The teacher needs to review the mail and to inform the students about the reception of the e-mail and the grade of the homework tasks.

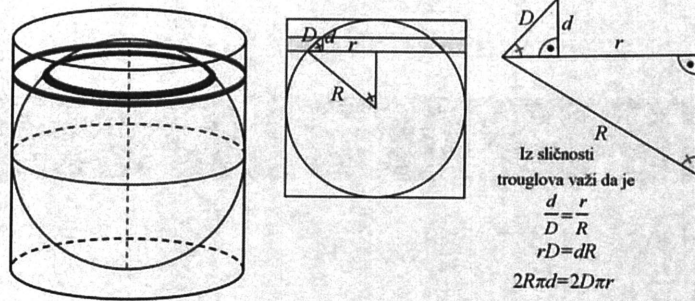


Figure 7: Surface of the sphere, 3th grade of high school.

The concept of surface  
**Elementary School**  
 4. grade  
 6. grade  
 7. grade  
 8. grade  
**High School**  
 3. grade  
 4. grade  
 Surface of arbitrary figure  $F$   
 Definite integral  
 Surface of flat figure  
 Surface of rotating figure  
 Interesting tasks  
 Literature

### Surface of flat figure

Using properties of definite integral and the conclusions that we have come in the previous page, we get the following useful formula:

Surface of Limited surface straight axis  $Ox$ , curve  $y = f(x)$  and a straight lines  $x = a \mid x = b$ , defined by the formula:

$$P = \int_a^b |f(x)| dx.$$

Note that if a function  $f(x) \leq 0$  then the above formula

$$P = \int_a^b f(x) dx.$$

And if in the interval  $[a, b]$  function  $f(x)$  changes sign at the point  $c$ , then the formula becomes

$$P = \int_a^c f(x) dx - \int_c^b f(x) dx.$$

If the surface is limited by two curves, and let the points  $x_1 \mid x_2$  solutions of the equation  $f_1(x) = f_2(x)$ , then the surface is:

$$P = \int_{x_1}^{x_2} f_2(x) dx - \int_{x_1}^{x_2} f_1(x) dx = \int_{x_1}^{x_2} |f_2(x) - f_1(x)| dx$$

Figure 8: Surface a flat figure, the 4th grade of high school.

Surface area of geometric figures

[Index](#) [Title](#) [Navigation](#) [Glossary](#) [Links](#) [contact](#)

📁 The concept of surface

### Elementary School

- 📁 4. grade
- 📁 6. grade
- 📁 7. grade
- 📁 8. grade
  - Surface of prism
  - Surface of pyramid
  - Surface of the cilinder
  - Surface of buy
  - Surface of the ball
  - Interesting tasks

### High School

- 📁 3. grade
- 📁 4. grade
- 📁 Literature

## Interesting task

Prism
Pyramid
Cylinder
Cup
Ball

1. What is the height of equal edges a three-sided pyramid edges  $9\text{ cm}$ ? Solution
2. Calculate the height of regular hexagonal pyramid if the side edge is length  $6\text{ cm}$  formed with a the plane base angle of  $45^\circ$ . Solution
3. Calculate the surface quadriateral regular pyramid if  $s = 35\text{ cm}$ ,  $h_a = 28\text{ cm}$ . Solution
4. The right four-way pyramid of main edge of the  $4\sqrt{2}\text{ cm}$  and height  $4\text{ cm}$ , divided by two diagonal cross-section of four pyramids. Calculate the surface of one of these parts. Solution
5. The base of the pyramid is a rhombus of  $6\text{ cm}$ , and obtuse angle from  $120^\circ$ . Calculate the surface and the height of the pyramid if the height of rhombus is equal to height of pyramid. Solution
6. Calculate the surface quadriateral pyramid, which is basically a rectangle, whose apex projects into the intersection of the diagonal basis and if  $a = 20\text{ cm}$ ,  $b = 14\text{ cm}$  a lateral height corresponding to the short side of the rectangle is  $26\text{ cm}$ . Solution
7. The base of the pyramid is a right-angled triangle, with cathetus  $8\text{ cm}$  |  $6\text{ cm}$ . Calculate the surface of the pyramid if its height is equal to half the hypotenuse, and its base is a right angle base threads.
8. Calculate  $P$  regular hexagonal pyramid if the area of the base is  $24\sqrt{3}\text{ cm}^2$  and  $H : a = 1 : 2$ .
9. The largest diagonal cross-section of regular hexagonal pyramid is an isosceles triangle. The expressions  $P$  and the function of the pyramid base edge  $a$ .
10. Is there a regular hexagonal pyramid with equal edges?
11. Side regular quadriateral pyramid is tilted toward the plane base at an angle of  $60^\circ$ . Calculate  $P$  in the function of the diagonal of the pyramid base.

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Figure 9: Interesting tasks.



Surface area of geometric figures

Index
Home
Maths
Geometry
Links
Contact

③ The concept of surface

**Elementary School**

③ 4. grade

③ 6. grade

③ 7. grade

③ 8. grade

Surface of prism

Surface of pyramid

Surface of the cylinder

Surface of buy

Surface of the ball

Interesting tasks

**High School**

③ 3. grade

③ 4. grade

③ Literature

### Interesting task

Prism Pyramid Cylinder Cup Ball

**1. What is the height of equal edges a three-sided pyramid edges 9 cm ?** Solution

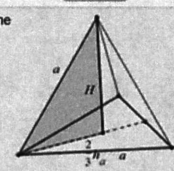
We can see the right-angled triangle on it and apply the Pythagorean Theorem:

$$H^2 = a^2 - \left(\frac{2}{3} h_a\right)^2$$

$$H^2 = 9^2 \text{ cm}^2 - \left(\frac{2\sqrt{3}}{3}\right)^2$$

$$H^2 = 81 \text{ cm}^2 - 27 \text{ cm}^2 = 54 \text{ cm}^2$$

$$H = 3\sqrt{6} \text{ cm}$$



Close solution

**2. Calculate the height of regular hexagonal pyramid if the side edge is length 6 cm formed with a the plane base angle of 45° .** Solution

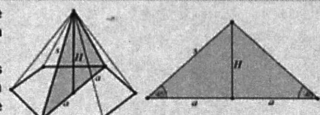
As in the previous task, draw a picture and then look at what it looks like a diagonal cross-section.

The angle at the top of the pyramid is straight, if you put the the height from the top of the pyramid it divides the right-angled triangle into two congruent isosceles triangles. Hence we can conclude that the height of the pyramid is equal to the basic edge  $a$  and using the Pythagorean theorem, we obtain:

$$s^2 = a^2 + H^2$$

$$6^2 \text{ cm}^2 = 2H^2$$

$$H^2 = 18 \text{ cm}^2, \text{ from here } H = 3\sqrt{2}. \text{ Close solution}$$



**3. Calculate the surface quadrilateral regular pyramid if  $s = 35 \text{ cm}$ ,  $h_a = 28 \text{ cm}$ .** Solution

Note the right-angled triangle and use the Pythagorean theorem to calculate  $a$ :

$$\left(\frac{s}{2}\right)^2 = s^2 - h_a^2$$

$$\left(\frac{s}{2}\right)^2 = 35^2 \text{ cm}^2 - 28^2 \text{ cm}^2$$

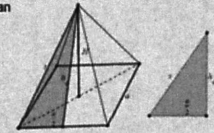
$$\left(\frac{s}{2}\right)^2 = 441 \text{ cm}^2$$

$$\frac{s}{2} = 21 \text{ cm}, \text{ and } a = 42 \text{ cm}$$

$$M = 4 \cdot \frac{a \cdot h_a}{2} = 2352 \text{ cm}^2$$

$$B = a^2 = 1764 \text{ cm}^2 \text{ and surface is:}$$

$$P = M + B = 2352 \text{ cm}^2 + 1764 \text{ cm}^2 = 4116 \text{ cm}^2 \text{ Close solution}$$



**4. The right four-way pyramid of main edge of the  $4\sqrt{2} \text{ cm}$  and height 4 cm, divided by two diagonal cross-section of four pyramids. Calculate the surface of one of these parts.** Solution

The base of the new pyramid is an isosceles right-angled triangle with kateto  $\frac{d}{2}$ , layer consists of two identical rectangular triangle with cathetus  $H$  |  $\frac{d}{2}$ , and the old side of the pyramid.

Calculate  $d$ .

$$d^2 = 2a^2, d^2 = 64 \text{ cm}^2 \text{ pa je } d = 8 \text{ cm}$$

To be able to calculate an area we need to determine  $h_a$

$$h_a^2 = \left(\frac{d}{2}\right)^2 + H^2$$

$$h_a^2 = 8 \text{ cm}^2 + 16 \text{ cm}^2, h_a = 4\sqrt{5}$$

And than:

$$B = \frac{1}{2} \left(\frac{d}{2}\right)^2 = 8 \text{ cm}^2$$

$$M = 2 \cdot \frac{H \cdot \frac{d}{2}}{2} + \frac{a \cdot h_a}{2} \text{ Close solution}$$

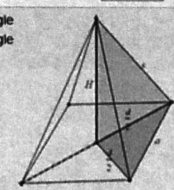


Figure 10: Interesting tasks.



Surface area of geometric figures

[Index](#) [Title](#) [Mathematics](#) [Geogebra](#) [Links](#) [Contact](#)

**The concept of surface**

**Elementary School**

- 4. grade
  - The need to measure
  - Surface of rectangular
  - Površina of parallelogram
  - Surface of triangle
  - Surface of trapezoid
  - Surface of the quadrilateral whose diagonal normal
  - Interesting tasks
  - Test of knowledge
  - Homework
- 7. grade
- 8. grade

**High School**

- 3. grade
- 4. grade
- Literature

### Test intermediate level

basic level
intermediate level
advanced level

1. Calculate the side  $b$  rectangle if  $P = 60\text{cm}^2$  i  $a = 12\text{cm}$   
Side  $b$  je   $\text{cm}$ .  
[8points]
2. Perimeter of the square is  $O = 7,2\text{dm}$ . Compute area of squares.  
Area of square is   $\text{dm}^2$ .  
[8points]
3. Surface area of parallelogram is  $P = 52\text{dm}^2$ , and side  $b = 10\text{dm}$ .?What is the height  $h_b$ ?  
Height  $h_b$  is   $\text{dm}^2$ .  
[8points]
4. Surface area of rhomb is  $14\text{cm}^2$ , and height is  $3,5\text{cm}$ . Calculate the perimeter of rhomb.  
Perimeter of rhomb is   $\text{cm}^2$ .  
[10points]
5. Surface right triangle is,  $900\text{mm}^2$  and one leg is  $4,5\text{cm}$ . What is the second leg?  
 1,5cm  
 2cm  
 2,5cm  
 3cm  
 [13points]
6. John wants to pave the kitchen tiles size  $20\text{cm}$  and  $15\text{cm}$ . How much tiles need if the dimensions of the kitchen are  $2\text{m}$  and  $6\text{m}$ ?  
He need  tiles.  
[13points]
7. Trapezoid Surface is  $P = 78\text{cm}^2$ , a trapezoid median is  $m = 12\text{cm}$ . Calculate the height of trapezoid.  
the trapezoid height is   $\text{cm}$ .  
[10points]
8. Carrots planted on the land forms a parallelogram of  $1,5\text{m}$  and height  $0,7\text{m}$ . On the land of the same form, of  $0,6\text{m}$  and height  $2,5\text{m}$ , planted spinach. What is the plant planted in a larger area?  
Larger area is planted with carrots? yes or no?   $\text{cm}^2$ .  
[10points]
9. Trapezoid Surface is  $336\text{dm}^2$ , one basis is  $29,4\text{cm}$ , and another  $10,6\text{cm}$ . Calculate the height.  
The height is   $\text{cm}$ .  
[10points]
10. Surface of rhomb was  $35,6\text{dm}^2$ , and a diagonal is  $5\text{dm}$ . Calculate other diagonal.  
Second diagonal is   $\text{dm}$ .  
[10points]

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Figure 11: Test of knowledge.

**Surface** area of geometric figures

Index Title Motivation Geogebra Links Contact

☐ The concept of surface

**Test intermediate level**

basic level **intermediate level** advanced level

**Elementary School**

- ☐ 4. grade
- ☐ 6. grade
  - The need to measure
  - Surface of rectangular
  - Surface of parallelogram
  - Surface of triangle
  - Surface of trapezoid
  - Surface of the quadrilateral whose diagonal normal
  - Interesting tasks
  - Test of knowledge
  - Homework
- ☐ 7. grade
- ☐ 8. grade

**High School**

- ☐ 3. grade
- ☐ 4. grade
- ☐ Literature

**Results**

1. 8
2. 8
3. 8
4. 0
5. 13
6. 0
7. 0
8. 10
9. 10
10. 10

Number of points : 67 od 100.  
Mark : (3)

The time solving: 1 min. i 28 sek.

Rešenja zadataka:

1. Solution is  $5\text{cm}^2$ .
2. Solution is  $3,24\text{dm}^2$ .
3. Solution is  $5,2\text{dm}^2$ .
4. Solution is  $16\text{cm}^2$ .
5. Solution is  $P = 2\text{cm}$ .
6. Solution is 400 tiles.
7. trapezoid height is  $6,5\text{cm}$ .
8. Solution is no.
9. Height is  $h = 16,8\text{cm}$ .
10. Dijagonal is  $d = 7,1\text{dm}$ .

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Figure 12: The solution of the test knowledge.

### 3. Conclusions

In this paper only a small part of the functionalities of GeoGebra and the possibilities that it offers are presented. We have demonstrated an interactive educational material related to the term surface area of a figure, in the elementary and high schools. Four aspects of created GeoGebra environment are presented: learning aspect, explanations, self- testing and homework. This approach can be applied to any mathematical topic. Teachers can use the GeoGebra educational software and the present abstract mathematical concepts in a virtual environment where students feel very safe. In this way, the GeoGebra environment turns into the place where students learn and play at the same time.

Surface
area of geometric figures

[Index](#)
[Title](#)
[Motivation](#)
[Gangetra](#)
[Links](#)
[Contact](#)

**The concept of surface**

**Elementary School**

- 4. grade
- 6. grade
  - The need to measure
  - Surface of rectangular
  - Surface of parallelogram
  - Surface of triangle
  - Surface of trapezoid
  - Surface of the quadrilateral whose diagonal normal
  - Interesting tasks
  - Test of knowledge
  - Homework
- 7. grade
- 8. grade

**High School**

- 3. grade
- 4. grade
- Literature

### First homework

First homework
Second homework
Third homework
Fourth homework

Your name

Your e-mail

Teacher's e-mail

---

1. Garden has the shape of a rectangle length  $a = 34m$  and width of  $b = 20m$ . Calculate the area of this garden in the fires. Surface of garden is   $m^2$ .
2. Tin roof forms of a rectangle of length  $320dm$ , and width  $12m$ , should be painted. How much is cost a painting of the roof if the coloring  $1m^2$  cost  $20din$ ? He need  din.
3. Side of the rectangle are  $a = 21cm$ ,  $b = 13cm$ , and the side of the square  $16cm$ . For how different are their surface? There are different for   $cm^2$
4. The basis of the house is a square volume of  $44m$ . Concrete path around the house takes  $1m$  wide. Surface of the path is   $m^2$
5. The book has a 100 sheets whose dimensions are  $21cm$  and  $30cm$ . How many  $m^2$  of paper needed to make 20 of these books? We need   $m^2$  paper.
6. If the area of the parallelogram  $P = 54cm^2$  and bases  $a = 12cm$ , is height  $h_a = 4.5cm$ ?
7. Calculate the area of rhomb if its scope is,  $96cm$  and the height is  $16cm$ . Surface of rhomb is   $cm^2$ .
8. The scope of the parallelogram is  $24cm$ , short side is two times smaller than a side long. If the height corresponding to the longer page is  $h_a = 2cm$ , to determine height corresponding to a short page. height is  $h_b$    $cm$ .

Complaint on homework

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Figure 13: Homework.

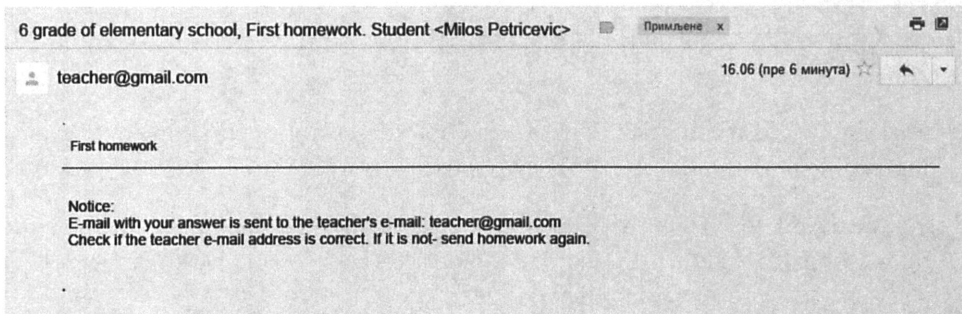


Figure 14: E-mail which student gets.

First homework, 6. grade of elementary school			
Student name:	Milos Petricevic		
Student e-mail:	milos.petricevic@gmail.com		
Text of problem	Corect answer	Student answer	
1. Vrt ima oblik pravougaonika dužine $a=34m$ i širine $b=20m$ . Izračunati površinu toga vrta u arima.	680m <sup>2</sup>	6,8	
2. Limeni krov oblika pravougaonika dužine 320dm, a širine 12m, treba obojiti. Koliko košta bojenje tog krova ako se za bojenje 1m <sup>2</sup> plaća 20din ?	7680	7680	
3. Stranice pravougaonika su $a=21cm$ , $b=13cm$ , a stranica kvadrata je 16cm . Za koliko se razlikuju njihove površine?	17cm <sup>2</sup>	17	
4. Temelj kuće je kvadrat obima 44m . Oko kuće vodi betonska staza širine 1m . Površina staze je ?	48m <sup>2</sup>	48	
5. Sveska ima 100 listova čije su dimenzije 21cm i 30cm. Koliko m <sup>2</sup> papira je potrebno da bi se napravilo 20 takvih svezaka?	1260m <sup>2</sup>	126	
6. Ako je površina paralelograma $P=54cm^2$ i osnovica $a=12cm$ , da li je visina $h_a=4,5cm$ ?	Da.	da	
7. Izračunati površinu romba ako mu je obim 96cm, a visina 16cm.	384cm <sup>2</sup>	65	
8. Obim paralelograma je 24cm. Kraća stranica je dva puta manja od duže stranice. Ako je visina koja odgovara dužoj stranici $h_a=2cm$ , odrediti dužinu visine koja odgovara kraćoj stranici.	4cm.	3	
Comments on homework			

Figure 15: E-mail that teacher gets.

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