

On Some Contributions of Professor Peter Rusev

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The 6th International Conference "Transform Methods and Special Functions' 2011", 20 - 23 October 2011 was dedicated to the 80th anniversary of Professor Peter Rusev, as one of the founders of this series of international meetings in Bulgaria, since 1994. It is a pleasure to congratulate the Jubiliar on behalf of the Local Organizing Committee and International Steering Committee, and to present shortly some of his life achievements and contributions to the areas of his research interests.

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Prof. Dr.Sc. Peter Rusev is born 28 August 1931 in the town of Shumen, so in 2011 we celebrated his 80th Jubilee.

Graduated (1953) at Dept. Math. and Physics - Sofia University, he has obtained a Ph.D. degree from Sofia University in 1957, for the thesis "*On the Distribution of Zeros of a Class of Entire Functions Represented in Integral Form*" ([T1]), under the guidance of Prof. Lyubomir Iliev.

His professional career in Institute of Mathematics and Informatics - Bulgarian Academy of Sciences (IMI – BAS) started in 1958 as a junior researcher, and with habilitation (Asso. Prof.) in 1968. The next degree - Dr. Sc. (Scientiae Doctor, Higher Doctorate), he received in 1979 at BAS for the thesis "*Representation of Analytic Functions by Means of Systems of Polynomials and Functions of Laguerre and Hermite of Second Kind*" ([T2]). Full Prof. from 1983, at the same Institute. Fellow of Alexander v. Humboldt Foundation - at Göttingen University (Sept. 1967 - June 1968, Sept. 1968 - June 1969). Emeritus Prof. of IMI – BAS, since 2002.

The *main research interests* of Prof. Rusev are in the topics: classical orthogonal polynomials, holomorphic functions and series expansions, special functions and integral transforms. He is *author of about 100 scientific articles, 2 scientific monographs and many other publications*, and member of the Editorial Board of the journal "*Fractional Calculus & Applied Analysis*", with a short CV at <http://versita.com/rusev/>.

Titles of his books:

[M1] *Analytic Functions and Classical Orthogonal Polynomials*. Bulgarian Mathematical Monographs **3**, Publ. House of Bulg. Acad. of Sci., Sofia, 1984, 135 pp.

[M2] *Classical Orthogonal Polynomials and Their Associated Functions in Complex Domain*. Bulgarian Mathematical Monographs **10**, Prof. Marin Drinov Acad. Publ. House, 2005, 278 pp.

Prof. Rusev has been *scientific advisor of 7 Ph.D. students* and their theses, as follows:

- *Ivan Ramadanoff* (Prof. Dr., Univ. de Caen, France, Directeur de Recherches); "Some problems of the theory of the Bergman function" (1974);
- *Ivanka Kasandrova* (Asso. Prof. Dr., Plovdiv Univ.): "Distribution and asymptotic properties of the zeros of a class of entire functions" (1977);
- *Johann Davidov* (Prof. D.Sc., IMI - BAS): "Holomorphic mappings and representative domains" (1977);
- *Valentin Hristov* (Asso. Prof., Dr., IMI - BAS): "On the pseudometrics of Caratheodory and Kobayashi" (1978);
- *Georgi Boychev* (Asso. Prof. Dr., Tracian Univ. at Stara Zagora): "Uniform convergence and summability of Jacobi, Laguerre and Hermite series" (1984);
- *Lyubomir Boyadjiev* (Prof. Dr.Sc., Techn. Univ. Sofia & Kuwait Univ.): "Analytic functions and Laguerre series" (1986);
- *Jordanka Paneva-Konovska* (Asso. Prof. Dr., Techn. Univ. Sofia): "Basicity and completeness of numerable systems of Bessel functions and polynomials" (1998).

In Institute of Mathematics and Informatics (IMI) - Bulgarian Academy of Sciences, he has been Scientific Secretary (1964-1969), Vice-Director (1971-1972), member of Scientific Council (1995-2007). Chairman of the Specialized Scientific Council on Mathematics and Mechanics at the Higher Attestation Commission, Bulgaria (1998-2004). Expert for National Science Fund and International foundation "St.St. Cyril and Methodius" on competitions for DAAD scholarships and other research projects.

Prof. Rusev has been advisor of 4 research projects (1989-2000), organizer of a series of international mathematical conferences and co-editor of proceedings of such conferences, namely: the 2nd Congress of Bulgarian Mathematicians (Varna 1967, Secretary), Internat. Conf. on Constructive Theory of Functions (Varna 1970, Vice-President), 3rd Congress of Bulgarian Mathematicians (Varna 1972, Vice-President), Internat. Conference on Complex Analysis

(Varna 1981, Progr. Comm.), Intern. Colloq. on Complex Analysis and Applications (Golden Sands 1983, Progr. Committee), Internat. Workshops on Transform Methods & Special Functions (Bankya 1994, Varna 1996, Blagoevgrad 1999, Chairman), Internat. Workshop "Transform Methods & Special Functions' 2003" (Progr. Committee), Jubilee Session for the 50th anniversary of IMI (1997, Org. Committee); Intern. Symposium "Geometric Function Theory and Applications' 2010" (member of Org. and Progr. Committees).

Prof. Rusev has taken essential role as editor and translator of several volumes, recently published by Bulgarian Academy of Sciences, with selected papers of other famous Bulgarian mathematicians as Nikola Obrechhoff, Lyubomir Iliev, etc. He has been also Ed.-in-Chief and author for the parts on special functions, complex analysis, theory of functions, in Physics-Mathematical and Technical Encyclopedia of Bulgarian Academy of Sciences, 1990 and 2000. Scientific Editor of the proceedings of the recent annual Spring Conferences of Union of Bulgarian Mathematicians.

Member of Amer. Math. Soc., Soc. for Didactic of Math. (Germany), Union of Bulgarian Scientists. Reviewer for Math. Reviews and mathematical journals as "Serdica (Bulg. Math. Journal)", "Math. Balkanica", "C.R. Acad. Bulg. Sci.", "Revista Technica" (Univ. del Zulia, Venezuela), "Kuwait J. Sci & Eng.", "Soochow J. of Math." (Taipei, Taiwan), "J. Math. Anal. and Appl.", "Bull. Belgian Math. Soc.", "Ann. Polonici Mat.", etc.

Visiting Professor at Trier University and Göttingen University (Germany), International Mathematical Banach Center - Warsaw (Poland), invited talks at many international conferences (Germany, Poland, Roumania, Russia, Turkey, Hungary, Serbia), etc.

Teaching activities (at Sofia University, Rousse University, Plovdiv University, Shumen University, IMI - BAS): lectures for undergraduate students (Complex Analysis, Linear Algebra and Analytic Geometry, Differential Equations), lectures for graduate students (Entire Functions, Conformal Mappings, Classical Orthogonal Polynomials, Functions of Several Complex Variables, Zeros of Polynomials), seminars, etc.

For his research, scientific and teaching activities, Prof. Peter Rusev has been awarded by: – Medal "Cyril and Methodius" (1974, 1984); Jubilee medal "1300 Years of Bulgaria" (1981); Jubilee medal "Marin Drinov" - on occasion of 100 years of Academy (1969); Memorial medals "10 years of Shumen University" (1981) and "25 years of Shumen University" (1996); Medal for contributions to IMI on occasion of its 60 years (2007). In 2002, he was promoted as "Doctor Honoris Causa" of the Shumen University, and as "Honorary Member of IMI – BAS".

Main directions of P. Rusev's scientific contributions

Prof. Peter Rusev has essential contributions in the following areas, related to the TMSF conferences: – distributions of zeros of entire functions, defined by means of Fourier transforms; – Bergman function. – classical orthogonal polynomials; – special functions and integral representations;

I. Distributions of zeros of entire functions, defined by means of Fourier transforms

This topic, considered as originated by the Hungarian mathematician G. Pólya, appeared as traditional one in the Bulgarian mathematics after a series of publications of Acad. Lyubomir Tchakaloff (On a class of entire functions, *J. of B.A.S.* **36** (1927), 51-92) and of Acad. Nikola Obrechhoff (On the zeros of polynomials and some entire functions, *Annuaire of Sofia Univ., Fac. Phys.-Math.*, **37** (1941/42), 1-115). Their successors in this direction in Bulgaria are: Acad. Lyubomir Iliev, Profs. Emil Bozhorov, Kiril Dochev, Dimitar Dimitrov, and others.

The contributions of Prof. Peter Rusev are included in his Ph.D. Thesis (1957) and publications [1]-[4], [10], [12], [13], [17], [22], [72], and recent ones, as [90] - [95]. The most essential are:

– Generalization of the modification of the Schur theorem (I. Schur, Über Potenzreihen, die im Innern des Einheitskreises beschränkt sind, *J. Reine Angew. Math.* 147 (1917), 205-232), proposed by L. Iliev in his Dr. Thesis "On the zeros of some classes of polynomials and entire functions" (Sofia, 1940). Application of this modification to the distribution of zeros of entire functions of the form (see [3])

$$U(f; z) = \int_0^1 f(t) \cos zt \, dt, \quad V(f; z) = \int_0^1 f(t) \sin zt \, dt.$$

An assertion, essentially more precise than Th. 2 from [3] was provided by K. Dochev as Th. 3 in his publication in *C.R. Acad. Bulg. Sci.*, **15**, No 3 (1962), 239-241. Commentaries on Dochev's arguments and a more direct proof of the same assertion are contained in Rusev's paper [17], pp. 46-48.

– Characterization of the asymptotic behaviour of the zeros of classes of entire functions of exponential type, bounded on the real axis and having only real zeros ([10], Ths. 6,7,8).

– Precision of the distribution of zeros of the entire function ([10], pp. 266-267)

$$\Phi(z) = \int_0^{\infty} \exp(-t^{2q}) \cos zt \, dt, \quad q \in N.$$

As it is well-known, the entire function Φ has only real zeros (G. Pólya, Über trigonometrische Integrale mit nur reellen Nullstellen, *J. Reine Angew. Math.* **158** (1927), 6-18).

– Sufficient conditions concerning the "symmetry" of the function f with respect to the point $1/2$ and the zeros of the polynomials $\sum_{k=0}^{n-1} f(k/2n-1) z^k$, so that the functions $U(f; z)$ and $V(f; z)$ to have only real zeros, infinitely many of which are joint ones ([17], Th. 3).

II. Bergman function

Rusev's interest to this topic has arisen during his 8 months scholarship in the Moscow State University, 1963. Especially, his attention was drawn to the relationship between: the analytical mappings (not necessarily single-valued) with single-valued Jacobians of domains in the space C^n which do not decrease the volume, and the Bergman functions of such domains.

His results are published in the papers [8] and [9]. Of them, as almost completed, can be considered the following ones:

- Analytical characteristics of bounded simply-connected poly-cylindrical domains of C^n ([8, Th.4], [9, Th.3]).
- Linearity of biholomorphic mappings preserving the volume of poly-cylindrical domains of C^n ([8, Th.6], [9, Th.5]).

III. Classical orthogonal polynomials

Rusev's interest in this area was born from his participation in the seminar of Acad. Nikola Obrechhoff and mostly, under the influence of his publication: N. Obrechhoff, On some orthogonal polynomials in the complex plane, *Izvestiya Math. Inst. B.A.S.* **2**, No 1 (1956), 45-67. As a result, in 1962 the first Rusev's publication [5] on this topic appeared.

An essential role to keep and make deeper the interests of Prof. Rusev in this area, was played by Hille's results on representation of holomorphic functions by means of series in the Hermite polynomials (E. Hille, (i) Contributions to the theory of Hermitian series, *Duke Math. J.* **5** (1939), 875-936; (ii) The representation problem, *Trans. A.M.S.* **47** (1940), 80-94), and by Pollard's results on representation of such functions by series of Laguerre polynomials (H.

Pollard, Representation of analytic functions by a Laguerre series, *Ann. Math.* **92**), 48 (1947), 358-363).

Another stimulation came from his unsatisfaction that not enough attention was paid in the classical Szegő's monograph (G. Szegő, *Orthogonal Polynomials*, AMS Colloq. Publ., 23, 1939, 4th Ed.: 1974) to clarify the character of convergence of the series in Laguerre and Hermite polynomials; as well as from the lack of attention to the role and importance of the associated functions (called also functions of second kind) of the classical orthogonal polynomials, and especially - of the Laguerre and Hermite polynomials.

In this way was formed the continuous interest of Rusev towards problems related to representations of holomorphic functions by series in classical orthogonal polynomials and their associated functions. The results of his long years' efforts in this area gave richness of fruits, like his Dr.Sc. thesis "*Representation of Analytic Functions by Means of Systems of Polynomials and Functions of Laguerre and Hermite of Second Kind*", [T2] (1979); his 1st monograph "*Analytic Functions and Classical Orthogonal Polynomials*" [M1] (1984); the 2nd monograph "*Classical Orthogonal Polynomials and Their Associated Functions in Complex Domain*", [M2] (2005); and many of his numerous papers, as: [5] - [6], [11], [18] - [21], [24] - [30], [32] - [39], [41] - [44], [48] - [52], [54], [56], [61], [67], [70], [73], [75], [78], [81], [83], [85], [86].

The most essential contributions in this direction are:

- Orthogonality of the Jacobi polynomials in the complex plane under most general assumptions on their parameters ([5], [M1]). The result from [5] was repeated later by other authors (e.g.: A.M. Krall, On complex orthogonality of Legendre and Jacobi polynomials, *Mathematica (Cluj)*, **25** (1980), 59-65).
- Inequality for the Laguerre polynomials and local uniform convergence of series in them in the convergence domain ([19], [M1]).
- Inequalities for the associated functions (functions of second kind) of Laguerre and Hermite and clarification of the character of convergence of series in them ([27], [34], [48], [M1]).
- Automorphisms by means of operators of fractional calculus of Riemann-Liouville type (Erdélyi-Kober fractional integrals) of the spaces of holomorphic functions admitting representations by series in Hermite polynomials and application of these operators to the problem for representation of holomorphic functions by series in Laguerre polynomials, under most general assumptions on the parameters ([42], [43], [51], [M1]).
- Characterization of the spaces of holomorphic functions representable by series in the polynomials of Laguerre and Hermite and their associated functions, by means of the classical integral transforms of Laplace, Fourier, Hankel

and Meijer ([36], [37], [44], [49], [50], [51], [M1]).

- Convergence and Cesaro's summability of series in the polynomials of Jacobi, Laguerre and Hermite on the boundary of their domain of convergence ([11], [26], [38], [56], [85]). An analogy with the classical results on convergence and $(C, 1)$ -summability of the Fourier trigonometric series is pointed out.

- Analogues of the classical results of Ostrovski and Hadamard for overconvergence and analytical uncontinuation of power series, for series in the polynomials of Laguerre and Hermite ([18], [M1]). The result of Rusev on overconvergence was repeated by Mu Lehua, Singularity and overconvergence of general Laguerre series, *J. of Math. Res. and Exposition*, **13**, No 3 (1903), 359-364.

- Tauber type theorem for summation by means of the Laguerre polynomials ([33])

- Theorems of Fatou and Abel type for Laguerre and Hermite series ([18], [24], [M1]). An analogy is established with the well known results on the power series.

To these latter mentioned contributions, one can add also Rusev's modification of a Szegő's result for analytical uncontinuation of a power series whose coefficients achieve finite number of values (G. Szegő, Über Potenzreihen mit endlich vielen verschiedenen Koeffizienten, *Sitzber. preuss. Akad. Wiss.*, 1922, 88-91); for series in more general systems of orthogonal polynomials ([15]); as well as Rusev's results on holomorphic extension of measurable functions and of locally Hölder functions ([73], [87]).

All these results are included also in Rusev's second monograph [M2], which is an extended and enlarged edition of the first one [M1].

Let us note that *due to the above-mentioned Rusev's contributions (III), the theory of the classical orthogonal polynomials in the complex domain obtained its closed form and completeness, comparable with the same theory in the real domain.*

IV. Special functions and integral representations

As it is well-known, the classical orthogonal polynomials of Jacobi, Laguerre and Hermite and their associated functions are, shortly speaking, "discrete" analogues of the functions of Gauss, Kummer, Tricomi and Weber-Hermite. This was the main motivation for Rusev's interest to the classical special functions, called also higher transcendental functions in the fundamental work of H. Bateman and A. Erdélyi, *Higher Transcendental Functions*, Vols. I,II,III; N. York, 1953.

The asymptotics of the classical orthogonal polynomials has been a subject of long-standing studies and efforts by many eminent experts. In particular,

for the polynomials of Jacobi, Laguerre and Hermite, the names of Darboux, Perron and Szegő should be mentioned for establishing of asymptotic formulas. With an exception for the Jacobi's case, the asymptotics of the associated functions of Laguerre and Hermite has almost not been studied. This fact and also the "doubt" in the proof of Th. 8.22.7, p. 219 in Szegő's monograph, became a motivation for Prof. Rusev to direct his attention to the asymptotics of the classical orthogonal polynomials and their associated functions, in complex domain.

Another stimulation was his desire to avoid using the known asymptotical formula of Perron for the Laguerre functions, obtained as a corollary of the results for confluent hypergeometric functions (O. Perron, Ueber das Verhalten einer ausgearteten hypergeometrischen Reihe bei unbegrenzten Vachstum eines Parameters, *J. Reine Angew. Math.*, **151** (1921), 63-78), as well as these for the Tricomi functions in the Bateman and Erdélyi's book, given as corollaries of the asymptotic expansions with respect to parameter of the solutions of differential equations. Thus, the papers [64], [69], [77], [79] on the asymptotics of the Weber-Hermite and Tricomi functions appeared, in which the main contributions can be summarized as follows:

- Asymptotical expansion of the Weber-Hermite functions, as a corollary of their integral representation that generalizes Cherry's formula and gives precise asymptotic formulas for the Hermite polynomials ([69]). The mentioned formula is given in Bateman and Erdélyi's Vol.II, with a reference to the publication of T.M. Cherry (Expansions in terms of parabolic cylinder functions, *Proc. Edinburgh Math. Soc.* (2), **8** (1949), 50-65). However, in this paper there is missing a proof of the validity of the integral representation (formula (1.7) in [69]) on which the used method of the saddle point is based.

- Precise asymptotic formulas of Perron's type for the polynomials of Laguerre, based on these for the Hermite polynomials ([77])

- Asymptotic formula for the functions related to the Tricomi functions, on the base of their integral representation ([79])

The polynomials of Laguerre and Hermite and their associated functions are numerable systems of functions of Kummer, Tricomi and Weber-Hermite. This was the natural motivation for Rusev to introduce and study integral representations of holomorphic functions involving these special functions. His main goal was to establish "continuous" analogues of the results for representation of holomorphic functions by series in polynomials and associate functions of Laguerre and Hermite. His publications, as [46], [47], [55], [60], [84] contain the following more essential achievements:

- "Uniform" asymptotics of the Kummer and Tricomi functions ([46],

[84])

- Inequalities for the Kummer and Tricomi functions ([46], [84])
- Analogues of the Abel lemma and Cauchy-Hadamard formula for improper integrals involving the Kummer and Tricomi functions ([84])
- Characterization of the spaces of holomorphic functions, admitting integral representations by means of the Kummer, Tricomi and Weber-Hermite functions, as images under the classical integral transforms of Hankel, Meijer and Fourier of suitable classes of entire functions ([47], [84])
- Uniqueness of the integral representations by means of the Kummer, Tricomi and Weber-Hermite functions ([84])

The interest of Prof. Rusev to the completeness of numerable systems of special functions in different functional spaces arose after his acquaintance with the publications of Yu.A. Kaz'min (On subsequences of Hermite and Laguerre polynomials, *Vestnik Moscow Univ., I - Math. & Mech.* **2** (1960), 6-9) and A.F. Leont'ev (On completeness of some systems of polynomials in domains of the complex plane, *Dokl. Acad. USSR* **126** (1959), 939-942) and also due to the influence of the results on completeness of systems of Laguerre and Hermite functions in spaces of functions with summable square from the book of Szegő (Th. 5.7.1). The results in this direction are contained in many papers, as: [57]-[59], [62]-[63], [65], [68], [70], [74], [78], [82]. The most representative of them are:

- Theorems of Müntz type for completeness in L^2 -spaces of numerable systems of Tricomi functions ([58])
- Completeness of numerable systems related to the Kummer and Weber-Hermite functions in L^2 -spaces ([74])
- Completeness of numerable systems of functions of Kummer and Weber-Hermite in spaces of holomorphic functions ([65])

To the above, one can add also Rusev' results on completeness in spaces of holomorphic functions of numerable systems of the Gauss hypergeometric functions ([68]), Tricomi functions ([78]), and associated Hermite functions ([82]).

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