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Abstracts

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THE INTUITIONISTIC DOUBLE NEGATION IS A MODALITY

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Some tautologies and rules of the intuitionistic double negation, e.g. $(\neg\neg A \wedge \neg\neg B) \Rightarrow \neg\neg(A \wedge B)$, $\frac{A \Rightarrow B}{\neg\neg A \Rightarrow \neg\neg B}$ show that we can consider it as a new unary operator satisfying the axioms of the "necessity" with some additional properties, e.g. $A \Rightarrow \neg\neg A$ and $\neg\neg(\neg\neg A) \Rightarrow \neg\neg A$. That is why it is interesting to obtain a full axiomatization of this new intuitionistic modality.

Let we denote $\neg\neg$ by $\Box$; IPL is the intuitionistic propositional logic; $I^{+}(\Box)$ is the positive (without negation) fragment of IPL extended by $\Box$ together with axioms $\Box(A \vee (A \Rightarrow B))$, $\Box(A \Rightarrow B) \Rightarrow (\Box A \Rightarrow \Box B)$, $A \Rightarrow \Box A$, $\Box(\Box A \Rightarrow A)$. If A is a formula of IPL without groups of odd number of adjacent symbols $\neg$ we shall denote by $A\frac{\neg\neg}{\Box}$ the "pairing" of the negations in A (i.e. the replacement of every double negation by $\Box$). Then the answer of our question is given by the following

Theorem. If A contains only even negations then it is provable in IPL iff $A\frac{\neg\neg}{\Box}$ is provable in $I^{+}(\Box)$.

$I^{+}(\Box)$ possesses an adequate Kripke semantics. Some superintuitionistic logics may be treated in the same style. E.g. the axiomatics of the double negation of Dummett's KC (with additional axiom $\neg A \vee \neg\neg A$) is $I^{+}(\Box)$ plus $\Box(A \vee B) \Rightarrow (\Box A \vee \Box B)$.