Gottfried Wilhelm Leibniz. *Sämtliche Schriften und Briefe.* Hrsg. von der Berlin-Brandenburgischen Akademie der Wissenschaften und der Akademie der Wissenschaften in Göttingen. Reihe 6. *Philosophische Schriften.* Hrsg. von der Leibniz-Forschungsstelle der Universität Münster. Bd. 4. 1677–Juni 1690. Akademie Verlag, Berlin 1999, XCI + 2950 pp.; *Verzeichnisse*, 500 pp. 1400 DM.

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I

The next volume of Leibniz's philosophical heritage appeared after a 20-year waiting. In fact, it contains three books of 1000 pages each plus 500 pages indexes. A preliminary edition, the so called *Vorausedition*, had been printed *ad usum collegialem* in 1982–1991.

From a technical point of view, the edition is a model of an academic approach to the scientific heritage of any classic: all variants and hesitations have been shown, with up to five modifications published, one over the other! This fact raises the question about a future re-edition of the older volumes in the same style. In the meanwhile, let us hope to see the next logical papers of Leibniz published soon. And, may we dream to read them in the Internet?

The volume covers the most productive period of Leibniz's logical researches: 1677–June 1690. Part A consists of Leibniz's papers on *scientia generalis*, *characteristica*, and *calculus universalis*. The first half of Part B includes Leibniz's *excerpta et notae marginales* to *scientia generalis*, etc., and the second one includes papers and marginalia on *metaphysica*. Part C is devoted to *philosophia naturalis*, *theologia*, *moralia*, and *scientia juris naturalis*. That is why I will limit my review with Part A which is the most interesting from the logical point of view. All Leibniz's considerations may be divided into three directions which I will present from the standpoint of today. Many of the papers had been published before (some of them partially) in *Die philosophischen Schriften von Gottfried Wilhelm Leibniz* (ed. C. Gerhardt, 7 volumes, Berlin, 1875–1890) or in *Opuscules et fragments inédits de Leibniz, éxtraits des manuscrits de la Bibliothèque royale de Hanovre par L. Couturat* (Alcan, Paris 1903; *reprint* Olms, Hildesheim 1961), and had been translated in *G. W. Leibniz, Logical Papers* (ed. G. Peterson, Clarendon Press, Oxford 1966). I will refer to these old editions marking them G, C, and P respectively.

II

The first direction I will name 'Logical Dreams'. It includes the well known sketches of the program of both philosophers which might take the quills in their hands and instead of disputing on a certain scientific question, could sit down at their abaci inviting each other in a friendly manner: *Calculemus*! According to this program, human reasoning had to be transformed into a kind of arithmetic using *characteristic numbers* instead of notions. In such a way, the logical truth of a proposition would turn into an arithmetical truth of a calculation. E.g., if the number of 'animal' were 2 and that of 'rational' were 3 (Leibniz's loved example) then the number of 'man' being by definition a 'rational animal' would be obtained by the multiplication $3 \cdot 2$. Then the answer of the question 'Is every man a rational being?' could be reduced to the fact that 6 is divisible by 3. Some of the most representative texts concerning this program are the following: *La vraie méthode* (C. 153-157); *De numeris characteristis ad linguam universalem constituendam* (G. 184-189); *De* *republica literaria* (G. 66-73); *Ad constitutionem scientiae generalis* (G. 127-156); *Recommandation pour instituer la science générale* (G. 157-173); *Elementa rationis* (C. 335-348); *De lingua philosophica* (C. 288-290); *De arte characteristica ad perficiendas scientias ratione ninentes* (G. 198-203); *Discours touchant la Méthode de la Certitude et de l'Art d'Inventer pour finir les disputes, et pour faire en peu de temps des grands progrés* (G. 174-183). The concrete realizations arithmetization of the reasoning will be analyzed bellow.

I have to add two notes here. First, all Leibniz logic operates with atoms of the kind 'A est B' and so is limited to syllogistic logic or, in the best case, to the monadic predicate calculus. Both of them are decidable. On contrary, we know now that the full predicate calculus containing dyadic relations is not decidable. Thus, even having relations reduced to properties using appropriate philosophical speculations (that Leibniz had insisted to be possible) no procedure of checking reasoning completely could be algorithmic. Second, no translation of arbitrary reasoning into arithmetic could be *ipso facto* a panacea for its correctness because of the undecidability of the arithmetic itself; the business of the computists is not more algorithmic than the business of the philosophers. Hence, the plan to load a few selected men to decompose (in a *quinquennium*!) all human knowledge to syllogistic relations connecting a list of primary notions and so to liquidate any disputation had been radically unrealizable. Nevertheless, the fragment of the arithmetic that Leibniz had had in mind to be used is decidable because it is the fragment containing only multiplication (without addition). At the end of the review we will see how certain translations of the monadic predicate calculus (with equality) into arithmetic of the integers with multiplication would be working.

III

The second direction of Leibniz's investigations I will entitle 'Algebraic Papers'. I include the abstract representation of the logical laws which (according to the initial language) leads to well known (now!) algebraic structures as partially ordered sets, semi-lattices, lattices, Boolean algebras, etc.[1] Here are some of the appropriate papers: *Specimen calculi universalis* (G. 218-221; C. 239-243; P. 33-39); *Ad specimen calculi universalis addenda* (G. 221-228; C. 249; P. 40-46); *De formis syllogismorum Mathematice definiendis* (C. 410-416; P. 105-117); *Generales Inquisitiones de Analysi Notionum et Veritatum* (C. 366-399; P. 47-87); *Specimina calculi rationalis* (C. 259-264); *Non inelegans specimen demonstrandi in abstractis* (G. 228-235; P. 122-143). I have given a full picture of all Leibniz's approaches, showing their exact place in a uniform algebraic scheme.

Let us begin with the traditional syllogistic. As we know from the result of J. Słupecki, the syllogisms not admitting empty terms are axiomatized by the following four axioms of J. Lukasiewicz:

$s\mathcal{A}s$, $s\mathcal{I}s$, $(m\mathcal{A}p)\&(s\mathcal{A}m) \Rightarrow (s\mathcal{A}p)$ (*Barbara*), and $(m\mathcal{A}p)\&(m\mathcal{I}s) \Rightarrow (s\mathcal{I}p)$ (*Datisi*).[2]

Consider $\mathcal{A}$ and $\mathcal{I}$ as binary relations $\leq$ and θ in a class of arbitrary objects, and take $\&$, $\Rightarrow$ in their informal sense. Then $\leq$ is *reflexive* and *transitive*, i.e., it is a *quasi-ordering relation*. In order to obtain an algebra, $\leq$ has to be *antisymmetric*, i.e., an additional axiom $(s\mathcal{A}p)\&(p\mathcal{A}s) \Rightarrow (s = p)$ is needed; strictly

[1] See my 'Various syllogistics from an algebraic point of view', presented at the 2nd Panhellenic Logic Symposium (Delphi, 13-17 June 1999: Proceedings, 197-200). The abstract of this paper is accessible also from my Web-sites: *http://www.math.bas.bg/~vlsot* and *http://www.geocities.com/sotirov.geo*.

[2] Jan Lukasiewicz, *Aristotle's syllogistic from the standpoint of modern formal logic*, 2nd ed. (Clarendon Press, Oxford, 1957).

speaking, $=$ here belongs to the extended language of syllogistic. In such a way, the set of terms turns into a *partially ordered structure*.[3] The second relation θ is *reflexive* and *monotonic* with respect to $\leq$. So the traditional syllogistic may be shortly characterized by the pair $(\leq, \theta)$. One cannot find this assumption in Leibniz's manuscripts, but it is notable that he was the first to introduce the three non-traditional syllogistic laws (e.g., in *Elementa Calculi*, C. 52).

In most of his manuscripts, Leibniz introduces into the syllogistic a *composition* of terms; let us denote it by $\circ$. Independently of what it is 'in reality', a conjunction or a disjunction, as a minimum it should be *idempotent*, *commutative*, and *associative*. To be formulated these three properties require only *term equality* to appear explicitly. Such a structure G. Birkhoff names *semi-lattice*. An ordering relation just we need in syllogistic may be defined in it by $x \leq y \leftrightarrow x = x \circ y$. Then $\circ$ will be an *inf* with respect to $\leq$ so introduced. The possibility to build up syllogistic in this way was noted many times by Leibniz. In fact, he lists idempotency and commutativity of the composition, defines $\mathcal{A}$ as it was shown (e.g., in *Primaria Calculi Logici fundamenta*, C. 235-236), and only associativity was not mentioned explicitly. However, it is described in examples (e. g., in *Elementa ad calculum condendum*, C. 258).

Of course, a second ordering relation may be defined by $x \leq y \leftrightarrow y = x \circ y$; then $\circ$ will be a *sup* with respect to it. If we continue in this direction, it would be natural to have *two* dual operations simultaneously, $\circ$ and $\bullet$, one of them playing the rôle of a term *conjunction*, and the other being a term *disjunction*. This means that one of them has to be an *inf* and the other to be a *sup* with respect to a *unique* ordering relation $\leq$. The coincidence of corresponding relations is ensured by the laws of *absorption* connecting $\circ$ and $\bullet$: $s \circ (s \bullet p) = s$ and $s \bullet (s \circ p) = s$. The resulting system $<=, \circ, \bullet>$ coincides with the algebraic structure named now *lattice*, a structure introduced and studied for the first time by C. S. Peirce. [4] Now, let us suppose an *extreme* element e exists in a structure $<=, \circ>$ with the sole property $x \circ e = e$ for any x. The last axiom gives $e \leq x$. So e is the *least* element of the structure. The second syllogistic relation θ can be defined by $x\theta y \leftrightarrow x \circ y \neq e$. In such a case, if only elements different from e are admitted, $x\theta x$ is exactly the requirement $x \neq e$. Leibniz uses literally the same definitions of $\leq$ and θ calling our $\neq e$ 'est Ens' (*Generales Inquisitiones*, Principles 164 and 169). Obviously, if term *negation* appears in the semi-lattice with some minimal suitable properties then the extreme element becomes definable by Leibniz's 'non-Ens': $e = b \circ (-b)$ for b a fixed element (e.g., *Specimina calculi universalis*, C. 259). The last basis may be characterized by the triple $<=, \circ, e(\circ, -)>$ because e is represented as a function of $\circ$ and $-$.

When the structure contains term negation in its full volume, the relation θ becomes definable by $\leq$: $x\theta y \leftrightarrow x \not\leq -y$ (also, $\leq$ by θ: $x \leq y \leftrightarrow \overline{x\theta - y}$). If empty and universal terms are prohibited then the full syllogistic of negative terms can be produced from the system $<\leq, ->$ using the axioms:

$$--x = x,\ x \leq y \rightarrow -y \leq -x,\ x \leq y \rightarrow x \not\leq -y$$

or, the last replaced with $x \not\leq -x$.[5] Leibniz lists the first three laws of negation, e.g., in *Generales Inquisitiones*, Principles 96, 93, 91 = 100; another exposition including the fourth formula has been given in *Fundamenta Calculi Logici* (C. 422).

The next combination $<=, \circ, ->$ includes term composition and term negation (besides equality) with appropriate axioms so to obtain a Boolean algebra of terms.

[3] G. Birkhoff, *Lattice theory*, 3rd ed. (Amer. Journ. Math., Providence, 1967).

[4] C. S. Peirce, 'On the algebra of logic', *Amer. Journ. Math.*, **3** (1880), 15-57; also in *Writings*, vol. 4 (Indianapolis), Indiana University Press, Indianapolis, 1989, 163-209).

[5] See A. Wedberg, 'The Aristotelian theory of classes', *Ajatus*, **15** (1948), 299-314; J. Shepherdson, 'On the interpretation of Aristotelian syllogistic', *Journ. Symbolic Logic*, **21** (1956), 137-147.

Then both relations $x \leq y$ and $x\theta y$ can be defined by $x \circ y = x$ and $x \circ (-y) \neq x$ respectively. These definitions are used by Leibniz, e.g., in *Primaria Calculi Logici fundamenta* (C. 236).

Finally, the *relation* $=$ can be replaced by a *property* '$= 0$', where 0 is the empty term. Then four syllogistic relations corresponding to the traditional categorical propositions can be defined on the base of $< \circ, -, = 0 >$. Using Leibniz's notation for the composition, they can be written in the following symmetric manner: $s\mathcal{A}p$ is $s(-p) = 0$, $s\mathcal{O}p$ is $s(-p) \neq 0$, $s\mathcal{E}p$ is $sp = 0$, and $s\mathcal{I}p$ is $sp \neq 0$. Leibniz used the last representations in *Generales Inquisitiones* (Principle 151). It is curious that precisely the same four equations have been proposed 160 years later by George Boole. [6]

IV

The third direction I call 'Arithmetical Models'; it includes some concrete translations of logic (or, more exactly, of syllogistic) into arithmetic. All Leibniz's 'arithmetical' papers (*Elementa Characterisicae universalis*, *Elementa calculi*, *Calculi universalis Elementa*, etc.) were produced in April 1679 (C. 42-92, 245-247; P/. 17-33). His first attempts to use simply divisibility of integers were unsuccessful. He explored 's is divisible by p' for 'Every s is a p' (like in the example above) but met difficulties using 's multiplied by x is divisible by p' for 'Some s is a p': some of the syllogisms cease being true in this interpretation. Only the more complicated method using pairs of co-prime numbers (*Calculus consequentiarum*; *Modus Examinandi Consequentias per Numeros*) was successful as Słupecki proved. [7] In this interpretation, if the pair of co-prime numbers (s_1, s_2) is assigned to s and (p_1, p_2) to p, then 'Every s is a p' is true when s_i is divisible by p_i (for $i = 1, 2$), and 'Some s is a p' is true when s_1 and p_2 as well as s_2 and p_1 are co-prime. This model obviously was more sophisticated than the one initially planed, but its larger shortcoming is that it could not envelop syllogistic of term negation or that of term conjunction. A correct reconstruction of Leibniz's primary plan which can be extended on the whole Boolean syllogistic is possible. [8]

In fact, Leibniz's ideas may obtain *two* dual arithmetical realizations. The first one I call *Scholastic*, following Leibniz himself. Terms are evaluated by integers greater than 1. If the same letters are used for the term values, $s\mathcal{A}p$ is replaced with 's is a divisor of p', and $s\mathcal{I}p$ with 'g.c.d.$(s, p) > 1$'. If empty terms are admitted, they are evaluated by 1. For the second interpretation named *Leibnizian* (being partially used by him) an arbitrary integer $u > 1$ is introduced and terms are evaluated by its proper divisors (i. e., less than u); $s\mathcal{A}p$ is replaced with 's is divisible by p', and $s\mathcal{I}p$ with 'l.c.m.$(s, p) < u$'. If empty terms are admitted, they are evaluated by u. When negation appears, the 'universe number' u *without multiple factors* must be introduced in both arithmetical semantics; terms are evaluated by divisors of u different from 1 and u (in the case of excluding empty and universal terms), and 'non-s is interpreted as $\frac{u}{s}$. If a term composition appears (in such a case, the full Boolean syllogistic is obtained), sp is modeled by g.c.d.(s, p) in the Scholastic arithmetical interpretation, and by l.c.m.(s, p) in the Leibnizian one.

For the pure monadic predicate calculus, the fact is used that any monadic proposition is equivalent to a monadic proposition with the same predicate symbols and *one* variable only. Let this sole variable be x. In addition, we may suppose

[6] G. Boole, *The mathematical analysis of logic, being an essay towards a calculus of deductive reasoning* (Macmillan, Cambridge 1847).

[7] Lukasiewicz (footnote), § 34.

[8] See my 'Aritmetizations of syllogistic à la Leibniz', *Journ. applied non-classical logics*, **9** (1999), n. 2–3, pp. 387–405.

it is not bound 'twice' anywhere. So, no subformula is a Boolean combination of two formulae $A(x)$ and B, one of them containing a free x and the other containing x bound; in $(Qx)A(x)$, where Q is a quantifier, a free x does occur in A. Let an arbitrary integer $u > 1$ *without multiple factors* be taken, and let its divisor d_i be associated with the predicate $P_i(x)$. Further, following the construction of the formula, a *divisor* of u will be associated with any subformula containing a free x, and a *statement* about divisors will be associated with the subformula when it does not contain a free x. If a and b are associated with $A(x)$ and $B(x)$, then g.c.d. (a, b) is associated with $A(x)\&B(x)$, $\frac{u}{a}$ with $\neg A(x)$, and so on for other Boolean connectives. The statements $a = u$ and $a > 1$ are associated with $(\forall x)A(x)$ and $(\exists x)A(x)$ respectively. If statements p and q are associated with subformulae A and B, then 'p and q' and 'not p' will be associated with $A\&B$ and $\neg A$, respectively. Finally, a certain statement comparing divisors of u with u and 1 will model the initial monadic proposition. This statement is an arithmetical truth for an arbitrary integer u *iff* the initial closed formula is a predicate tautology.

Finally, for the monadic predicate calculus *with equality*, any individual variable x_i must be evaluated by a *prime* divisor of u, say, by $d(x_i)$; the interpretation of any predicate P_i may be arbitrary divisor $d(P_i)$ (possibly 1 or u). Denote by $AR[F]$ the arithmetical model of the formula F. Then for atomic formulae, $AR[P_i(x_j)]$ is '$d(x_j)$ divides $d(P_i)$', $AR[x_i = x_j]$ is '$d(x_i) = d(x_j)$'; for a subformula G, $AR[(\forall x)G]$ is 'for any prime divisor d, $AR^x_d[G]$' where AR^x_d differs from AR in attaching d to x.[9]

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[9] See my 'Monadic predicate calculus with equality arithmetized à la Leibniz', *C. r. Acad. bulgare Sci.*, **54** (2001), n. 1, 9-10.