Abstracts

Part I: Plenary Lectures

Bernstein-Durrmeyer operators with arbitrary weight functions

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Let ρ be a non-negative bounded regular Borel measure on $[0,1]$ that satisfies the following positivity assumption: $\int_0^1 p\,d\rho > 0$ for all polynomials p such that $p \geq 0$ on $[0,1]$, $p \not\equiv 0$. The Bernstein basis polynomials of degree $n \in \mathbb{N}$ are defined for $0 \leq k \leq n$ by

$$p_{n,k}(x) := \binom{n}{k} x^k (1-x)^{n-k}, \qquad x \in [0,1].$$

We introduce the Bernstein-Durrmeyer operator with weight ρ

$$\mathbf{M}_{n,\rho} f := \sum_{k=0}^{n} \frac{\int_0^1 f\, p_{n,k}\, d\rho}{\int_0^1 p_{n,k}\, d\rho}\, p_{n,k}$$

for $f \in L^q_\rho[0,1]$, $1 \leq q < \infty$, or $f \in C[0,1]$. The operator $\mathbf{M}_{n,\rho}$ is linear and positive, and it reproduces constant functions. It generalizes the well-known Bernstein-Durrmeyer operators with Jacobi weights. The motivation for this generalization comes from learning theory.

We investigate the operator $\mathbf{M}_{n,\rho}$ and its multivariate version on the d-dimensional simplex. In particular, we make first steps in understanding convergence of the operator.

Joint work with Kurt Jetter (University of Hohenheim).

The reduced basis method and greedy algorithms

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The Reduced Basis Method is a technique for simultaneously solving a large family of parametric PDEs. We shall show how this method can be viewed as approximating the elements of a compact set in a Hilbert space by a suitable n-dimensional space. We shall introduce a greedy procedure for finding the finite dimensional space and analyze the convergence of this greedy algorithm.

Zeros of polynomials

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We shall report some recent results on zeros of polynomials. We shall discuss topics concerning polynomials with real zeros, the so-called hyperbolic polynomials, and some applications related to the Riemann xi-function. Various results on zeros of classical orthogonal polynomials will be considered too. Finally, we shall formulate some interesting open problems on these topics.

On the Gasca-Maeztu conjecture in $\mathbb{R}^2$ and $\mathbb{R}^3$

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The talk is focused on recent progress in the Gasca-Maeztu conjecture.

Sub-exponentially localized kernels and frames with applications to numerical analysis

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We construct *sup-exponentially localized* kernels and frames (needlets) in the context of classical orthogonal expansions, namely, expansions in Jacobi polynomials, spherical harmonics, orthogonal polynomials on the ball and simplex, and Hermite and Laguerre functions. Further on, such rapidly decaying needlets are developed in the context of *tensor product* orthogonal polynomials. These tools are employed to the development of the theory of weighted Triebel-Lizorkin and Besov spaces on *product domains*, whose components are among the d_1-dimensional cube $[-1,1]^{d_1}$, the unit ball in $\mathbb{R}^{d_2}$, the unit sphere in $\mathbb{R}^{d_3}$, the d_4-dimensional simplex, etc.

A method for fast evaluation of spherical polynomials at many scattered points on the unite 2-dimensional sphere is presented as an application of the sup-exponentially localized needlets. It is fast, local, memory efficient, numerically stable and with guaranteed (prescribed) accuracy. The method can be also applied for approximation on the sphere, verification of spherical polynomials and for fast generation of surfaces in computer-aided geometric design.

Existence of a solution to the 3D primitive equations "in the large"

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For the system of primitive equations describing large-scale ocean dynamics, existence and uniqueness of a solution "in the large" is proved. This system is obtained from 3D Navier-Stokes equations with taking into account smallness of depth with respect to horizontal size. So the equation for the velocity in vertical direction is modified and two more nonlinear equation for salinity and temperature are added. Formulation of the main result is the following: for arbitrary 3D domain being a cylinder with a piece-wise smooth boundary, for any viscocity coefficient, any sufficiently smooth initial condition and arbitrary time interval a solution to this problem exists and is unique and the L_2 norm of the solution gradient is continuous in time. So the result is just the same as was supposed by Leray for the Navier-Stokes problem.

Greedy algorithms and the best m-term approximation

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Let H be a real separable Hilbert space. We say a set $\mathcal{D} \subset H$ is a dictionary if each $g \in \mathcal{D}$ has norm one ($\|g\| = 1$) and $\overline{\text{span}\mathcal{D}} = H$. One of the main problem of Nonlinear Approximation is to construct for arbitrary $f \in H$ an m-term approximation

$$f \to \sum_{k=1}^{m} c_k(f) g_k(f), \quad c_k(f) \in \mathbb{R},\ g_k(f) \in \mathcal{D}.$$

Greedy algorithms offer an effective procedure for computing such approximations. In the talk we discuss the two probably most popular kind of Greedy Algorithms: Pure Greedy Algorithm (PGA) and Orthogonal Greedy Algorithm (OGA), and compare their performance (the rate of convergence) with the best m-term approximation with regard to the dictionary $\mathcal{D}$:

$$\sigma_m(f, \mathcal{D}) := \inf_{c_k \in \mathbb{R},\ g_k \in \mathcal{D},\ 1 \le k \le m} \|f - \sum_{k=1}^{m} c_k g_k\|.$$

We pay a special attention to recent results on the efficiency of Greedy Algorithms with regard to dictionaries with small coherence $\mu(\mathcal{D})$

$$\mu(\mathcal{D}) := \sup_{\phi, \psi \in \mathcal{D},\ \phi \ne \psi} |\langle \phi, \psi \rangle|,$$

and discuss their applications to Compressed Sensing.

Derivative estimates for meromorphic functions and functions of exponential type

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We shall present some inequalities for meromorphic functions with restricted poles and for functions of exponential type in a half-plane. Their relationship to certain Bernstein type inequalities for rational functions will be discussed. We also intend to mention a few problems which we would have liked to solve but have not been able to.

Landau-Kolmogorov inequality revisited

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The main guideline in studying the Landau-Kolmogorov inequality in the max-norm on a finite interval is Karlin's conjecture: among all the functions f with $\|f\| = 1$ and $\|f^{(n)}\| = \sigma$, the maximal value $\|f^{(k)}\|$ of the norm of intermediate derivative is attained by an appropriate Zolotarev polynomial or spline. So far, this conjecture has been proved for all $\sigma > 0$ for $n \leq 4$, and for particular $\sigma = \sigma_n$ for all n. Using a new approach, we prove Karlin's conjecture in several further subcases. First of all, we close the "polynomial" case ($\sigma \leq \sigma_n$, all n and k). Secondly, in the spline case ($\sigma > \sigma_n$), we advanced up to the second intermediate derivative ($k = 1, 2$, all n). And, finally, we obtained nearly complete solution (all σ, almost all k) for (not very) small $n \leq 20$.

Extending and modifying classical operators to the weighted case

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Linear operators like Bernstein, Szász-Mirakyan, Kantorovich play an important role in approximation theory, both from theoretical and numerical point of view. Our purpose here is to enlarge the class of functions where these operators act. For this aim we introduce weights, both on finite and infinite intervals. These weights may also have singularities at fixed inner points. The corresponding function classes are such that they allow a high rate of unboundedness of the functions not only at the endpoints, but at inner points as well. Boundedness of the operators, convergence, error estimates, Voronovskaya type limit relations and saturation will also be considered.

Extremal problems in approximation theory and Lagrange principle

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Essential part of Borislav Boyanovs scientific biography was devoted to exact solution of extremal problems. In the report devoted to the memory of this prominent mathematician, extremal problems will be considered from the point of view of one general principle of the extremal theory, which we call the Lagrange principle. According to this principle, when one searches a necessary condition of an extremal problem with equality constraints in which smoothness is interlaced with convexity, it is sufficient to construct the Lagrange function of the problem and then to apply necessary conditions for a minimum of the Lagrange function as if the variables are independent. The equations that one will find, combined with the given equations, will serve to determine all the unknowns. This general method will be illustrated by a series of concrete problems of approximation, interpolat1ion, recovery, Kolmogorov-type inequalities etc.

Part II: Regular Talks

Old and new results on the Favard operator

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In 1944, J. Favard introduced the operator F_n given by

$$(F_n f)(x) = \frac{1}{\sqrt{\pi n}} \sum_{\nu=-\infty}^{\infty} f\left(\frac{\nu}{n}\right) \exp\left(-n\left(\frac{\nu}{n} - x\right)^2\right)$$

which is a discrete analogue of the Gauss–Weierstrass singular convolution integral. The sequence $(F_n f)$ converges to f for continuous functions defined on $\mathbb{R}$ which have polynomial growth at $\pm\infty$. We report some known properties such as saturation theorems in certain polynomial weight spaces and present Kantorovich and Durrmeyer variants.

In fact, we treat a slight generalization F_{n,σ_n} which was introduced and studied by Gawronski and Stadtmüller. While in previous investigations functions of polynomial or exponential growth were considered, here we deal with the larger class of functions satisfying a growth condition $f(u) = O\left(e^{Ku^2}\right)$ as $|u| \to \infty$.

We study the local rate of convergence for smooth functions. The main result is a complete asymptotic expansion for the sequence $(F_{n,\sigma_n} f)$ as n tends to infinity. Furthermore, we consider a truncated version of these operators which possesses the same asymptotic properties. All results were proved also for simultaneous approximation.

Some recent results are joint work with Prof. Paul L. Butzer.

On the convergence of trigonometric Fourier series

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For continuous functions of two variables modified modulus of continuity and modulus of variation are considered. In terms of such modulus a sufficient conditions of uniform convergence of double trigonometric Fourier series is given.

An inequality of Duffin–Schaeffer type

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Denote by H_m the m-th Hermite polynomial, and for $n \in \mathbb{N},\ n \geq 2$, let $a = a(n)$ be the rightmost zero of H_n. We prove that if f is an algebraic polynomial with real coefficients of degree not exceeding n, such that $|f| \leq |H_n|$ at the zeros of H_{n+1}, then for $k = 1, \ldots, n$,

$$|f^{(k)}(x+iy)| \leq |H_n^{(k)}(a+iy)| \quad \text{for every} \quad (x,y) \in [-a,a] \times \mathbb{R},$$

and the equality occurs only when $f = \pm H_n$. This is an analogue of the Duffin and Schaeffer refinement of Markov's inequality.

On Fournier-Gagliardo mixed norm spaces

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We study mixed norm spaces $\mathcal{V}(\mathbb{R}^n)$ that arise in connection with Sobolev spaces $W_1^1(\mathbb{R}^n)$. We prove embeddings of $\mathcal{V}(\mathbb{R}^n)$ into Lorentz type spaces defined in terms of iterative rearrangements. Basing on these results, we introduce a scale of mixed norm spaces $\mathcal{V}^p(\mathbb{R}^n)$. We prove that $\mathcal{V}(\mathbb{R}^n) \subset \mathcal{V}^p(\mathbb{R}^n)$ and we discuss some questions related to this embedding.

Lower bounds for eigenvalues by nonconforming FEM on convex domain

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In this work we analyze the approximations of second order eigenvalue problems (EVPs). The nonconforming piecewise linear finite element with integral degrees of freedom is used. We prove that the eigenvalues computed by means of this element on convex domain are smaller that the exact ones if the mesh size is small enough. Some numerical results are also given.

Locally monotone approximations of real functions on graphs

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The idea of using monotonicity as a concept of smoothness within the Approximation Theory originates in the works of Sendov and Popov, e. g. [4]. The properties of the modulus of nonmonotonicity introduced by them are studied by many of their students and collaborators, including in the work of Borislav Bojanov, [2]. In this work we present an extension of this theory to a multidimensional setting with applications to signal processing where locally monotone approximations appear naturally as an output of filters, e.g. for noise from signal separation. The LULU operators, well known in the multi-resolution analysis of sequences [3], are filters which are particularly intended to extract a signal of prescribed local monotonicity. The abstract mathematical problem is approximation of real functions defined on a connected graph by a set of locally monotone functions on the same domain. The LULU operators suitably extended to this general setting as in [1] have shape preserving properties important for the processing of signals of arbitrary dimension. In addition to that, we prove that they produce locally monotone approximations which are *nearly optimal* in the sense that the error of the approximation in any l_p norm, $p \in [1, \infty]$, is bounded by a constant multiple of the error of any other approximation by functions from the same set.

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Polynomials that deviate the least from zero in measure and in the uniform norm on compact sets of a given measure and other related problems

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We will discuss several interrelated extremal problems for trigonometric polynomials that deviate the least from zero in the following three senses:

1) in measure; more precisely, with respect to the functional
$$\mu(f_n) = \operatorname{mes}\{t \in [0, 2\pi] : \ |f_n(t)| \geq 1\};$$
2) in the uniform norm on compact sets of a given measure;
3) with respect to the integral functionals
$$\int_{-\pi}^{\pi} \varphi(|f_n(t)|)\, dt$$
over the class of functions φ nondecreasing and nonnegative on the half-line $[0, +\infty)$

We will discuss corresponding problems for algebraic polynomials on the unit circle with zeros on the circle.

Most of the results are obtained by the author jointly with A. S. Mendelev [1].

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On a class of local average sampling expansions in subspaces of $L_2(\mathbb{R})$

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Let τ be a sampling set on $\mathbb{R}$, ϕ is a function with certain regularity and a non-vanishing property in a neighborhood of the origin for its Fourier transform $\hat{\phi}$ and $\{\rangle f, \phi(. - \tau_n)\langle\}$ is a sequence of sampled data for some square-integrable function f. Then there exists a space V_ϕ^τ such that every function $f \in V_\phi^\tau$ is uniquely determined by and stably reconstructed from the above sample set. As the reconstruction formula involves evaluating the inverse of an infinite matrix we consider a partial reconstruction formula suitable for numerical implementation and provide an estimate to the corresponding error.

A reduced space interpolation method for the time derivative applied to transport in domains with heterogeneous geometric dimensions

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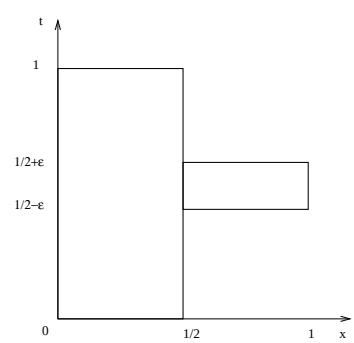
In this work, we consider transport phenomena by convection or by conduction taking place in 2D domains of heterogeneous dimensions. Such problems arise when for example two time scales are considered. In what follows, an heterogeneous domain is presented.



When $\epsilon = 0$, when look for a function u solution of:

$$(\mathbf{T}_0)\begin{cases} 1_{[0,\frac{1}{2}]}(x)\partial_t u(x,t) + \partial_x u(x,t) + u(x,t) = f(x,t);\ dans\,\Omega \\ \partial_t u(x,t) = 0; \quad dans \quad \frac{1}{2} < x < 1 \\ u(x,0) = u_0(x) \qquad 0 < x < \frac{1}{2}; \quad u(0,t) = 0; \end{cases}$$

By using a numerical method based on a projection method, error estimations are given for the space discretized problem, and the efficiency of the method is checked with an example.

Scattered data approximation in high dimensions

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Given a data set $X \subset \mathbb{R}^d$ of N data points x^i along with values $y^i \in \mathbb{R}^{d'}$, $i = 1, ..., N$, and viewing the y^i as values $y^i = f(x^i)$ of some unknown function f, we wish to return for any query point $x \in \mathbb{R}^d$ an approximation $\tilde{f}(x)$ to $f(x)$. Here the spatial dimension d should be thought of as large. We wish to emphasize that we do not seek a representation of $\tilde{f}$ in terms of a fixed set of trial functions but define $\tilde{f}$ through recovery schemes which, in the first place, are designed to be fast and to deal efficiently with large data sets. For this purpose we propose new methods based on what we call *sparse occupancy trees* and piecewise linear schemes based on simplicial subdivisions.

On local behavior of holomorphic functions along complex submanifolds of $\mathbf{C}^{\mathbf{N}}$

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In the talk I establish some general results on restrictions of holomorphic functions to complex submanifolds of $\mathbb{C}^N$. The subject pertains to the area of the, so-called, polynomial inequalities for analytic and plurisubharmonic functions that includes, in particular, Bernstein, Markov and Remez type inequalities.

Smooth convex resolution of unity on general partitions of multidimensional domains using generalized expo-rational B-splines, and application to multivariate approximation and Hermite interpolation on scattered point sets

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Consider a scattered point set in a multidimensional domain and a general partition of the domain in subdomains such that: (i) every subdomain corresponds to one, and only one, element of the scattered point set (henceforward called 'its point'); (ii) the subdomains are either disjoint or may overlap in such a way that each subdomain contains in its interior only 'its point' and no other element of the scattered point set (the boundary of the subdomain can contain other elements of the scattered point set); the subdomains are bounded and simply connected. Starting from two families of radial-basis or tensor-product generalized expo-rational B-splines (GERBS) one of which has Hermite interpolation property and the other one forms a smooth convex resolution of unity of specific type associated with the original subdomain partition, we design an explicit algebraic construction of a new family of basis functions which combines the properties of smooth convex resolution of unity (associated with the original subdomains) with Hermite interpolation in the elements of the original scattered point set. Hermite interpolation of multivariate multidimensional vector fields in the scattered point set is achieved by a linear combination of the new basis functions where the coefficient of each basis function is the (tensor-product) Taylor polynomial centered at 'its point' and including all partial derivatives up to the total order of Hermite interpolation, which order may vary at different elements of the scattered point set. Once the vector field has been Hermite-interpolated, it is very easy to switch from interpolatory to Bezier form of the presentation, by changing the monomial bases in each variable used in the tensor-product Taylor polynomial to Bernstein polynomial bases, where the Bernstein polynomials are scaled to the support of the original tensor-product GERBS families (with respective modification to the radial-basis case). The construction is readily parallelized. The assumptions on the partition subdomains are very general and include disjoint convex covers such as Voronoi tilings as well as overlapping star-shaped covers such as the star-1 neighbourhoods of the vertices in a triangulation (simplectification) in dimensions 2, 3 and higher. Replacing the local Taylor polynomials by polynomials of the same total degree which are optimal with respect to a local least-squares or K-functional criterion, in combination with the resolution of unity, provides high-quality data fitting and multivariate approximation. In the case of Hermite interpolation, the respective generalized Vandermonde matrix is always in Jordan normal form.

Some extremal problems for algebraic polynomials on the Euclidean sphere

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Let $\mathbb{R}^m$, $m \geq 2$, be the m-dimensional real Euclidean space; let $\mathbb{S}^{m-1}$ be its unit sphere. For a number h, $-1 < h < 1$, we define the spherical cap $\mathbb{C}(h) = \mathbb{C}(h, e_m) = \{x = (x_1, \ldots, x_m) \in \mathbb{S}^{m-1} : \ x_m \geq h\}$ centered at the "north pole" $e_m = (0, \ldots, 0, 1)$ of the sphere. We denote by χ_h the characteristic function of the cap $\mathbb{C}(h)$. Our interest is the best approximation $e_{n,m}(\chi_h) = \inf\{\|\chi_h - P_n\|_{L(\mathbb{S}^{m-1})} : P_n \in \mathcal{P}_{n,m}\}$ of the characteristic function χ_h of the cap $\mathbb{C}(h)$ in the space $L(\mathbb{S}^{m-1})$ by the set $\mathcal{P}_{n,m}$ of algebraic polynomials in m real variables with real coefficients of total degree at most n.

In particular, we will give the value of $e_{n,m}(\chi_h)$ for $m = 3$ and for all $h \in (-1, 1)$. For $h = \cos \frac{\pi j}{n+1}$, $1 \leq j \leq n$, the result was obtained by the author earlier [1]. The problem of finding the value $e_{n,m}(\chi_h)$ is connected, in particular, with the problem about the best constant in the Jackson–Nikol'skii inequality between the uniform and integral norms of an algebraic polynomial on the sphere. We use an approach of A. G. Babenko and Yu. V. Kryakin [2].

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Exact performance of (ρ, θ)-Hough transform for star chain images processing

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The well known (ρ, θ)-interpretation [1] of Hough transform (HT) [2] is a projection technique that is most often treated in image processing considering its facilities to localize long stretched objects in a given image [3]. The definition of *"exact HT"* has been introduced for the both given grids $(x_{size} \times y_{size})$ and $(\rho_{size} \times \theta_{size})$ of the input image and of the HT result image respectively, considering the (ρ, θ)-HT like a Radon transform [4]. A few iterative approaches have been also proposed to approximate the exact HT through a balance among the *inner-noise-of-performance*'s level and the respective software effectiveness - programming duration and/or processing speed [4]. Unfortunately, these approaches become less and less acceptable with increasing of the input image grid as in the case of astronomical images, e.g. when HT is applied for identification of flare objects in chain plate images [5]. In this work an analytic solution for the exact performance of (ρ, θ)-HT is proposed based on the results in [4]. Being not iterative the solution is enough simply programmable and is expected to be much more effective than the previous ones already mentioned. This research is partially sponsored by the National Astroinformatics project, Grant # DO-02-275/2008 of the National Science Fund at Bulgarian Ministry of Education and Science (`www.astroinformatics.eu`).

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Splines with constraints

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It is classically known that the interpolating cubic spline can be viewed as a solution of an optimal control problem. We build upon this observation by adding constraints to the problem such as convexity, monotonicity and restricted range, and show how to handle the constraints both theoretically and numerically by applying various versions of the Lagrange principle.

On estimating the rate of best trigonometric approximation in L_p by a modulus of smoothness

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We present a characterization of best trigonometric approximation in L_p, $1 \leq p \leq \infty$, by a modulus of smoothness, which is invariant under translation of the approximated function by a trigonometric polynomial of a given degree like best trigonometric approximation itself. This characterization is quite similar to the one given by the classical modulus of smoothness. The modulus possesses properties similar to those of the classical one.

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Asymptotic behavior of Carleman orthogonal polynomials*

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Let L be an analytic Jordan curve in the complex plane $\mathbb{C}$. Polynomials that are orthonormal with respect to area measure over the interior domain of L were first considered by Carleman, who established a strong asymptotic formula for the polynomials valid on certain open neighborhood of the closed exterior of L. Here we extend the validity of Carlemans asymptotic formula to a maximal open set, every boundary point of which is an accumulation point of the zeros of the polynomials.

* Joint work with Erwin Miña-Díaz, University of Mississippi

Optimal adaptive sampling recovery based on quasi-interpolant representations

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We propose a new approach to study optimal adaptive sampling algorithms for recovery of functions by sets of a finite capacity which is measured by their cardinality or pseudo-dimension. Let $W \subset L_q,\ 0 < q \le \infty$, be a class of functions on $[0,1]^d$. For B a subset in L_q, we define a sampling recovery method with the free choice of sample points and recovering functions from B as follows. For each $f \in W$ we choose n sample points. This choice defines n sampled values. Based on these sampled values, we choose a function from B for recovering f. The choice of n sample points and a recovering function from B for each $f \in W$ defines a sampling algorithm $S_n^B(f)$ for recovery by functions in B. An efficient sampling algorithm should be adaptive to f.

Given a family $\mathcal{B}$ of subsets in L_q, we consider optimal adaptive sampling algorithms for recovery of functions in W by B from $\mathcal{B}$ in terms of the quantity

$$R_n(W,\mathcal{B})_q := \inf_{B\in\mathcal{B}} \sup_{f\in W} \inf_{S_n^B} \|f - S_n^B(f)\|_q.$$

Denote $R_n(W,\mathcal{B})_q$ by $e_n(W)_q$ if $\mathcal{B}$ is the family of all subsets B of L_q such that the cardinality of B does not exceed 2^n, and by $r_n(W)_q$ if $\mathcal{B}$ is the family of all subsets

B in L_q of pseudo-dimension at most n. Let $0 < p, q, \theta \le \infty$ and $\alpha > d/p$. Then for the d-variable Besov class $B^\alpha_{p,\theta}$, we proved that

$$e_n(B^\alpha_{p,\theta})_q \asymp r_n(B^\alpha_{p,\theta})_q \asymp n^{-\alpha/d}. \tag{1}$$

In comparing with the asymptotic order of optimal non-adaptive sampling recovery $n^{-\alpha/d+(1/p-1/q)_+}$, the convergence rate (1) is better for $p < q$. To construct asymptotically optimal adaptive sampling algorithms we use a B-spline quasi-interpolant representation in Besov spaces associated with some equivalent discrete quasi-norm. The main results of this talk are published in [1] and [2].

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Subspace correction method for discontinuous Galerkin discretizations of linear elasticity equations

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In this talk we will present a preconditioning techniques for certain classes of discontinuous Galerkin (DG) methods, so-called interior penalty (IP) finite flement methods, for linear elasticity problems in primal (displacement) formulation. We will recall some of their stability and approximation properties and comment on their suitability as a discretization tool for problems with nearly incompressible materials. Next we propose a natural splitting of the DG space, which gives rise to uniform preconditioners. The presented approach was recently introduced by B. Ayuso and L. Zikatanov in the context of designing subspace correction methods for scalar elliptic equations and is extended here to linear elasticity, i.e., a class of vector field problems. Similar to the scalar case the solution of the linear algebraic system corresponding to the IP DG method is reduced to a solution of a problem arising from discretization by nonconforming Crouzeix-Raviart elements plus the solution of a well-conditioned problem on the complementary space.

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On interpolation in the unit disk based on both Radon projections and function values

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There are important problems in medicine, materials science, radiology, archeology, biology, geophysics, oceanography, for which the relevant data comes as line integrals along a finite set of segments and tomographic reconstruction is applied using such kind of information. Recently many mathematicians have investigated methods for solving various approximation problems using Radon projections type of data.

Here we consider interpolation of function in two variables on the unit disk by bivariate polynomials, based on its Radon projections and function values. We prove a necessary and sufficient condition for regularity of a scheme of chords and points of the unit circle. Results of some numerical experiments are presented too.
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Strong approximation of two-dimensional Walsh Fourier series

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We study the exponential uniform strong approximation of two-dimensional Walsh-Fourier series. In particular, it is proved that the two-dimensional Walsh-Fourier series of the continuous function f is uniformly strong summable to the function f exponentially in the power $1/2$. Moreover, it is proved that this result is best possible.

Bases in Banach spaces of smooth functions on Cantor-type sets

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We present explicitly for a Cantor-type set K a Schauder basis in the Banach space $C^p(K)$ of p times differentiable on K functions as well as in the Whitney space $\mathcal{E}^p(K)$. In the construction we use local Taylor expansions of functions.

Simultaneous approximation by Bernstein operators in Hölder norms I & II

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In these two talks we present a quantitative result on simultaneous approximation by classical Bernstein operators, considered as mapping the space $C^{m,\alpha}[0,1]$ into $C^{r,\beta}[0,1]$. Here $C^{m,\alpha}[0,1]$ denotes all m-times continuously differentiable functions on $[0,1]$ whose m-th derivative satisfies a Hölder-Lipschitz condition with exponent α. $C^{r,\beta}[0,1]$ is defined analogously. The main result is the following theorem.

Theorem. *Let $r, m \in N_0, 0 \leq \alpha, \beta \leq 1, r \leq m$, $r + \beta \leq m + \alpha$. Then for $f \in C^{m,\alpha}$ and $n > m + 1$ one has*

$$||B_n f - f||_{r,\beta} \leq c_r \cdot (n - r - 1)^{\max\left\{-1, \frac{r+\beta-m-\alpha}{2}\right\}} \cdot ||f||_{m,\alpha}.$$

Here c_r is a constant depending only on r, α, β.

m-term approximation in tensor products of Sobolev and Besov spaces

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Tensor products yield a quite easy approach towards functions in many variables and high-dimensional approximation. Here we shall be concerned with associated tensor product spaces of isotropic Sobolev and Besov spaces, which turn out to be closely related to function spaces of dominating mixed smoothness.

Within this framework, the problem of m-term approximation with respect to tensor product wavelet systems is considered. Our main interest lies in the asymptotic behaviour of the error of this method. To this end, several explicit constructions are investigated, including the (linear) approximation from hyperbolic crosses.

The obtained results extend and complement earlier ones by Temlyakov, Dinh Dung and Nitsche.

Strong converse results on Baskakov-Durrmeyer operators and their quasi-interpolants

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We consider the Durrmeyer modification of the Baskakov operators and the corresponding quasi-interpolants. We present a strong converse result of type A for the operators and one of type B for the quasi-interpolants in the terminology of Ditzian and Ivanov.

Near-best approximation by a de la Vallée Poussin-type interpolatory operator

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Practically, it is always an interesting problem to construct a discrete linear operator, which approximates for instance continuous functions in near-best order. It is a natural idea to use some kind of de la Vallée Poussin-type means for proving best, or near-best order of approximation. After the investigations of eg. R. Bojanic, O. Shisha, and G. Freud, in 1974 J. Szabados gave the discrete version of de la Vallée Poussin means in the trigonometric. The considerable generalization of this result, which was simultaneously an answer of a question of G. Freud and A. Sharma, was given by O. Kis and J. Szabados.

In 1999 H. N. Mhaskar and J. Prestin established a result on bounded quasi-interpolatory operators, which also based on de la Vallée Poussin-type means. Their discretizing method based on the quadrature formula, and not on some integral approximating sum, as in the previous papers. However it deals with rather general weighted spaces, (applying the results to generalized Jacobi and Freud weights) the interpolatory property had been lost. Generalized de la Vallée Poussin means in Jacobi-weighted L^p-spaces was first treated by Nguyen Xuan Ky, and in 2008 further results were given by G. Mastroianni and W. Themistoclakis.

In this talk we should like to give a de la Vallée Poussin-type interpolatory process in some Jacobi weighted spaces, and deal with the question of approximation of continuous functions. The utility of this method is in the simplicity of the nodes.

Numerical investigation of fluxon solutions in stackes of Josephson junctions

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The Sakai-Bodin-Pedersen model - a system of perturbed sine-Gordon equations - is used to study numerically the dynamics of Josephson phases in stacks of inductively coupled long Josephson Junctions. The boundary conditions correspond to a stack of overlap geometry. We investigate solutions with static and moving fluxons in each junction and find their regions of stability and existence with respect to the parameters. Different physical characteristics are calculated and interpreted. To solve numerically the above problems Finite Element Method and Finite difference method are used.

Boolean invertible matrices identified from two permutations, their corresponding unbalanced Haar matrices and an application to data encryption

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We construct all $m \times m$ boolean invertible matrices (called Z matrices) satisfying the following property: when we perform the Hadamard product operation on the set of rows of a Z matrix we produce either a member of the same set or the zero row. It turns out that all Z matrices come from column permutations of upper unitriangular boolean matrices and they are uniquely determined by a pair of permutations of the set $\{1, 2, \ldots, m\}$. As the Gram Schmidt orthonormalization process of the set of row vectors of Z matrices produces unbalanced Haar matrices, Haar matrices can be coded by a pair of permutations as well. We develop a new encryption scheme based on the identification property of Z matrices.

Functions approximated by any sequence of interpolating generalized polynomials

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Let $\{u_0, u_1, \ldots, u_n, \ldots\}$ be an inifinite complete Chebyshev system of $C[a, b]$. Let

$$X_n : (a \leq) x_0^{(n)} < x_1^{(n)} < \cdots < x_{k_n}^{(n)} (\leq b), n \in \mathbb{N}$$

be any prescribed system of nodes, where $\lim_{n\to\infty} k_n = \infty$ and let $p_{f,X_n}(x), n \in \mathbb{N}$ be the generalized polynomials in $\mathrm{Span}\{u_0, u_1, \ldots, u_{k_n}\}$ that interpolates the function f at the nodes of X_n. Then, we consider properties of functions $f \in C[a, b]$ which are uniformly approximated by any sequence $\{p_{f,X_n}(x)\}$ of interpolating generalized polynomials to f.

Universal construction of orthogonal double shift functions and applications

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A subset $\{\nu_1, \nu_2, \ldots, \nu_n\}$ of a vector space V, with the inner product $\langle \cdot, \cdot \rangle$ is called orthonormal if $\langle \nu_i, \nu_j \rangle = 0$ when $i \neq j$. That is, the vectors are mutually perpendicular. Moreover, they are all required to have length one, i.e., , $\langle \nu_i, \nu_i \rangle = 1$. An orthonormal set must be linearly independent, and so it is a vector space basis for the space it spans. Such a basis is called an orthonormal basis. In signal processing orthogonal basis functions have many applications. The simplest example is the standard basis e_1 for the Euclidean space $\mathbb{R}^n$. A rotation $R(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$ through the origin will send an orthonormal set to another orthonormal set. In fact, given any orthonormal basis, there is a rotation, or rotation combined with a flip, which will send the orthonormal basis to the standard basis. These are precisely the transformations which preserve the inner product, and are called orthogonal transformations. Example, Haar matrices, all orthogonal wavelet and multiwavelet transforms, as well as all other orthogonal matrices and transforms. These matrices can be represents by unitary matrix. The simplest and most popular way to represent a 2×2 unitary matrix is by a rotation $R(\phi)$. Factorization of a matrix $R(\phi)$ of two Jacobi reflectors matrices leads to *universal representation for orthogonal double shift transformations*: $R(\phi) = J(\alpha)J(\beta)$ with $b - a = \phi \mod 2\pi$. This $R(\phi)$ can be represented as a rank-2 modification to the identity matrix.

On the Schur-Szegő composition of polynomials

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The Schur-Szegő composition of the polynomials $P_j := \sum_{i=0}^{n} C_n^i b_{i,j} x^i$, $j = 1, 2$, is defined by the formula $P_1 * P_2 = \sum_{i=0}^{n} C_n^i b_{i,1} b_{i,2} x^i$. Any polynomial $(x+1)Q$, where Q is monic and $\deg Q = n-1$, is representable as Schur-Szegő composition of $n-1$ polynomials of the form $(x+1)^{n-1}(x+a_k)$. The mapping between the tuple of coefficients of the polynomial Q and the elementary symmetric polynomials of the quantities a_k can be viewed as an affine mapping in the space of monic degree $n-1$ polynomials. It preserves the set of polynomials the real parts of whose roots are all positive. We discuss this and other geometric properties of this mapping and of its analog for functions of the form $e^x Q$ with Q as above.

Infinite-dimensional generalization of Kolmogorov-Gelfand theory of widths and applications to Compressive Sensing

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Based on the notion of multivariate Chebyshev system (cf. `http://arxiv.org/pdf/0808.2213v1`)
we introduce a new concept of widths (called "harmonic widths") which is an infinite-dimensional generalization of the theory of Kolmogorov and Gelfand. This new approach allows to obtain non-trivial widths for subsets of $\mathbb{C}^{\infty}(D)$ for domains D in $\mathbb{R}^n$. In particular, we consider the widths of the set

$$K = \left\{ f \in \mathbb{C}^{\infty}(B) : \int_B |\Delta^p f(x)| \, dx \leq 1 \right\}$$

where $B \subset \mathbb{R}^n$ is the open unit ball; for $n = 1$ the widths of K were studied by Kolmogorov in his original paper of 1937. Similar sets are interesting for Image Processing applications. As in the case of the usual widths (cf. e.g. A. Cohen, W. Dahmen, and R. DeVore, Compressed sensing and best $k-$term approximation, available in
`http://www.math.sc.edu~devore/publications/CDDSensing\_6.pdf`),
we seek for an application of these new harmonic widths to Compressive Sensing by means of appropriate Wavelet Analysis. Research partially sponsored by Project Astroinformatics, DO-02-275, `www.astroinformatics.eu`.

Zero distribution of sequences of rational approximants

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Let $I := [-1, 1]$ and let the function f be continuous and real- valued on the interval I. Given a pair (n, m) of non-negative integers, set $r_{n,m}$ for the best real Chebyshev approximant to f on I with respect to $\mathcal{R}_{n,m}$ $(= p/q; \deg p \leq n; \deg q \leq m)$. As is known, the rational function $r_{n,m}$ is unique and characterized by the alternation condition for the error curve $f - r^*_{n,m}$; namely: writing $r_{n,m} = p_n/q_m$, where p_n and q_m have no common factors, let

$$d_{n,m} := \min\{n - \deg p_n;\ m - \deg q_m\},$$

then there exist $m+n+2-d_{n,m}$ points $x_k^{(n,m)}$,

$$-1 \le x_1^{(n,m)} < \cdots < x_{m+n+2-d_{n,m}}^{(n,m)} \le 1,$$

which satisfy

$$\lambda(-1)^k (f - r_{n,m})(x_k^{(n,m)}) = \|f - r_{n,m}\|_I,$$

where $\lambda = -1$ or $\lambda = 1$ is fixed.

For definiteness, let us consider a sequence $\{m_n\}$ with

$$m_n \le n, \quad m_n \le m_{n+1} \le m_n + 1.$$

Assume now that f has a singularity at the point $z_0 \in I$. Our interest is devoted to the behavior of the zeros of the sequence $\{r_{n,m_n}\}$ around the point z_0. E. B. Saff has announced the hypothesis that z_0 attracts zeros of $\{r_{n,m_n}\}$ as $n \to \infty$. In the present talk, we provide results about corroboration of this hypothesis for some functional classes, as well as more precise statements about the zeros distribution. The results are joint with H. P. Blatt.

On Markov, Bernstein and Chebyshev type inequalities for k-monotone polynomials

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In a recent joint paper with József Szabados we initiated the study of Markov type inequalities for the so called **k-monotone** polynomials, whose first k derivatives are nonnegative in the interval considered. For the first derivative of k-monotone polynomials the exact constant of Markov type inequality was found for uniform and L_1 norms. This exact constant was given in terms of largest zeros of certain Jacobi polynomials. Moreover it was shown that in general for derivatives of order j the sharp order of magnitude of Markov constants is $\left(\frac{n^2}{k}\right)^j$.

Subsequently we also established asymptotically sharp Bernstein type inequalities for derivatives of k-monotone polynomials, when their size is measured on $[-1,1]$ in weighted uniform or L_1 norms with weight $\varphi(1-x)$, where φ is such that $\varphi(x)/x$ is decreasing. It turns out that the order of magnitude of constants in Bernstein type inequalities for derivatives of order j is $\left(\varphi\left(\frac{k}{n^2}\right)\frac{n^2}{k}\right)^j$.

In addition we also give an exact solution to the Chebyshev type extremal problem of finding monic k-monotone polynomials of minimal uniform and L_1 norms. Just as in the case of Markov type inequalities for k-monotone polynomials these extremal polynomials are related to certain Jacobi polynomials.

Weak convergence of greedy algorithms in Banach spaces

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We give sufficient conditions for weak convergence of the X-greedy algorithm. We also consider some modified greedy algorithms. The results are joint with S. Dilworth, K. Shuman, V. Temlyakov and P. Wojtaszczyk.

Discontinuous Galerkin finite element method of elliptic problems

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In the last decade, especially after the fundamental paper of Arnold, Brezzi, Cockburn, and Marini, [1], the discontinuous Galerkin method (DG FEM) has emerged as a powerful technique for solving PDE's that features flexibility in meshing and polynomial spaces, easily controlled stability, local conservation properties, and possibility to glue mixed, standard Galerkin, nonconforming, and other finite element approximations. However, DG FEM uses excessive number of degrees of freedom. Hybridization is one possible way to substantially reduce the number of degrees of freedom.

The first hybridization of a finite element method was proposed in 1965 by Fraejis de Veubeke as an implementation of numerical methods for solving the equations of linear elasticity. In 1985 Arnold and Brezzi showed that the hybridization is more than an implementation trick. They proved that the new unknown, which plays a role of a Lagrange multiplier for restoring the interelement continuity of the normal derivative across the interfaces, contains *extra* information about the exact solution and used this for postprocessing the finite element solution. After another two decades, a new perspective on hybridization emerged [2] with the characterization of the approximate trace as the solution of certain interface problem. The main ideas of the hybridization combined with the technique of lifting operators from discontinuous Galerkin (DG) approximations led to a unified hybridization technique [3] for DG, mixed, nonconforming, and conforming finite element approximations of second order elliptic problems.

In the talk we shall discuss this general hybridization framework for second order elliptic problems, which is characterized by: (1) the finite element spaces of

the local solutions, (2) the numerical traces of the solution and the flux, and (3) the space of the Lagrange multiplier. We first show, that this framework reproduces the known hybridization of the standard mixed finite element approximations, in particular those of [2]. Next we study the hybridization of the interior penalty (IP) DG method and show that the proposed procedure fully characterizes the class of hybridizable IP DG schemes. We demonstrate that from the known IP DG methods only the IP scheme due to Ewing, Wang, and Yang, [4], is hybridizable. Finally, we show that most of the LDG schemes are hybridizable and fully characterize them.

The talk is based on the joint work [3] of the author with Cockburn and Gopalakrishnan.

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On functions of bounded p-variation

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We obtain estimates of the total p-variation ($1 < p < \infty$) and other related functionals for a periodic function $f \in L^p[0,1]$ in terms of its L^p-modulus of continuity. These estimates are sharp for any rate of the decay of $\varnothing(f;)_{.p}$. Moreover, the constant coefficients in them depend on parameters in an optimal way.

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Bandlimited one-sided approximations to the Gaussian

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Let B_τ be the class of entire functions of exponential type $\leq \tau$. We consider the problem of best $L^1(\mathbb{R})$ and best one-sided $L^1(\mathbb{R})$ approximation by functions from B_τ to locally integrable real-valued f whose (distributional) Fourier transform $\widehat{f}$ is given for $|t| \geq \tau$ by

$$\widehat{f}(t) = \int_0^\infty \lambda^{-1/2} e^{-\pi\lambda^{-1}t^2} d\nu(\lambda), \tag{2}$$

where ν is a (not necessarily finite) positive Borel measure on $[0, \infty)$ satisfying certain growth conditions.

Functions of this form include $x \mapsto |x|^s$ where $s > 0$ and s not an even integer, the functions $x \mapsto x^{2n} \log|x|$ with $n \in \mathbb{N}$, and $x \mapsto \log(\frac{x^2+a}{x^2+b})$ for $0 < a < b$. We obtain the best approximations from B_τ to the Gaussian, and use these to obtain best approximations to functions satisfying (2). The approximation errors are given in terms of Jacobi theta functions.

Rational functions with fixed poles deviated least from zero on several arcs with zeros on those arcs

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On the set $\Gamma_\mathcal{E} = \{z \in \mathbb{C} : z = e^{i\varphi}, \varphi \in \mathcal{E}\}$, where $\mathcal{E} = [\alpha_1, \alpha_2] \cup \cdots \cup [\alpha_{2l-1}, \alpha_{2l}]$, $0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_{2l} < 2\pi$, we shall consider functions $R_N(z) = P_N(z)/\sqrt{D(z)}$, where $D(z)$ is a polynomial of degree $2a$, $z^{-a}D(z) > 0$ for $z \in \Gamma_\mathcal{E}$; $P_N(z)$ is a monic polynomial with zeros on $\Gamma_\mathcal{E}$. The class of such functions will be denoted by $\mathcal{R}_N^D(\mathcal{E})$.

Theorem (joint with S.V. Tyshkevich) *If for any j, $j = 1, \ldots, l$, the sum of the harmonic measures of $\Gamma_{\mathcal{E}_j}$ with respect to the zeros of the polynomial $D(z) = e^{ia\varphi}\mathcal{A}(\varphi) = \prod_{j=1}^{m^*}(z - z_j)^{m_j}$ is a natural number, $\overline{\omega}(z, x)$ is the density of the harmonic measure, then the minimum in the extremal problem*

$$\max_{z\in\Gamma_\mathcal{E}} |R_N^*(z)| = \min_{R_N \in \mathcal{R}_N^D(\mathcal{E})} \max_{z\in\Gamma_\mathcal{E}} |R_N(z)|$$

is given by $R_N^*(e^{i\varphi}) =$

$$A_N^* \varepsilon e^{i\frac{N-a}{2}\varphi} \cos\left(\frac{\pi}{2} \int\limits_{\mathcal{E}\cap[b,\varphi]} \left((N-a)(\overline{\omega}(\infty,\xi)+\overline{\omega}(0,\xi)) + \sum_{j=1}^{m^*} m_j \overline{\omega}(z_j,\xi)\right) d\xi\right),$$

$|\varepsilon| = 1$, *with suitable finite constant* A_N^*.

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Harmonic analysis and approximation by radial functions

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We characterize the radial functions whose scattered shifts form a fundamental system in the space $L_p(\mathbb{R}^d)$. In particular, we show that for any even function h from the space $L_p(\mathbb{R},\mu)$, $1 \le p \le 2$, the space formed by all possible linear combinations of shifted radial functions $h(\|x+a\|)$, $a \in \mathbb{R}^d$, is dense in the space $L_p(\mathbb{R}^d)$, if and only if the function h is not a polynomial.

Computing multivariate Fekete and Leja points by numerical linear algebra[1]

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We present two greedy algorithms, that compute discrete versions of Fekete-like points for multivariate compact sets by basic tools of numerical linear algebra. The first gives the so-called *Approximate Fekete Points* by QR factorization with column pivoting of Vandermonde-like matrices. The second computes *Discrete Leja Points* by LU factorization with row pivoting. We also discuss the asymptotic distribution of such points when they are extracted from *Weakly Admissible Meshes*. These meshes are nearly optimal for least-squares approximation and contain interpolation sets nearly as good as Fekete points of the domain. These allow us to replace a continuous compact set by a discrete version, that is "just as good" for all practical purposes. Some examples will be presented in order to show the effectiveness of the algorithms.

[1]This is a joint work with: Len Bos, University of Verona and Alvise Sommariva and Marco Vianello, University of Padova.

Acknowledgments. We were supported by the "ex-60%" funds and by the project "Interpolation and Extrapolation: new algorithms and applications" of the University of Padova, and by the INdAM GNCS.

Markov interlacing property for exponential polynomials

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Let U_n be an extended Tchebycheff system on the real line. Given a point $\bar{x} = (x_1, \ldots, x_n)$, where $x_1 < \cdots < x_n$, we denote by $f(\bar{x}; t)$ the polynomial from U_n, which has zeros $x_1, \ldots, x_n$. (It is uniquely determined up to multiplication by a constant.) The system U_n has the Markov interlacing property (M) if the assumption that $\bar{x}$ and $\bar{y}$ interlace implies that the zeros of $f'(\bar{x}; t)$ and $f'(\bar{y}; t)$ interlace strictly, unless $\bar{x} = \bar{y}$.

We formulate a general condition which ensures the validity of the property (M) for polynomials from U_n. We also prove that the condition is satisfied for some known systems, including exponential polynomials $\sum_{i=0}^{n} b_i e^{\alpha_i x}$ and $\sum_{i=0}^{n} b_i e^{-(x-\beta_i)^2}$. As a corollary we obtain that property (M) holds true for Müntz polynomials $\sum_{i=0}^{n} b_i x^{\gamma_i}$, too.

Min-max polynomials on the ball in several dimensions

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We consider the unit ball in $\mathbb{R}^d$, $d \geq 2$, and the problem of finding a best uniform approximation on the unit ball to a homogeneous polynomial in d variables with real coefficients by polynomials of lower degree. We call the error function of best approximation a min-max polynomial on the unit ball. More precisely, for $d = 2$ we introduce a new class of min-max polynomials to the bivariate monomials $x^n y^m$ for all $n, m \in \mathbb{N}$. Then we consider the weighted analogue of this problem and give explicit solutions for two types of weight functions positive on the unit disc.

[2]Supported by the Austrian Science Fund FWF, project number: P20413-N18.

Furthermore, some results are presented for $d \geq 3$, their simplest case being min-max polynomials on the unit ball to the monomials $x_1 ... x_{d-1} x_d^m$ for all $m \in \mathbb{N}$.

The talk is based on joint work with Franz Peherstorfer.

Some properties of Durrmeyer modification of the Meyer-König and Zeller operators and other composite linear operators

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We study some asymptotic and localization result for the several Durrmeyer type modifications of the classical Meyer-König and Zeller operators. We also consider and analyze other sequences of operators that can be obtained by composition (notice that the Meyer-König and Zeller sequence derives by composition from the Baskakov operators on $[0, \infty)$).

On summability almost everywhere by Cesáro and Abel-Poisson methods of series with respect to block-orthonormal systems

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Block-orthonormal systems were introduced by Gaposhkin. He proved, that the Menshov-Rademacher‘s theorem and the strong law of large numbers are valid for such systems in certain conditions. We obtained some results on convergence and summability of series with respect to block-orthonormal systems.In particular, Menshov-Rademacher's and Gaposhkin's theorems were generalized and the exact Weyl multipliers for the convergence and summability almost everywhere of series with respect to block-orthonormal systems were established.

The necessary and sufficient conditions on the length of blocks were obtained, when the Kacmarz‘s theorem is valid for the block-orthonormal series.Using proved

theorems it is possible determine exact Weyl multipliers for the Cesáro (C,1) summability almost everywhere of block-orthonormal series in the case, when the Kacmarz's theorem is not true. We obtained the sufficient conditions on the length of blocks guaranteeing $(C,\alpha)(-1<\alpha<0)$-summability almost everywhere of block-orthonormal series.

Now the sufficient conditions on the blocks are established, when the block-orthonormal series are $(C,\alpha)(\alpha>0)$ and Abel-Poisson's summable almost everywhere; Also equivalence of the methods $(C,\alpha)(\alpha>0)$ and Abel-Poisson's are established in certain conditions for the block-orthonormal series.

Markov type inequalities for oscillating exponential polynomials

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Let $\mathcal{V}_n(\bar{\alpha})$ be the set of all exponential polynomials of the form

$$v(x)=\sum_{i=0}^{n} b_i e^{-\alpha_i x},\ 0<\alpha_0<\cdots<\alpha_n,$$

which have n simple zeros in $(0,\infty)$. Let $h_j(v),\ j=0,\ldots,n$ be the absolute values of the local extrema of a polynomial $v\in\mathcal{V}_n(\bar{\alpha})$.

We prove that for every $v\in\mathcal{V}_n(\bar{\alpha})$, $k\in\mathbb{N}$, and for every convex and strictly increasing on $[0,\infty)$ function ψ such that $\psi(0)=0$, the quantities $h_j(v^{(k)}),\ j=0,\ldots,n$ and the integral $\int_0^\infty \psi\left(|v^{(k)}(x)|\right)dx$ are monotone increasing functions of $h_0(v),\ldots,h_n(v)$. As corollaries we obtain various Markov type inequalities for polynomials from $\mathcal{V}_n(\bar{\alpha})$.

Schauder fractal bases of $\mathcal{L}^p(I)$

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In former papers we have defined fractal functions associated to any continuous mapping defined on a bounded real set. In particular one can consider perturbed polynomials, trigonometric bases, etc. The construction is made via an iterated function system and its corresponding fractal attractor.

In this work the procedure is extended to p-integrable ($1 \leq p < +\infty$) functions defined on a compact interval I. Every map in endowed with a family of fractal functions that contains the original as a particular element. This correspondence enables the construction of new spanning families for $\mathcal{L}^p(I)$. Depending on the value of the coefficients of the iterated function system, one obtains Schauder sequences and bases for this space, composed by fractal functions close eventually to the classical.

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Universal phenomena in the complex plane

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The most famous universal phenomenon in the complex plane, discovered by Voronin in 1975, is the Riemann Zeta-Function ζ which satisfies also the following well-known properties:

(i) It is holomorphic in the complex plane except for a single pole at $z = 1$ with residue 1.
(ii) The symmetry relation $\zeta(z) = \overline{\zeta(\bar{z})}$ holds for $z \neq 1$.
(iii) The functional equation $\zeta(z)\Gamma(z/2)\pi^{-z/2} = \zeta(1-z)\Gamma((1-z)/2)\pi^{-(1-z)/2}$ holds.

Let A denote the set of all functions satisfying (i)-(iii). By constructing "universal" approximants of ζ it is shown that –in a topological sense– the majority of the functions in A fails to satisfy Riemann's Conjecture.
Moreover, we consider universal meromorphic approximation on Vitushkin sets.

On the behavior of ultraspherical polynomials in the complex plane

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It is well-known that the squared modulus of every function f from the Laguerre–Polya class $\mathcal{L}-\mathcal{P}$ of obeys a MacLaurin series representation

$$|f(x+iy)|^2 = \sum_{k=0}^{\infty} L_k(f;x)y^{2k}, \quad (x,y) \in \mathbb{R}^2,$$

which reduces to a finite sum when f is a polynomial having only real zeros. The coefficients $\{L_k\}$ in this formula are representable as non-linear differential operators acting on f, and by a classical result of Jensen $L_k(f;x) \geq 0$ for $f \in \mathcal{L}-\mathcal{P}$ and $x \in \mathbb{R}$. Here, we prove a conjecture formulated by the first-named author in 2005, which states that for $f = P_n^{(\lambda)}$, the n-th ultraspherical polynomial, the functions $\{L_k(f;x)\}_{k=1}^n$ are monotone decreasing on the negative semi-axis and monotone increasing on the positive semi-axis. We discuss the relation between this result and certain polynomial inequalities, which are close in spirit to the celebrated refinement of the classical Markov inequality, found by R. J. Duffin and A. C. Schaeffer in 1941.

On ψ-direct sums and some inequalities

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Sequence spaces with absolute and normalized norms correspond to a class of convex functions. Recently some authors succeed in getting results (for instance geometric properties) about such spaces, called ψ-direct sums, by investigation the corresponding function ψ. We consider some two dimensional cases and get some inequalities (for instance, of Beckenbach-Dresher type).

The talk is based on a joint work with L.-E. Persson and S. Varošanec.

A Goodman-Sharma variant of the Meyer-König and Zeller operators

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The uniform weighted approximation errors of a Goodman-Sharma variant of the Meyer-Konig and Zeller operators are characterized for functions from $C(w)[0,1]$ with weight of the form $x^{\alpha_0}(1-x)^{\alpha_1}$ for $\alpha_0, \alpha_1 \in [-1,0]$. Direct and strong converse theorems are proved in terms of the weighted K-functional.

The talk is based on a joint work with Kamen Ivanov (Institute of Mathematics and Informatics of Bulgarian Academy of Sciences).

On extended cubatures of Turán type for the ball

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We consider two generalizations of the Turán type $(0,2)$ quadrature for the unit ball in R^n that approximate the integral of a function u over the unit ball by linear combination of surface integrals over the unit sphere of k normal derivatives of u and surface integrals of u and $\Delta^{k+1}u$ over m concentric spheres centered at the origin. We prove that these two cubatures are the only ones that are exact for all $(2m+k+1)$-harmonic functions. We derive explicitly the weights and the nodes of these cubatures for general k.

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An inequality for self reciprocal polynomials

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Let $\mathcal{P}_n$ be the class of all polynomials of degree at most n. Polynomials $f \in \mathcal{P}_n$ which satisfy the condition $z^n f(1/z) \equiv f(z)$ are called self-reciprocal and form the sub-class $\mathcal{P}_n^*$ of $\mathcal{P}_n$. For any $\rho > 0$, let $\mathcal{M}_\infty(f\,;\,\rho) := \max_{|z|=\rho} |f(z)|$ and

$$\mathcal{M}_p(f\,;\,\rho) := \left(\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(\rho e^{i\theta})|^p \,\mathrm{d}\theta\right)^{1/p},\ 0 < p < \infty.$$

If $f \in \mathcal{P}_n$ then $\mathcal{M}_p(f'\,;\,\rho) \le n\rho^{n-1}\,\mathcal{M}_p(f\,;\,1)$ for any $p > 0$ and $\rho \ge 1$, whereas, if $f \in \mathcal{P}_n^*$ then $\mathcal{M}_p(f'\,;\,\rho) \le (n/2)\rho^{n-1}\,\mathcal{M}_p(f\,;\,1)$ for any $p > 0$ and $\rho \ge 1$. Lately, it has been noted that at least for $p \ge 1$, there exists a positive number ρ_n strictly less than 1 such that $\mathcal{M}_p(f'\,;\,\rho) \le n\rho^{n-1}\,\mathcal{M}_p(f\,;\,1)$ for $\rho \ge \rho_n$ if $f \in \mathcal{P}_n$. By analogy, it has been asked if there was a positive number $\rho_n^* < 1$ such that $\mathcal{M}_p(f'\,;\,\rho) \le (n/2)\rho^{n-1}\,\mathcal{M}_p(f\,;\,1)$ for all $\rho \ge \rho_n^*$ and any $f \in \mathcal{P}_n^*$. We propose to discuss this question.

A new extremal property of the Chebyshev polynomials: Maximizing pairs of coefficients of bounded polynomials

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Let $T_n(x) = \sum_{k=0}^{n} t_{n,k} x^k$ denote the n-th Chebyshev polynomial of the first kind, and let $P_n(x) = \sum_{k=0}^{n} a_k x^k$ denote any real polynomial of degree not exceeding n which satisfies $|P_n(x)| \le 1$ for $|x| \le 1$ or merely $|P_n(\cos((n-i)\pi/n)| \le 1$ for $0 \le i \le n$. For the sum of the moduli of two coefficients of $P_n(x)$ then holds

(i) $|a_{k-1}| + |a_k| \le |t_{n,k}|$ if $n \equiv k \mod 2$ (G. Szegő's coefficient inequality, which implies V. A. Markov inequality $|a_k| \le |t_{n,k}|$, with a special case the P. L. Chebyshev inequality $|a_n| \le 2^{n-1}$ (see [1, p.673], [2]);

(ii) $|a_0| + |a_n| \le 2^{n-1}$, if n is odd (see [3]).

In an attempt to generalize these sharp estimates for pairs of coefficients we ask: Is $|t_{n,k}|$ large enough to sharply majorize $|a_j| + |a_k|$, where $j < k \leq n$; $j \not\equiv n \equiv k \mod 2$? We obtain an affirmative answer which covers the two inequalities (i) and (ii) above, but there remain exceptional cases. We also provide an answer, in terms of $|t_{n,k}|$, to the analogous question concerning the alternative constellation of pairs $|a_k| + |a_j|$ $(k < j < n;\ j \not\equiv n \equiv k \mod 2)$, in particular $|a_k| + |a_{k+1}|$. The results can be extended to $P_n(x)$ bounded symmetrically on $[-1, 1]$ in the sense of [4], [5]. For a related issue in the complex domain see [6].

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Bernstein operators for functions in the unit ball of euclidean space

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In this talk we present a new class of Bernstein operators for functions in several variables based on the Laplace-Fourier expansion of a function. The Bernstein operator B_n should have the following characteristics: (i) For a given function f defined on the unit ball, $B_n(f)$ is a polynomial, (ii) $B_n(f) = f$ for harmonic polynomials f of degree $\leq n$, (iii) For the computation of $B_n(f)$ one needs to know the function values of f on concentric spheres with center 0 and of suitable radii. The main idea is to define a multivariate Bernstein operator by introducing suitable univariate Bernstein operators acting on the Laplace-Fourier coefficients of the function f. By a variable transformation these Bernstein operators correspond to Bernstein operators for exponential polynomials recently studied by J.M. Aldaz, O. Kounchev and the author.

Conjectures and results on the multivariate Bernstein inequality on convex bodies

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We survey the findings on the Bernstein inequality over the past twenty years or so.

If K is a centrally symmetric convex body in $\mathbb{R}^d$ or even in any Banach space X, then in 1991 Y. Sarantopoulos (and later by a different method also M. Baran) proved that for any interior point $x \in K$ and any polynomial p we have

$$\|\nabla p(x)\| \leq \frac{\deg p \sqrt{\|p\|^2 - p^2(x)}}{\sqrt{1 - \|x\|_K^2}}.$$

Thus both the *inscribed ellipse method* of Sarantopoulos and the *pluripotential theory* approach of Baran give a sharp Bernstein type inequality for convex *centrally symmetric* bodies. For not necessarily symmetric convex bodies, our best estimates (in a joint paper with A. Kroó) use the so-called *generalized Minkowski functional*, and are within a $\sqrt{2}$ constant factor of the truth, but a seemingly natural conjecture, formulated jointly with Y. Sarantopoulos in 2002, fails, as was shown by an insightful construction of N. Naidenov in 2006.

In the case of the standard simplex in $\mathbb{R}^d$, the exact yield of the inscribed ellipse method was computed jointly with L. Milev, in 2005. Subsequently realizing that Baran's method gives exactly the same, I conjectured that these methods are equivalent for all convex bodies.

In a paper still in print we proved this conjecture jointly with D. Burns, N. Levenberg and S. Ma'u. Now it remains to see if the two equivalent estimates do really provide sharp bounds? We will explain the underlying idea of a program, started in the last years by Naidenov and Milev, to decide this ultimate question.

On the double Fourier series of functions of bounded partial Λ-variation

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Let f be a real function of two variables of period 2π with respect to each variable. Given interval $I = (a, b)$ and points x, y from $T := [0, 2\pi]$ we denote

$$f(I, y) := f(b, y) - f(a, y), \qquad f(x, I) = f(x, b) - f(x, a).$$

Let Ω be the set of all collections $E = \{I_j\}$ of non-overlapping intervals from T ordered in arbitrary way.

For the increasing sequence of positive numbers $\Lambda = \{\lambda_n\}_{n=1}^{\infty}$ we set

$$\Lambda V_1(f) = \sup_y \sup_{E\in\Omega} \sum_j \frac{|f(I_j, y)|}{\lambda_j}, \qquad \Lambda V_2(f) = \sup_x \sup_{E\in\Omega} \sum_j \frac{|f(x, I_j)|}{\lambda_j}.$$

We say that the function f has *Bounded Partial Λ-variation*, if

$$P\Lambda V(f) := \Lambda V_1(f) + \Lambda V_2(f) < \infty.$$

The necessary and sufficient conditions on the sequence $\Lambda = \{\lambda_n\}$ are obtained for the convergence of Fourier series of functions of bounded partial Λ-variation. Similar problems for the convergence of Cesàro means of negative order are considered.

Extreme domains associated with a complex polynomial

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With every complex polynomial, some closed domains on the complex plane $\mathbb{C}$ are associated. The motivation is to formulate a sharp analog of the Rolle's theorem for complex polynomials.

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Key words: *zeros and critical points of polynomials, apolarity, apolar locus, polar derivative, polar locus, complex Rolle's theorem.*

Relative linear n-widths of sets of differentiable functions

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Let X be a linear normed space, V be a cone in X and A be a subset of X, such that $A \cap V \neq \emptyset$. Relative linear n-width of set A in X with constraint V defined by

$$\delta_n(A \cap V, V)_X = \inf_{L_n(V) \subset V} \sup_{f \in A \cap V} \|f - L_n f\|_X,$$

the left-most infimum being taken over all continuous linear operators L_n with finite rank at most n, such that $L_n(V) \subset V$.

Denote by $C^k[0,1]$, $k \geq 0$, the space of all real-valued and k-times continuously differentiable functions on [0,1]. Let D^i be the i-th differential operator. Let $\sigma = (\sigma_i)_{i \geq 0}$ be a sequence with $\sigma_i \in \{-1, 0, 1\}$, and let k be an integer with $\sigma_0 \cdot \sigma_k \neq 0$. Let

$$C_{0,k}(\sigma) = \{f \in C^k[0,1] \ : \quad \sigma_i \cdot D^i f \geq 0, \quad i = 0, \ldots, k\}.$$

We will present estimations of relative linear n-widths of some sets of differentiable functions in $C[0,1]$ with shape-preserving constrains $C_{0,k}(\sigma)$.

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Modified product cubature formulae with finite-differences error representation

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The product cubature formulae are the usual tool for approximation of a double integrals over rectangular domain. In this paper we consider a modification of the product cubature formulae, based on interpolation of bivariate functions with quasi-polynomials. We consider bounded and measurable functions and obtain the error representation of cubature formulae with finite differences.

Interpolation by spherical polynomials on the sphere

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Let

$$H_N(\theta,\lambda)=\sum_{n=0}^{N}\sum_{m=0}^{n}\{c_{nm}P_{nm}(\cos\theta)\cos m\lambda+s_{nm}P_{nm}(\cos\theta)\sin m\lambda\},$$

be a spherical polynomial of degree N, where P_{nm} are the Legendre's associated functions. The interpolation problem

$$H_N(\theta_p,\lambda_q)=f_p^q,\quad p,q=0,1,...,N.$$

has an unique solution for the angles

$$\theta_p=\frac{(2p+1)\pi}{2N+2},\quad \lambda_q=\frac{2\pi q}{N+1},\ p,q=0,1,...,N.$$

Multidimensional Jackson inequality in mean-square norm

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Consider the space $L_2=L_2(\mathbb{T}^N)$, $\mathbb{T}^N=\mathbb{R}^N/(2\pi\mathbb{Z}^N)$, of complex-valued functions of N variables with the usual mean-square norm

For a set of complex numbers $M=\{\mu_j\}_{j\in\mathbb{Z}}$ a μ_j such that $0<\sum_{j\in\mathbb{Z}}|\mu_j|<\infty$, $\sum_{j\in\mathbb{Z}}\mu_j=0$ and a vector $\mathbf{t}$, we assign a difference operator $\Delta_{\mathbf{t}}^M\colon\ L_2\to L_2$ of the form $\Delta_{\mathbf{t}}^M f(\mathbf{t})=\sum_{k\in\mathbb{Z}}\mu_k f(\mathbf{x}+k\mathbf{t})$. For a compact set $T\subset\mathbb{R}^N$ containing an open neighborhood of zero and a function $f\in L_2$, we define the modulus of continuity $\omega_M(f,\delta)=\sup_{\mathbf{t}\in\delta T}\|\Delta_{\mathbf{t}}^M f\|,\quad \delta\geq 0.$

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Let Λ be some subset of $\mathbb{Z}^N$ containing the origin. Denote by $E_\Lambda(f)$ the value of the best approximation of the element $f \in L_2$ by functions from L_2 with spectrum in the set Λ; i.e., $E_\Lambda(f) = \inf_{\{c_\mathbf{s}\}_{\mathbf{s}\in\Lambda}\subset\mathbb{C}} \left\| f(\mathbf{x}) - \sum_{\mathbf{s}\in\Lambda} c_\mathbf{s} e^{i\mathbf{s}\mathbf{x}} \right\|$.

In the present paper, we establish the Jackson inequality in the space L_2 for periodic functions of many variables with the least sharp constant.

Theorem. *There exists a number $\gamma > 0$ depending only on the sets M and T such that the inequality*

$$E_\Lambda(f) \le \frac{1}{\sqrt{\sum\limits_{j\in\mathbb{Z}} |\mu_j|^2}}\, \omega_M\left(f, \frac{\gamma}{r(\Lambda)}\right)$$

holds for all $f \in L_2$.

Error bounds for scattered data interpolation in $\mathbb{R}^3$ by smooth cubic curve networks

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We consider the problem of interpolating scattered data in $\mathbb{R}^3$. We assume that the data are sampled from a smooth bivariate function F. For a fixed triangulation T associated with the projections of the data onto the plane Oxy we consider a smooth interpolating cubic curve network S defined on the edges of T. We show that $\|F - S\|_{L_2(T)} \le C(T)\, \|F^{(4)}\|_{L_2(T)}$. The dependence of the term $C(T)$ on the triangulation T is analysed and evaluated.

Asymptotic expansions for a Durrmeyer variant of Baskakov and Meyer-König and Zeller operators and quasi-interpolants

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We consider a Durrmeyer variant of Baskakov and Meyer-König and Zeller operators. The main results include the local rate of convergence which is based on the eigenstructure of the operators. We present a complete asymptotic expansion for the Baskakov-Durrmeyer operators and their quasi-interpolants in terms of certain differential operators. Furthermore, we state analogues results for the Meyer-König and Zeller operators and introduce their quasi-interpolants.

What condition can correctly generalize monotonicity in L^1-convergence of sine series?

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In classical Fourier analysis, the integrability of trigonometric series is considered as an interesting but difficult topic. In particular, the integrability of sine series have not been touched in dozens years since Boas, Heywood published their classical results, meanwhile the generalizations of (deceasing) monotonicity have been developed from various quasimonotonicity and bounded variation conditions, finally, to the mean value bounded variation condition, an essential ultimate condition in most sense, and applied to various convergence problems extensively including uniform convergence, mean convergence, L^p integrability and best approximation etc.. The difficulty of the research can be seen from this point. We may need another point of view now. Given a sine series $\sum_{n=1}^{\infty} a_n \sin nx$, its sum function can be written as $g(x)$ at the point x where it converges. However, it is usually a very hard job to verify if the sum function or the sine series itself belongs to $L_{2\pi}$ or not. On the other hand, in studying L^1-convergence problems, people usually need a requirement that $g \in L_{2\pi}$, which also becomes a hard condition to check or a prior condition to set in most cases. For instance, the well-known classical results for L^1-convergence says that (see, Zygmund [14]), *let the real even (odd) function $f \in L_{2\pi}$, and its*

Fourier cosine (sine) coefficients $\{a_n\}_{n=1}^{\infty} \in \mathbf{MS}$*, then* $\lim_{n\to\infty} \|f - S_n(f)\|_{L^1} = 0$ *if and only if* $\lim_{n\to\infty} a_n \log n = 0$. On the other hand, the classical result of uniform convergence of sine series (Chaudy- Jolliffe theorem, see [14]) says that, *let* $\{a_n\}_{n=1}^{\infty} \in \mathbf{MS}$*, then the sine series* $\sum_{n=1}^{\infty} a_n \sin nx$ *uniformly converges if and only if* $\lim_{n\to\infty} na_n = 0$. The difference of two types of classical theorems is very clear: one need a prior condition $f \in L_{2\pi}$, the other does not require that $f \in C_{2\pi}$. Mathematicians surely prefer the latter to the former in mathematical sense. The reason that the prior condition in L^1 case cannot be avoided mainly arises from the much more computation complexity in the integrable space than the continuous space, and technical benefit to turn the computation from infinity into finiteness by setting the integrability. Furthermore, people also note that the first important problem in the integrable space should be L^1-convergence, which may be achieved by the series itself (by coefficients) without mentioning the sum function. Based upon these reasons, why do we not try to find a more direct and clean version of L^1- convergence? The present paper is arranged as follows. First we construct a non-negative sequence which shows that although it is quasi-monotone and the inequality $\sum_{n=1}^{\infty} n^{-1} a_n < \infty$, the corresponding sine series does not converge in L^1-norm (Theorem 1). Then, we raise a new correct condition which can guarantee the above mentioned condition to be necessary and sufficient for which the corresponding sine series does converge in L^1-norm (Theorem 2), and also discuss the relationships with other related sets (Theorem 3 and 4). Finally, we prove the newly raised condition cannot be weakened further in this L^1-convergence case in some sense (Theorem 5). As a whole, we give a complete solution to this topic.

Keywords: trigonometric series, convergence, integrability, monotonicity

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Some canonical decompositions of discontinuous Galerkin finite element spaces and preconditioning

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We introduce a natural decomposition of the discontinuous Galerkin Finite Element spaces. For the lowest order case this decomposition is a direct sum of of the Crouzeix-Raviart non-conforming finite element space and a subspace that contains functions discontinuous at interior faces. We will also indicate how to construct such decompositions for higher order elements. Based on these decompositions we develop iterative and preconditioning techniques for the solution of the linear systems resulting from several discontinuous Galerkin (DG) Interior Penalty (IP) discretizations of elliptic problems. We analyze the convergence properties

of these algorithms for both symmetric and non-symmetric IP schemes. We also present numerical examples confirming the theoretical results. Further extension to problems with jumps in the coefficients and linear elasticity will also be discussed.

The talk is based on joint works with Blanca Ayuso de Dios from Centre de Recerca Matematica (CRM), Spain.