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ON OPTIMAL MONOSPLINES

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In this paper we discuss some of the most recent results on optimal monosplines These results deal primarily with the characterization of such minimal monosplines. These monosplines arise naturally when one is interested in finding optimal integration formulae. Examples of settings where they occur are the Hardy space H_2, and L_2 (B), where B is the unit ball. In this connection polynomial monosplines will also be discussed. Finally some open problems in this area will be formulated.

• We considered the problem of finding the optimal polynomial monospline of minimal uniform norm of the following form:

$$M(x)=x^n+\sum_{i=0}^{n-1} a_i x^i+\sum_{i=1}^{t}\sum_{j=0}^{m_i-1} a_{ij}(x-\xi_i)_+^{n-1-j}, \tag{1}$$

where n is a fixed positive integer, each m_i is a fixed positive odd integer with $m_i \leqq n-1$; $i=1,\ldots,t$, $0<\xi_i<\xi_{i+1}<1$; $i=1,\ldots,t-1$.

In 1960 [4] R. Johnson, for the special case where $m_i=1$; $i=1,\ldots,t$, proved that there existed one and only one monospline of the form (1) of least uniform norm over [0,1]. J. Schoenberg and Z. Ziegler [6] and C. Micchelli [5] had conjectured that this uniqueness result is valid for arbitrary m_i subject to the restrictions noted above. In [1] R. Barrar and H. Loeb had shown that there exists such a monospline of least uniform norm. In a recent paper [2] R. Barrar and H. Loeb have been able to demonstrate that such an optimal monospline is unique. This optimal monospline $M(x)$ is uniquely characterized by a set of points $0=x_0<x_1<\cdots<x_N=1$, where $M(x_i)=-M(x_{i+1})$; $i=0,1,\ldots,N-1$ and $\max\{|M(x)|: x\in[0,1]\}=M(x_N)$ with $N=n+\Sigma_{i=1}^{t}(m_i+1)$. The proof involves the use of the differential equation approach of C. Fitzgerald and L. Schumaker [3].

In some new work accomplished in Eugene and Munster we have been able to prove uniqueness results for Extended Totally Positive Kernels with respect to the uniform norm. Special cases of these results include the Cauchy Kernel, $1/(1-xy)$ and the Bergmann Kernel, $(1-xy)^{-2}$.

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