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SOME IMPLICIT FUNCTION THEOREMS IN LINEAR SPACES WITH A DISTINGUISHED CLASS OF CONVERGENT SEQUENCES

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Summary. In the pa er the linear spaces with a distinguished class of convergent sequences (L-spaces) were defined and s me convergence conditions of stationary one-step iterative methods for solution of equations in these spaces were given.

Moreover, the application which the theorem on mean value (in a new version) may have for research on convergence of the iterative methods, was shown.

The results obtained were used in formulating some implicit function theorems.

In mathematical analysis a situation arises in which the convergence of a sequence of elements in space of one kind is determined by the convergence of a sequence of elements in space of some other kind and the latter sequence is generated by special "majorant" functions.

The paper proves that the problem of existence and uniqueness of a fixed point of mapping which transforms into itself the linear space with a distinguished class of convergent sequences (without norm), can be solved in a similar manner.

A convenient method of a majorant determination based on a new mean value theorem was also established. The results obtained were further used to prove certain implicit function theorems. Other particular applications of the results were also discussed.

This work presents part of the research carried out in the field of analysis in L-spaces. Other results on this subject are contained in [7] and in the author's doctoral dissertation.

1. Definition of the L-spaces. The L-spaces introduced in this section are some particular linear pseudotopological spaces [5, 3]. Some class of sequences will substitute the class of filters determined in definition of linear pseudotopological space. This substitution is performed, because the iteration processes are of sequential type and this is the subject of this paper.

Let the real linear space E be given and suppose that some classes of sequences from this space are distinguished (the so-called convergent sequences), and for each sequence $\{x_n\}$ of this class exactly one element (limit) is determined $x=\lim_{n\to\infty} x_n$ in such a way that the following conditions are satisfied:

(1.1) a) If $\lim_{n\to\infty} x_n = x$ and $k_1, k_2, \ldots, k_n, \ldots$ is a strictly increasing sequence of naturals, then $\lim_{n\to\infty} x_{k_n} = x$,

b) if $x_n = x$ for each n, then $\lim_{n\to\infty} x_n = x$,

c) if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = x$, then the sequence $x_1, y_1, x_2, y_2, \ldots, x_n, y_n, \ldots$ is convergent with a limit x,

d) if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then $\lim_{n\to\infty} x_n + y_n = x + y$,

e) if $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} \lambda_n = \lambda (\lambda, \lambda_n \in \mathsf{R})$ then $\lim_{n\to\infty} \lambda_n x_n = x$.

Definition. 1. *Space E with the distinguished class of convergent sequences that satisfy the conditions* (1.1) *we are going to denote by E and call the L-space with the carrier* E.

Some considerations in the paper [5] imply that for every L-space E there exists locally convex linear topological space E^0 with the same carrier and the following property

(1.2) $$x = \lim_{n\to\infty} x_n \text{ in } E \Rightarrow x \overset{\text{top}}{=} \lim_{n\to\infty} x_n \text{ in } E^0.$$

On the other hand, one can observe that the extracting class of all convergent sequences in some linear topological space E^0, the set of sequences that satisfy condition (1.1), is defined. Thus there is defined some L-space with the same carrier as E^0, while the condition (1.2) is satisfied.

Definition 2. *For given L-spaces E_1 and E_2 the mapping $f\colon E_1 \to E_2$ will be called continuous if*

(1.3) $$x = \lim_{n\to\infty} x_n \textit{ in } E_1 \Rightarrow f(x) = \lim_{n\to\infty} f(x_n) \textit{ in } E_2.$$

2. The mean-value theorem. Let E_1 and E_2 be L-spaces and let $r\colon E_1 \to E_2$ then to the mapping r one might arrange a new function Θr defined in such a way

(2.1) $$\Theta r(\lambda, x) = \begin{cases} r(\lambda x)/\lambda & \text{while } \lambda \neq 0, \\ 0 & \text{while } \lambda = 0. \end{cases}$$

Employing the function Θr will introduce an important definition of the "small" mapping:

2.2. Definition 3. *The mapping $r\colon E_1 \to E_2$ is called small and one will denote $r \in R(E_1\colon E_2)$ iff*

a) $r(0) = 0$,

b) *if $x_n \to x_0$; $x_n, x_0 \in E_1$, $\lambda_n \to \lambda_0$; $\lambda_n, \lambda_0 \in \mathsf{R}$ and $\lambda_0 x_0 = 0$ then $\Theta r(\lambda_n, x_n) \to 0$ in E_2.*

The conception of the "small" mapping introduced by the above definition one can apply for defining the derivative of the operator $f\colon E_1 \to E_2$ in point $z \in E_1$.

Definition 4. *If there exists linear and continuous mapping $e\colon E_1 \to E_2$ such that the mapping r defined as follows: $r(h) = f(a+h) - f(a) - l(h)$ is "small", then the mapping f is called differentiable at the point a.*

Mapping l (which is called the derivative of operator f in a point a) is defined uniquely and as was shown in [7] it preserves a vast majority of properties of the "strong" Fréchet derivative. Also in [7], using the idea of N. Bourbaki the mean value theorem is proved. Before formulating the theorem, we will make some initial assumptions:

If the domain of the function f is L-space R with the class of convergent sequences in natural topology, then $\lim\limits_{\varkappa\to 0} \frac{f(\alpha+\varkappa)-f(\alpha)}{\varkappa}$ (if it exists at all) will be denoted by $f^{\cdot}(\alpha)$. It is easy to show if the mapping $f\colon \mathsf{R}\to F$ is differentiable at the point $\alpha\in\mathsf{R}$, then there exists $f^{\cdot}(\alpha)$ and holds

$$(f'(\alpha))\varkappa=\varkappa f^{\cdot}(\alpha),\ f^{\cdot}(\alpha)=f'(\alpha)\cdot 1.$$

Let us assume that two mappings are given

$$f\colon \mathsf{R}\supset[\alpha,\beta]\Rightarrow E,\ \varphi\colon \mathsf{R}\supset[\alpha,\beta]\to\mathsf{R}$$

and let us consider the conditions:

(2.2) E^0 is linear topological Hausdorff space,

(2.3) mappings f and φ are continuous,

(2.4) B is some closed, convex subset of the space E^0,

(2.5) for almost every $t\in[\alpha,\beta]$ mappings f and φ are differentiable at a point t, and

(2.6) $$f^{\cdot}(t)\in\varphi^{\cdot}(t)\cdot B,\ s<t\Rightarrow\varphi(s)\leqq\varphi(t).$$

T h e o r e m 1 (of mean value). *If conditions* (2.2)—(2.6) *are satisfied then* $f(\beta)-f(\alpha)\in(\varphi(\beta)-\varphi(\alpha))\cdot B$.

If between the functions φ and f there are relations described by the thesis of the theorem 1, then we say that φ is a "majorant" of the mapping f (over the interval $[\sigma,\beta]$).

Methods of integrating a "majorant" were given by L. K a n t o r o v i c h [6] (defined in a bit different manner, but with similar properties), for mappings acting in partially ordered linear spaces. He employed the Newton-Leibniz formula of integral calculus:

$$f(x_0+h)-f(x_0)=\int\limits_{x_0}^{x_0+h} f'(x)\cdot h\,dx.$$

His result was used then for proving the convergence of Newton's method.

As shown in [7], the mean-value theorem 1 one can apply for the same goal but with far weakly assumptions imposed on functions f and φ.

3. Majorant principle for one-step iterative methods in *L*-spaces. Applying W. C. R h e i n b o l d t's idea [11] we will formulate the general principle of majorant, which will allow us to state if some iterative process is convergent in L-space or it is not.

Since that moment we will assume that E^0 is sequentially complete locally convex topological Hausdorff space and the additional condition is satisfied:

(3.1) $$x_n\to x \text{ in } E\Leftrightarrow x_n\to x \text{ in } E^0.$$

In addition to this let B be some convex, bounded and closed subset of the space E^0, which contains the origin. We do not assume that B is a neighbourhood of zero, because in such a case E^0 would be the Banach space, and our considerations were restricted to a "classical" case.

Definition 5. *Let in space* E^0 *the sequence* $\{x_n\}$ *be given, then the nondecreasing sequence of real numbers* $\{t_n\}$, *such that* $t_n \geqq 0$ *for* $n=0, 1, 2, \ldots$, *will be called the B-majorant of the sequence* $\{x_n\}$, *if* $x_{n+1}-x_n \in (t_{n+1}-t_n) \cdot B$, for $n=0, 1, 2, \ldots$. For the B-majorant of the sequence $\{x_n\}$ and for integers $m>n \geqq 0$ there holds

$$x_m - x_n = \sum_{i=n}^{m-1} (x_{i+1}-x_i) \in \sum_{i=n}^{m-1} (t_{i+1}-t_i) \cdot B \subset (t_m - t_n) \cdot B. \tag{3.2}$$

So if there exists $\lim_{n\to\infty} t_n = t^* \in \mathsf{R}$, then from the sequential completeness of space E^0 and from condition (3.1) follows the existence of $x^* = \lim_{n\to\infty} x_n \in E$.

In addition to this we get the "estimation"

$$x^* - x_n \in (t^* - t_n) \cdot B \text{ for } n=0, 1, 2, \ldots. \tag{3.3}$$

Let now the operator G: $E \to E$ be given. Let us consider the equation $G(x)=x$ and the sequence that corresponds to this equation, which is defined as follows

$$x_{n+1} = G(x_n) \text{ for } n=0, 1, 2, \ldots, \tag{3.4}$$

while x_0 is given in advance.

We are looking for B-majorizing sequence $\{t_n\}$ for the sequence $\{x_n\}$, among the solutions of some difference equations of the form

$$t_{n+1} - t_n = \psi(t_n - t_{n-1}, t_n, t_{n-1}) \tag{3.5}$$

with fixed bounded values t_0 and t_1.

Definition 6. *Function* ψ: $Q \subset \mathsf{R}^p \to \mathsf{R}^1$ *is said to be of class* $\Gamma^p(Q)$ *if it has the following properties*:

a) *The domain* Q *is a hypercube* $Q = J_1 \times J_2 \times \cdots \times J_p$, where each J_i *is an interval on* $[0, \infty)$ *containing* 0,

b) ψ *is nonnegative and isotone on* Q_1, *i. e., if* $(z_1^i, \ldots, z_p^i) \in Q (i=1, 2)$ *and* $z_j \leqq z_j^2$ *(for* $j=1, 2, \ldots, p$*), then*

$$0 \leqq \psi(z_1^1, \ldots, z_p^1) \leqq \psi(z_1^2, \ldots, z_p^2).$$

Let $\psi \in \Gamma^3(Q)$ and $Q = J_1 \times J_2 \times J_3$. Then the solution $\{t_n\}$ of the difference equations (3.5) is said to exist for given t_0, t_1, if $t_{n+1} - t_n \in J_1$, $t_n \in J_2 \cap J_3$ for all $n \geqq 0$, i. e. if the entire sequence $\{t_n\}$ defined by (3.5) exists.

In paper [7] the following theorem is proved

Theorem 2. *Let the continuous operator (in the sense of definition 2) be given* G: $E \supset D \to E$ *and some subset* $D_0 \subset D$. *Suppose there exists the function* $\psi \in \Gamma^3(Q)$ *and the point* $x_0 \in D$, *such that: if* $G(x)$, $x \in D_0 \subset D$ *then*

$$G(G(x)) - G(x) \in \psi(t-p, t, p) \cdot B, \tag{3.6}$$

whenever $(t-p, t, p) \in Q$, $G(x) - x \in (t-p) \cdot B$, $x - x_0 \in p \cdot B$, $G(x) - x_0 \in t \cdot B$.

Let further for t_0 *and* $t_1 \geqq t_0$, *selected in such a way that* $G(x_0) - x_0 \in t_1 \cdot B$, *exist a solution of difference equations* (3.6). *Then*

— if the elements of the sequence defined by (3.4) *belong to* D_0, *and for every* n, $(t_n - t_{n-1}, t_n, t_{n-1}) \in Q$, *then* $\{t_n\}$ *is a B-majorant of the sequence* (x_n);

— if there exists $\lim_{n\to\infty} t_n = t^*$, *then there also exists* $\lim_{n\to\infty} x_n = x^*$

and the estimation (3.3) *holds;*

— if $x^* \in D$ and G is continuous mapping (in the sense of definition 2) in the point x^*, then $x^* = G(x^*)$.

In applications of theorem 2 one should always show that the sequence $\{x_n\}$ is included in the set D_0. The following lemma that guarantees the existence of mentioned inclusion might be proved with the help of induction.

Lemma 1. *Let the assumptions of theorem 2 hold. If $x_0, \dots, x_m \in D_0$ and with fixed m $S_m^n := \{x: x - x_m \in (t_n - t_m) \cdot B\} \subset D_0$ (for $n = m, m+1, \dots$) then $x_n \in D_0$* for *$n \geq m$. If in addition to this there exists* $\lim_{n\to\infty} t_n = t^*$, *then the condition $x_n \in D_0$ for $n \geq m$ is satisfied, if $\bar{S}_m := \{x: x - x_m \in (t^* - t_m) \cdot B\} \subset D_0$.*

In our interest there is this particular situation when there exists the function $\varphi: \mathbb{R} \supset J \to \mathbb{R}$ (while $J = \bigcap_{i=1}^{3} J_i$) such that

$$\psi(u - v, u, v) = \varphi(u) - \varphi(v) \text{ if } u, v \in J, v \leq u. \tag{3.7}$$

It is then easily seen that $\{t_n\} \subset J$ is a solution of (3.5) with $t_0 = 0$, $t_1 = \varphi(0)$ if and only if

$$t_{n+1} = \varphi(t_n) \text{ for } n = 0, 1, \dots . \tag{3.8}$$

Assuming function φ to be continuous over J, then from (3.8) follows that $t^* = \varphi(t^*)$ if $\lim_{n\to\infty} t_n = t^* \in J$.

In further considerations we will employ the following result obtained by L. Kantorovich:

Theorem 3. *Let $\varphi: [t_0, s_0] \subset \mathbb{R}^1 \to R^1$ be continuous and isotone, and $\varphi(t_0) \geq t_0$, $\varphi(s_0) \leq s_0$. Then the sequences $t_{n+1} = \varphi(t_n)$, $s_{n+1} = \varphi(s_n)$, $n = 0, 1, \dots$ satisfy $t_0 \leq t_n \leq t_{n+1} \leq \lim_{n\to\infty} t_n = t^* \leq s^* = \lim_{n\to\infty} s_n \leq s_{n+1} \leq s_n \leq s_0$, where t^* is the smallest and s^* the largest fixed point of φ in $[t_0, s_0]$.*

In the case of our considerations $t_0 = 0$.

The conditions imposed on the function ψ guarantee a monotonicity of the function φ. Hence in virtue of Kantorovich's theorem, we conclude that $t < \varphi(t)$ for $0 < t < t^*$, and t^* (if it exists) is the smallest fixed point of the operator φ in J.

At present, the theorem on uniqueness can be formulated

Theorem 4. *Suppose that the assumptions of theorem 2 are satisfied, excluding the inclusion* (3.6), *that will be replaced by a more general condition: let for every $x, y \in D_0$*

$$G(y) - G(x) \in \psi(t - p, t, p) \cdot B, \tag{3.9}$$

if only $(t - p, t, p) \in Q$, $y - x \in (t - p) \cdot B$, $y - x_0 \in t \cdot B$, $x - x_0 \in p \cdot B$. Let for bounded values $t_0 = 0$, $t_1 = \varphi(0)$, then equalities (3.7) *and* (3.8) *hold, and let* $\lim_{n\to\infty} t_n = t^* = \varphi(t^*) \in J$ *exist. Then the only fixed point of operator G in the set $D_0 \cap S_0$ is x^* (where $\bar{S}_0 := \{x: x - x_0 \in t^* \cdot B\}$.*

Proof: Assume $y^* = G(y^*) \in D_0 \cap \bar{S}_0$. Then $y^* - x_0 \in t^* \cdot B = (t^* - t_0) \cdot B$. With the help of induction we will show that $y^* - x_n \in (t^* - t_n) \cdot B$ for all $n \geq 0$. In fact, since $\{t_n\}$ is B-majorant of the sequence $\{x_n\}$, so

$$y^* - x_{n+1} = G(y^*) - G(x_n) \in \psi(t^* - t_n, t^*, t_n) \cdot B = (t^* - t_{n+1}) \cdot B,$$

so $y^* = \lim_{n\to\infty} x_n = x^*$.

Applying again the Kantorovich theorem 3 we can state a more general form of the theorem 4, where the region of uniqueness is spread.

Theorem 5. *Suppose the assumptions of the theorem 4 are satisfied. Let there exist the point $\widehat{t}\in J$ such that $\widehat{t}>t^*$ and $\varphi(t)<t$ for $t^*<t<\widehat{t}$. Then x^* is the only fixed point of operator G in the set*

$$D_0\cap\bigcup_{s_0<\widehat{t}}\{x:\ x-x_0\in s_0\cdot B\}.$$

Proof: Let $y^*=G(y^*)\in D_0\cap\{x:\ x-x_0\in s_0\cdot B$ and $s_0<\widehat{t}\}$. If $s_0\leq t^*$, then the uniqueness follows from theorem 4. If $y^*-x_0\in s_0\cdot B$, $t_0=0$, $t^*<s_0<\widehat{t}$, then by induction we show that $y^*-x_{n+1}\in(s_{n+1}-t_{n+1})\cdot B$, where $s_{n+1}=\varphi(s_n)$, $n=0,1,2,\ldots$.
Let $y^*-x_n\in(s_n-t_n)\cdot B$. Then

$$y^*-x_0=(y^*-x_n)+(x_n-x_{n-1})+\cdots+(x_1-x_0)\in(s_n-t_n)\cdot B+(t_{n-1}-t_n)\cdot B$$
$$+\cdots+(t_1-t_0)\cdot B\in s_n\cdot B.$$

Since (3.9) and $x_n-x_0\in t_n\cdot B$ we have at present

$$y^*-x_{n+1}=G(y^*)-G(x_n)\in\psi(s_n-t_n,s_n,t_n)\cdot B=(\varphi(s_n)-\varphi(t_n))\cdot B=(s_{n+1}-t_{n+1})\cdot B.$$

In virtue of the Kantorovich theorem 3 there is $\lim_{n\to\infty}t_n=t^*=\lim_{n\to\infty}s_n$, and hence $x^*=\lim_{n\to\infty}x_n=y^*$.

4. Implicit function theorem. The results obtained are applied at present for proving a few theorems on implicit function.

Further by $E_1\times E_2$ we are going to denote the L-space with a carrier $E_1\times E_2$ and a class of convergent sequences distinguished in the following way

$$(x_n\to x_0)\Leftrightarrow(x_n^1\to x_0^1\ \text{in}\ E_1\ \text{and}\ x_n^2\to x_0^2\ \text{in}\ E_2).$$

Theorem 6. *Let E_1 and E_2 be two L-spaces and X_0 be the subset of E_2. Let us consider the transformations x_0: $E_1\to X_0$ and G: $(x_0(E_1)+X_0)\times E_1\to E_2$ and suppose that for each fixed $z\in E_1$ the mapping $G(\cdot,z)$: $x_0(z)+X_0\to E_2$ satisfies the conditions of theorems 4 and 5 with $D_{0(z)}:=x_0(z)+X_0$, $x_0:=x_0(z)$ and the function φ and the set B are independent of the choice of the point z. Let also $X_0\subset\bigcup_{s_0<t}(s_0\cdot B)$. Then there exists a uniquely defined function x: $E_1\to(x_0(E_1)+X_0)$ such that $G(x(z),z)=x(z)$. Let in addition x_0 and G be continuous mapping (in topological sense), then x is a continuous function that transforms E_1^0 into E_2^0.*

Proof: The existence and uniqueness of function x follows directly from theorems 4 and 5.

Now let us suppose that functions x_0 and G are continuous (in topological sence). Then one can show, by induction, that continuity of the function $x_n(z)$ implies the continuity of the mapping $x_{n+1}(z)=G(x_n(z),z)$ which is the superposition of continuous mappings. So x_n is a continuous function for every n.

At present let V be any neighbourhood of origin in E_2^0, then for n large enough (dependent on B and φ only, and independent of z) by virtue of (3.3), assumptions on the set B, there holds the following $x_n(z)-x(z)\in V$.

So x is a continuous function as a limit of uniformly convergent sequence of continuous functions.

At present, employing the mean-value theorem, we will state other versions of theorem 6. We start with introducing some new notions.

Similarly as in § 3, we assume that $J \subset \mathbb{R}$ is an interval of the form $[0, a]$, $[0, a)$ or $[0, +\infty)$. The set $S=[y, z]$ of $x \in E$, such that $x=y+\lambda(z-y)$, where $0 \leq \lambda \leq 1$, $\lambda \in \mathbb{R}$, we are going to call interval.

D e f i n i t i o n 7. *Let the interval $S \subset E$ that contains points x and y, be given. Let, further the functions $G: E \to E$ and $\varphi: J \to J$ be given, such that the mapping $f: [0, 1] \to E$ defined by equality $f(\lambda)=G(x+\lambda(y-x))$ and the function $\psi: [0, 1] \to \mathbb{R}$ defined by $\psi(\lambda)=\varphi(p+\lambda(t-p))$ satisfy the assumptions of mean-value theorem 1, while*

$$(t-p, t, p) \in Q=J \times J \times J, \quad y-x_0 \in t \cdot B_{x_0},$$
$$y-x \in (t-p) \cdot B_{x_0}, \quad x-x_0 \in p \cdot B_{x_0}$$

(where x_0 is a given point in E and B_{x_0} is dependent on x_0 convex, bounded and closed subset of E^0). Then the function φ is called the B_{x_0} majorant of the mapping G over interval S.

From the mean value theorem 1 for the B_{x_0} majorant of the mapping G over interval $[x, y]$ we conclude that $G(y)-G(x)=f(1)-f(0) \in (\psi(1)-\psi(0)) \cdot B_{x_0}=(\varphi(t)-\varphi(p)) \cdot B_{x_0}$.

D e f i n i t i o n 8. *Let D_0 be some convex subset of the space E. If on every interval contained in D_0 the function $\varphi: J \to J$ is a B_{x_0} majorant of the mapping $G: D_0 \to E$, then we say that φ is a B_{x_0} majorant of the mapping G on D_0.*

Now from theorems 1, 4, 5 follows

T h e o r e m 7. *Let E_1 and E_2 be two L-spaces, and let X_0 be a convex set. Further, there are defined transformations*

$$x_0: E_1 \to X_0, \quad G: (x_0(E_1)+X_0) \times E_1 \to E_2$$

and the function $\varphi: J \to J$, that for every $z \in E_1$ is a B-majorant of the mapping $G(\cdot, z): x_0(z)+X_0 \to E_2$ over the set $x_0(z)+X_0$ (while the function φ and set B are independent of z and x_0). Let also $X_0 \subset \bigcup_{s_0<\hat{t}}(s_0 \cdot B)$. Then there exists a uniquely defined function $x: E_1 \to (x_0(E_1)+X_0)$ such that $G(x(z), z)=x(z)$.

If, in addition, x_0 and G are continuous functions (in a topological sense) then $x(z)$ is a continuous mapping.

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