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ON DISCRETE LINEAR OPERATORS IN THE FUNCTION SPACE A

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Summary. Answering a question of T. Kővári we prove that for any system of nodes $z_{k,n}(1\leq k\leq n,\ n=1, 2, \ldots, |z_{k,n}|=1)$ and of algebraic polynomials $p_{k,n}$ there exists a function f continuous on the closed unit disc and analytic in its interior so that the sequence of the functions

$$L_n f=\sum_{k=1}^{n} f(z_{k,n}).p_{k,n}$$

does not converge to f uniformly on the unit circle as n tends to infinity. Some other similar questions are treated as well.

1. Introduction. We denote by $T=\{z:|z|=1\}$ the complex unit circle and by $C=C(T)$ the Banach space of continuous functions defined on T. (C is endowed with the maximum norm.) The closed subspace A of C consists of the restrictions to T of the functions continuous on $\{z:|z|\leqq 1\}$ and analytic in $\{z:|z|<1\}=D$. The character z^k is denoted by $e_k(z)$ and the corresponding Fourier coefficient of $f\in C$ is

$$a_k(f)=\int_T e_{-k}.fd\lambda \quad (k=0,\ \pm 1,\ \pm 2, \ldots,\ d\lambda=\frac{d\arg z}{2\pi}).$$

Let $C_{j,k}$ denote for each pair of integers $j\leqq k$ the subspace of C linearly spanned by the set $\{e_m:j\leqq m\leqq k\}$. Instead of $C_{-k,k}$ and $C_{0,k}$ we use the notations C_k and A_k, respectively. C_0 denotes the set of all complex numbers as well.

If X and Y are subspaces of C, then $B(X, Y)$ denotes the normed space of bounded linear operators mapping X into Y. The norm of an $L\in B(X, Y)$ is $\|L\|=\sup\|Lf\|:f\in X,\ \|f\|\leqq 1\}$. For every $z\in T$ the functional $L(z)$ is defined on X as follows: $L(z)f=(Lf)(z)$.

It is known that there exist operator sequences $\{L_n\}_{n=1}^{\infty}$, where L_n is of the form

(1) $$L_n f=\sum_{k=1}^{n} f(z_{k,n}).p_{k,n};\quad z_{k,n}\in T,\ p_{k,n}\in C \text{ for } 1\leqq k\leqq n,\ n=1, 2, \ldots,$$

and

(2) $$\lim_{n\to\infty}\|L_n f-f\|=0$$

holds for all $f \in C$. (The operators of the form in (1) are usually called discrete.) For the well-known examples due to S. Bernstein $\|L_n f - f\| \leq d_1 . \omega(f, 1/n)$; $f \in C$, $n=1, 2, \ldots$ even with $p_{k,n}$ belonging to C_n. ($d_1, d_2, \ldots$ are absolute positive constants, $\omega(\delta, f) = \max\{|f(z_1)-f(z_2)| : z_1, z_2 \in T, |z_1 - z_2| \leq \delta\}$).

The algebraic polynomials form a dense set in A, so it is natural to ask whether there is a sequence $\{L_n\}_{n=1}^{\infty}$ of the type (1) in which the $p_{k,n}$ are algebraic polynomials of z and (2) holds for all $f \in A$. This question was raised by T. Kővári (see P. Turán [1], Problem XLV). We treat a more general problem.

We say that the operator $L \in B(X, Y)$ is determined on the set $H \subset T$ if $f, g \in X$, $f|_H = g|_H$ imply $Lf = Lg$. ($f|_H$ is the restriction of f to H.) Suppose that $\{H_n\}_{n=1}^{\infty}$ is a sequence of closed subsets of T. We ask if there is an $\{L_n\}_{n=1}^{\infty} \subset B(C, A)$ satisfying (2) for every $f \in A$, where L_n is determined on H_n $(n=1, 2, \ldots)$. In § 2 we prove that the answer to the latter question is yes if and only if $\lambda(H_n) > 0$ holds for each n large enough. In particular, for operator sequences having the representation (1) $H_n = \{z_{k,n} : 1 \leq k \leq n\}$ $(n=1, 2, \ldots)$ can be chosen, thus the answer to Kővári's question is negative. In § 3 we investigate an analogous problem concerning the pointwise convergence of function sequences $\{L_n f\}_{n=1}^{\infty}$.

2. On the Uniform Convergence. Theorem 1. *Let $H_n \subset T$ $(n=1, 2, \ldots)$ be closed sets of angular Lebesgue measure (a. L. m.) zero. Suppose that $L_n \in B(C, A)$ and L_n is determined on H_n; $n=1, 2, \ldots$. Then there is an $f \in A$ for which* (2) *does not hold.*

Proof. We begin with the observation

$$\|L_n\| = \|L_n|_A\|, \tag{3}$$

where $L_n|_A$ means the restriction of L_n to A. $\|L_n\| \geq \|L_n|_A\|$ follows from $A \subset C$. Suppose now that $f \in C$, $\|f\| \leq 1$. By the Rudin-Carleson theorem (see T. Gamelin [2], p. 58.) there exists a $g \in A$ satisfying $f|_{H_n} = g|_{H_n}$ and $\|g\| \leq 1$. Hence we have $\|L_n|_A\| \geq |L_n g| = |L_n f|$, and taking the supremum in f we obtain $\|L_n|_A\| \geq \|L_n\|$.

Suppose that (2) is true for all $f \in A$. Then the principle of the uniform boundedness and (3) yield

$$\|L_n\| < K; \quad n=1, 2, \ldots \tag{4}$$

with some $K > 0$ independent of n. It is a known fact that the norms of the functions f_m defined by

$$f_m(z) = \sum_{\substack{k=-m \\ k \neq 0}}^{m} k^{-1} z^k = 2i . \sum_{k=1}^{m} k^{-1} \sin(k . \arg z)$$

have a common upper bound d_2. So

$$\|f_{m,\varrho}\| \leq d_2; \quad m=1, 2, \ldots \tag{5}$$

holds for $f_{m,\varrho}(z) \stackrel{\text{def}}{=} f_m(\varrho . z)$ too, where $\varrho \in T$ is an arbitrary parameter. (4) and (5) imply

(6) $$\|L_n f_{m,\varrho}\| \leq d_2 . K; \quad n, m=1, 2, \ldots, \quad \varrho \in T.$$

L_n is linear, therefore we have the relation

$$L_n f_{m,\varrho} = \sum_{\substack{k=-m \\ k \neq 0}}^{m} k^{-1} \varrho^k . L_n e_k,$$

in particular

(7) $$g_{n,m}(z) = (L_n f_{m,1/z})(z) = \sum_{\substack{k=-m \\ k \neq 0}}^{m} k^{-1} z^{-k} . (L_n e_k)(z)$$

is valid for every m, n and $z \in T$. (6) and (7) yield

(8) $$\|g_{n,m}\| \leq d_2 . K$$

for the function

(9) $$g_{n,m} = \sum_{k=1}^{m} k^{-1}(e_{-k}) . L_n e_k + \sum_{k=1}^{m} (-k)^{-1} e_k . L_n e_{-k}.$$

Using (2) with $f=e_k$ we obtain

(10) $$\lim_{n\to\infty} a_0(e_{-k} . L_n e_k) = 1; \quad k=1, 2, \ldots,$$

furthermore $L_n e_{-k} \in A$ implies

(11) $$a_0(e_k . L_n e_{-k}) = 0; \quad k=1, 2, \ldots .$$

It follows from (9), (10) and (11) that $|a_0(g_{n,m})| > \log m$ holds for each fixed m and n large enough contradicting to (8) if $\log m > d_2 . k$. Thus Theorem 1 is proved.

Remarks. 1. After verifying (3), the proof can be completed in different ways. What we have to do is essentially contained in the proof of Rudin's theorem stating that there is no continuous linear projection of C onto A. (See [3] or K. Hoffman [4].) The above simple argument was suggested by G. Halász [5] and differs from my original one.

2. First I proved Theorem 1 for finite sets H_n only. I. Joó proposed to me to use the Rudin-Carleson theorem in the general case.

Theorem 2. *Let $H \subset T$ be a closed set of positive a. L. m. Then for every positive integer m there exists an operator $L_m \in B(C, A_{2m-1})$ determined on H satisfying*

(12) $$\|L_m f - f\| \leq 4 . E_m(f) + \frac{1}{m} . \|f\|; \quad f \in A,$$

where $E_m(f) = \inf\{\|f-g\| : g \in C_m\}$.

Corollary. *If the sequence $\{H_n\}_{n=1}^{\infty}$ consists of closed subsets of T and $\lambda(H_n) > 0$ holds for $n=1, 2, \ldots$ then there exists a sequence $\{L_n\}_{n=1}^{\infty} \subset B(C, A)$ so that L_n is determined on H_n; $n=1, 2, \ldots$ and (2) is true for each $f \in A$.*

The question answered in this Corollary was raised by I. Joó and we solved it together in the partial case $\lim_{n\to\infty} \lambda(H_n) = 1$.

Lemma 1. *Let $k \geq 0$ be an integer, $\varepsilon > 0$ furthermore H as in Theorem 2. Then there is an $F_{\varepsilon,k} \in B(C, C_0)$ determined on H satisfying*

$$(13) \qquad |F_{\varepsilon,k}f - a_k(f)| \leq \varepsilon \,.\, \|f\|$$

for all $f \in A$.

Proof. The functionals $F_{\varepsilon,0}(\varepsilon > 0)$ will be constructed by means of a method due to G. M. Golusin and V. I. Krilov (see [7], Ch. II, § 5.9). We define a step function u, on T as follows:

$$u(z) = \lambda(H)^{-1}(1 - \lambda(H)) \,.\, \log(1/\varepsilon), \quad \text{if} \quad z \in H,$$

$$(14) \qquad u(z) = -\log(1/\varepsilon), \quad \text{if} \quad z \in T \setminus H.$$

The Poisson integral of u is a real-valued, bounded and harmonic function $\tilde{u}$ in D. By a theorem of Fatou the radial limits of $\tilde{u}$ exist and equal u a.e. on T (see e. g. [4]). (14) implies $\tilde{u}(0) = 0$. We denote by $\tilde{v}$ the harmonic conjugate of $\tilde{u}(\tilde{v}(0) = 0)$ and put $\tilde{g} = \exp(\tilde{u} + i\tilde{v})$, $\tilde{g}$ is bounded and analytic in D, hence its radial limits define a bounded and measurable function g on T again by Fatou's theorem. Using $\tilde{g}(0) = 1$, we obtain

$$(15) \qquad a_0(f) = \int_T f \,.\, g d\lambda; \quad f \in A.$$

It is easy to verify that $|g| = \varepsilon$ holds a. e. on $T \setminus H$, thus (15) yields for each $f \in A$ $|a_0(f) - \int_H f \,.\, g d\lambda| \leq \|f\| \int_{T/H} |g| d\lambda \leq \|f\| \,.\, \varepsilon$. Hence $F_{\varepsilon,0}$ defined by $F_{\varepsilon,0}f = \int_H f \,.\, g d\lambda$ $(f \in C)$ satisfies (13) for each $f \in A$.

We construct $F_{\varepsilon,1}, F_{\varepsilon,2}, \ldots$ by recursion. Suppose that $k > 0$ and $F_{\varepsilon,k-1}$ has already been defined for every $\varepsilon > 0$. Then we put

$$(16) \qquad F_{\varepsilon,k}f = F_{\frac{\varepsilon}{3}, k-1}(e_{-1} \,.\, (f - F_{\delta,0}f)); \quad f \in C,$$

where $\delta = \delta(\varepsilon, k) \overset{\text{def}}{=} \varepsilon/3(\|F_{\varepsilon/3, k-1}\| + 1)$.

$F_{\varepsilon,k}$ belongs to $B(C, C_0)$ and is determined on H because $F_{\varepsilon/3, k-1}$ and $F_{\delta,0}$ have the same properties. We use in (16) the identity

$$(17) \qquad e_{-1} \,.\, (f - F_{\delta,0}f) = (a_0(f) - F_{\delta,0}f) \,.\, e_{-1} + e_{-1} \,.\, (f - a_0(f)).$$

From now up to the end of this proof $f \in A$ is always assumed. $|a_0(f) - F_{\delta,0}f| \leq \delta \,.\, \|f\|$, therefore

$$(18) \qquad |F_{\varepsilon/3, k-1}((a_0(f) - F_{\delta,0}f) \,.\, e_{-1})| \leq \|F_{\varepsilon/3, k-1}\| \,.\, \delta \,.\, \|f\| \leq (\varepsilon/3) \,.\, \|f\|$$

is true by the definition of δ. $e_{-1} \,.\, (f - a_0(f)) \in A$, $a_{k-1}(e_{-1} \,.\, (f - a_0(f))) = a_k(f)$ and $\|e_{-1} \,.\, (f - a_0(f))\| \leq 2\|f\|$ are valid, thus we have by the induction

$$(19) \qquad |F_{\varepsilon/3, k-1}(e_{-1} \,.\, (f - a_0(f))) - a_k(f)| \leq (\varepsilon/3) \,.\, 2\|f\|.$$

(16)-(19) imply (13) for each $f \in A$. Q. e. d.

Proof of Theorem 2. De la Vallée Poussin's theorem implies

$$(20) \qquad \|f - f_m\| \leq 4 \,.\, E_m(f) \quad (f \in A),$$

$$\text{where } f_m = \sum_{k=0}^{m} a_k(f) \,.\, e_k + \sum_{k=m+1}^{2m-1} m^{-1}(2m - k) \,.\, a_k(f) \,.\, e_k.$$

Lemma 1 yields in particular

(21) $$|a_k(f)-F_{1/2m^2,k}f|\leq\|f\|/2m^2;\quad 0\leq k<2m,\quad f\in A.$$

It follows from (20) and (21) that L_m having the representation

$$L_m f=\sum_{k=0}^{m}(F_{1/2m^2,k}f).e_k+\sum_{k=m+1}^{2m-1}m^{-1}(2m-k).(F_{1/2m^2,k}f).e_k;\quad f\in C$$

satisfies (12). Thus Theorem 2. is proved.

3. **On the Pointwise Convergence.** Let $V_n\in B(C, C_{2n-1})$ be defined as

(22) $$(V_nf)(z)=(1/3n).\sum_{j=0}^{3n-1}f(z_{j,n}).Q_n(z/z_{j,n}),$$

where $z_{j\,n}=\exp(2j\pi i/3n); 0\leq j<3n$ and $Q_n=\Sigma_{k=-n}^{n}e_k+\Sigma_{n<|k|<2n}(2n-|k|).e_k/n.$ V_n is a discrete version of the de la Vallée Poussin operator. J. Szabados [6] proved that $V_n|C_n=\mathrm{id}$, $\|V_n\|\leq 3$ and consequently

(23) $$\|V_nf-f\|\leq 4.E_n(f);\quad f\in C$$

are valid. Now we consider sequences in $B(C, A)$ in some sense similar to $\{V_n\}_{n=1}^{\infty}$.

Theorem 3. *Suppose that $\{L_n\}_{n=1}^{\infty}$ satisfies the conditions of Theorem 1, furthermore $L_n|A_n=\mathrm{id}$ and $\mathrm{Im}\,L_n\subset A_{Km}$ hold for $n=1, 2, \ldots$, where K is a positive integer independent of n. Then*

$$\lambda(\{z: z\in T,\ \sup_n\|L_n|_A(z)\|=+\infty\})>0.$$

Corollary. *There exists a divergent sequence $\{(L_nf)(z)\}_{n=1}^{\infty}$, where $f\in A$ and $z\in T$.*

Proof of Theorem 3. One can see that similarly to (3)

(24) $$\|L_n(z)\|=\|L_n|_A(z)\|;\quad n=1, 2, \ldots$$

holds for every $z\in T$. We quote

Lemma 2 (G. Halász, [5]). *Let $E\subset T$ be closed, $1>\delta^*>0$, $m>0$ an integer. Suppose that the polynomial*

$$g^*=1+\sum_{\delta^*m\leq j\leq m}b_j.e_j;\quad \forall b_j\in C_0$$

satisfies $|g^*|\leq 1/4$ on E. Then there exist positive numbers δ' and δ'' depending on δ^* and E only so that

(25) $$\lambda(\{z: z\in T,\ |g^*(z)|>\exp(m\delta'')\}|\geq\delta|$$

is true whatever g^* and m be.

We shall use the method of the proof of Theorem 1 in [5]. According to (24) we may assume

(26) $$\|L_n(z)\|<\Lambda;\quad z\in E,\quad n=1, 2, \ldots,$$

where $E\subset T$ is a closed set of positive a. L. m. and Λ is a positive number independent of n.

Let us consider the functions $f_{n,\varrho,\delta}=\Sigma_{\delta n\leq k\leq n}k^{-1}\varrho^k . e_k$, where the positive integer n, $0<\delta<1$, and $\varrho\in T$ are parameters. (5) implies the inequality $\|f_{n,\varrho,\delta}\|\leq 2d_2$ uniformly in n, ϱ and δ. Thus we have

$$(27) \qquad |(L_n f_{n,\varrho,\delta})(z)|\leq 2d_2 . \| L_n(z)\|; \quad z\in T$$

for all n, ϱ and δ. Using the fact that $L_n|_{A_n}=\mathrm{id}$ we obtain

$$L_n f_{n,\varrho,\delta}=\sum_{\delta n\leq k\leq n} k^{-1}\varrho^k . e_k+\sum_{-n\leq k\leq -\delta . n} k^{-1}\varrho^k . L_n e_k ,$$

which yields for each n and δ

$$(28) \qquad g_{n,\delta}(z)\overset{\text{def}}{=}(L_n f_{n,\,1/z,\,\delta})(z)=\sum_{\delta n\leq k\leq n} k^{-1}+\sum_{\delta n\leq k\leq n}(-k^{-1})z^k . (L_n e_{-k})(z)$$

(27) and (28) imply for all n and δ

$$(29) \qquad |g_{n,\delta}(z)|\leq 2d_2 . \| L_n(z)\|; \quad z\in T$$

and in particular by (26)

$$(30) \qquad |g_{n,\delta}(z)|\leq 2d_2 . \Lambda; \quad z\in E.$$

It follows from (28) and from the condition $IML_n\subset A_{K.n}$ that $g_{n,\delta}$ is an algebraic polynomial of degree not exceeding $(K+1)n$, furthermore $a_k(g_{n,\delta})=0$ holds for $1\leq k<\delta . n$. Now we choose and fix the number δ satisfying the inequality

$$(31) \qquad \log(1/\delta)>8d_2 . \Lambda$$

and put

$$g_n^*=g_{n,\delta}/a_0(g_{n,\delta})=g_{n,\delta} . (\sum_{\delta n\leq k\leq n} k^{-1})^{-1}.$$

It follows from (30) and (31) that $|g_n^*|\leq 1/4$ holds on E for n large enough, thus Lemma 2 can be applied with the above E, $\delta^*=\delta . (K+1)^{-1}$, $g^*=g_n^*$ and $m=(K+1) . n$. (25) yields

$$(32) \qquad \lambda(E_n')\geq\delta'; \quad n \text{ is large enough,}$$

with the set E_n' defined as

$$(33) \qquad E_n'=\{z : z\in T, \quad |g_n^*(z)|>e^{(K+1) . n . \delta''}\}.$$

Using (29), (33) and the definition of g_n^* we obtain

$$\| L_n(z)\|\geq\log(1/\delta) . e^{(K+1) . n . \delta''}/3d_2; \quad z\in E_n'$$

for n large enough, and hence by (32) the proof of Theorem 3 can be completed in a standard way.

Theorem 4. *Let $\{m_n\}_{n=1}^{\infty}$ be a sequence of integers satisfying $m_n\geq n$ $(n=1, 2, \ldots)$ and*

$$(34) \qquad \lim_{n\to\infty} m_n/n=+\infty.$$

Then there exists a sequence of discrete operators $\{L_n\}_{n=1}^{\infty}$ *having the properties* $L_n|_{A_n}=\mathrm{id}$ *and* $L_n \in B(C, A_{m_n})$ *and fulfilling*

$$\lim_{n\to\infty}(L_n f)(z)=f(z); \quad z\in T,\ f\in C. \tag{35}$$

Proof. We shall define L_n as a suitable modification of V_n described in (22). To perform this we need the following form of V_n:

$$V_{nf}=\sum_{k=-n}^{n} a_{k,n}(f)\,.\,e_k+\sum_{n<|k|<2n} n^{-1}(2n-|k|)\,.\,a_{k,n}(f)\,.\,e_k, \tag{36}$$

where

$$a_{k,n}(f)\overset{\text{def}}{=}1/3n\,.\sum_{j=0}^{3n-1} f(z_{j,n})\,.\,(z_{j,n})^{-k}; \quad k=0,\ \pm 1,\ \pm 2,\ldots.$$

The proof of Theorem 2 in [5] contains the following

Lemma 3. (G. Halász). *Suppose that* $0<\alpha<1/4$, *m and q are positive integers, furthermore* $0<q-m\leq d_3\,.\,\alpha^2 m$ *and* $\alpha>d_4\,.\sqrt{m^{-1}\log m}$ *hold. Then there exists such a* $p_q\in A_{m-1}$ *that*

$$\|e_q-p_q\|_{[\alpha,\,2\pi]}\overset{\text{def}}{=}\max_{\alpha\leq\arg z\leq 2\pi}|z^q-p_q(z)|\leq m^{-2} \tag{37}$$

is fulfilled.

It follows from (34) that $m_n>2n$ and $\alpha_n<1/4$ are valid for $n>n_0$ with

$$\alpha_n\overset{\text{def}}{=}2\,.\max\{d_4\,.\sqrt{m_n^{-1}\log m_n},\ \sqrt{2n/d_3\,.\,m_n}\}.$$

In case $n\leq n_0$ we put

$$L_n f=\sum_{k=0}^{n} a_{k,n}(f)\,.\,e_k; \quad f\in C. \tag{38}$$

In case $n>n_0$ we apply Lemma 3 with $\alpha=\alpha_n$, $m=m_n$ and $m_n<q<m_n+2n$.

According to (37), the polynomials $p_q\in A_{m_n}$ (depending on n as well) satisfy

$$\|e_q-p_q\|_{[\alpha_n,\,2\pi]}\leq(m_n)^{-2}; \quad m_n<q<m_n+2n. \tag{39}$$

A transformation in (39) yields

$$\|e_k-p_k^*\|_{[\alpha_n,\,2\pi]}\leq(m_n)^{-2}<1/2n^2; \quad -2n<k<0, \tag{40}$$

where $p_k^*(z)\overset{\text{def}}{=}z^{m_n}\,.\,\overline{(P_{-k+m_n}(z))}$. Let $L_n\ (n>n_0)$ be defined by

$$L_n f=\sum_{k=-n}^{-1} a_{k,n}(f)\,.\,p_k^*+\sum_{k=0}^{n} a_{k,n}(f)\,.\,e_k \tag{41}$$

$$+\sum_{k=n+1}^{2n-1} n^{-1}(2n-k)\,.\,(a_{k,n}(f)\,.\,e_k+a_{-k,n}(f)\,.\,p_{-k}^*); \quad f\in C.$$

It can be seen easily that $L_1, L_2, \dots$ are discrete operators. $a_{k,n}(f)$ equals zero if $f \in A_n$ and $k \notin [0, n]$, hence by (36), (38) and (41) $L_n|_{A_n} = V_n|_{A_n} = \text{id}$ holds for $n = 1, 2, \dots$. $Im L_n \subset A_{m_n}$ follows from $n \leqq m_n$ if $n \leqq n_0$ and from $2n < m_n$, $p_k^* \in A_{m_n}$ $(-2n < k < 0)$ if $n > n_0$. Using (36), (40), (41), and $|a_{k,n}(f)| \leqq \|f\|$; $-2n < k < 0$ we obtain

$$|(L_n f)(z) - (V_n f)(z)| \leqq \|f\| \cdot \sum_{k=-2n+1}^{-1} |p_k^*(z) - z^k| \leqq \|f\|/n;$$

$$f \in C, \quad n > n_0, \quad \alpha_n \leqq \arg z \leqq 2\pi.$$

(35) easily follows from (23), (42) and $\lim_{n\to\infty} \alpha_n = 0$. Thus Theorem 4 is proved.

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