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ESTIMATES OF AGMON-MIRANDA TYPE IN PLANE DOMAINS WITH CORNERS AND EXISTENCE THEOREMS

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Summary. The main purpose of the paper is to prove estimates

$$\| u \|_{C^{m-1}(G)} \leqq c \left[\| u \|^{\Gamma}_{m-1} + \| u \|_{C(G)} \right]$$

of Agmon-Miranda type for classical solutions of the Dirichlet problem for a uniformly strongly elliptic partial differential operator $\mathscr{L}$ of order $2m>2$ in bounded plane domains with corners. The estimate is proved under two conditions (A) and (B). It follows from estimates like $\| u \|_{C^{m-1}(G)} \leqq c [\| u - \tilde{u} \|_{W^m(\Omega)} + \| \tilde{u} \|_{C^{m-1}(G)}]$, which were proved by J. Nečas. The function $\tilde{u}$ is constructed by means of Poisson kernels in half-planes and by multiple layer potentials in the quadrant or its exterior. The estimates are applied to get Fredholm-like existence theorems for Dirichlet data. The condition (B) seems to be rather heavy and guarantees that an integral equation which arises from a multiple layer potential in the quadrant is uniquely solvable in L_∞ and other function spaces. The estimates hold at least for fourth order operators in domains which have right angles at the corners, if the principal part of the operator is Δ^2 near the corners.

0. Introduction. A linear partial differential operator

$$\mathscr{L} = \sum_{|\alpha| \leqq 2m} a_\alpha(x) D^\alpha$$

of order $2m$ is called *properly elliptic* on the closure G of a bounded domain $\Omega \subset R^2$, if for any two orthogonal vectors $\mathbf{n}$ and $\mathbf{t}$ in R^n the polynomial

$$\tau \longmapsto \mathscr{L}_0(x, \tau\mathbf{n} + \mathbf{t}) = \sum_{|\alpha| = 2m} a_\alpha(x)(\tau\mathbf{n} + \mathbf{t})^\alpha$$

has m zeros in the upper open complex half-plane as well as in the lower one. If the boundary Γ of Ω is a smooth $(n-1)$-dimensional manifold, then it is well-known that for a properly elliptic operator $\mathscr{L}$ and solutions $u \in C^{m-1}(G) \cap C^{2m}(\Omega)$ of the homogeneous equation $\mathscr{L}u = 0$ in Ω an a priori estimate

(1)
$$\| u \|_{C^{m-1}(G)} \leqq c \left[\| u \|^{\Gamma}_{m-1} + \| u \|_{C(G)} \right]$$

of Agmon-Miranda type holds (see [1, 8, 15, 18, 19]). In this estimate the summand $\| \cdot \|^{\Gamma}_{m-1}$ denotes

$$\| u \|_{m-1}^{\Gamma} = \sup_{|\alpha|<m} \| D^\alpha u \|_{C(\Gamma)} .$$

This semi-norm on $C^{m-1}(G)$ or $C^{m-1}(R^n)$ is equivalent to the semi-norm

$$u \longmapsto \sup_{0 \leq k < m} \| D_\nu^k u \|_{C^{m-1-k}(\Gamma)},$$

where $D_\nu^k u$ denotes the k-th order derivative of u in normal direction on the boundary. The estimate (1) is trivial for $m=1$, but we are interested in the case $m>1$.

There is a lot of a priori estimates in several function spaces for solutions of general elliptic boundary value problems. These estimates play a crucial role in proving existence theorems or some Fredholm theory. Usually function spaces like Hilbert spaces, spaces with L_p norms or of Hölder continuous functions are preferred to spaces C^k, since those function spaces have better properties than C^k has. To ask for the estimate (1) it seems, however, to be rather naturally and, furthermore, of some interest from a numerical point of view. For domains with corners and edges much less is known than for domains with a smooth boundary.

In this paper Agmon-Miranda type estimates will be proved for solutions of the Dirichlet problem for uniformly strongly elliptic equations in bounded plane domains with a piecewise smooth boundary, if two additional conditions are satisfied. The results are summarized in Sect. 3. Some details of the proofs are treated in [6].

1. The Idea of the Proof. For boundary values f which are induced by a function $F \in C^{m-1}(G)$ a function $\tilde{u} \in C^{m-1}(G)$ with the Dirichlet data f will be constructed in such a way that

$$\| \tilde{u} \|_{C^{m-1}(G)} \leqq c \| f \|_{m-1}^{\Gamma} \tag{2}$$

(several positive constants will be denoted by c). If the boundary values are induced by an F of $C^m(G)$, then the function $\tilde{u}$ even belongs to $W_p^m(\Omega)$ $(p>1)$, in particular to $W^m(\Omega) = W_2^m(\Omega)$ and satisfies

$$|(\mathscr{L}^*\varphi, \tilde{u})| \leqq c \| f \|_{m-1}^{\Gamma} \| \varphi \|_{W^m}, \quad \varphi \in C_0^\infty(\Omega). \tag{3}$$

Partial integrations in the L_2 scalar product $(\mathscr{L}^*\varphi, \tilde{u})$, in which $\mathscr{L}^*$ denotes the formally adjoint operator of $\mathscr{L}$, provide a sesquilinear form $B_{\mathscr{L}}$ on $W^m(\Omega)$ associated with $\mathscr{L}$. The operator $\mathscr{L}$ is called $\mathring{W}^m$ *elliptic*, if there is a positive constant c_0 such that

$$c_0 \| \varphi \|_{W^m}^2 \leqq | \mathscr{B}_{\mathscr{L}}(\varphi, \varphi) |, \quad \varphi \in C_0^\infty(\Omega). \tag{4}$$

The operator $\mathscr{L}$ on G is called *uniformly strongly elliptic,* if there is a positive constant c_0' such that $\operatorname{Re} \Sigma_{|\alpha|=2m} a_\alpha(x)\xi^\alpha \geqq c_0' |\xi|^{2m}$. Uniform strong ellipticity of $\mathscr{L}$ (with continuous coefficients in $\mathscr{L}_0$) is characterized by Gårding's inequality.

If $\mathscr{L}$ is $\mathring{W}^m$ elliptic, then for $\tilde{u} \in W^m(\Omega)$ there is exactly one variational solution u of $\mathscr{L}u = 0$ such that $u - \tilde{u} \in \mathring{W}^m(\Omega)$. Nečas [16, Chap. 7] proved that this u belongs to $C^{m-1}(G)$ and has the same Dirichlet data as $\tilde{u}$ if $\tilde{u} \in C^{m-1}(G)$. He also proved an estimate which was modified by Sändig [17] to

$$\| u \|_{C^{m-1}(G)} \leqq c[\| u - \tilde{u} \|_{W^m} + \| \tilde{u} \|_{C^{m-1}(G)}].$$

Now the estimates (2), (3) and (4) for $\varphi = u - \tilde{u}$ provide

$$\| u \|_{C^{m-1}(G)} \leqq c \| f \|^{\Gamma}_{m-1}.$$

If $\mathscr{L}$ is uniformly strongly elliptic, then there is a $\lambda > 0$ such that $\mathscr{L} + \lambda$ is $\mathring{W}^m$ elliptic because of the Gårding inequality. We already know that there is an $u_\lambda \in C^{m-1}(G) \cap W^m(\Omega)$ satisfying $\mathscr{L}u + \lambda u = 0$, $u_\lambda - \tilde{u} \in \mathring{W}^m(\Omega)$ and (5) with u_λ instead of u. If the Dirichlet data f are induced by a variational solution u of $\mathscr{L}u = 0$, then $v = u - u_\lambda \in \mathring{W}^m(\Omega)$ is a variational solution of $\mathscr{L}v + \lambda v = \lambda u$. Applying again results of [16], we obtain $v \in C^{m-1}(G)$ and

$$\| v \|_{C^{m-1}(G)} \leqq c[\| u \|_{W^{1-m}} + \| v \|_{W^m}] \leqq c_1 \| u \|_{C(G)}.$$

This estimate together with (5) provides the estimate (1) for $u = v + u_\lambda$.

Up to now we have proved (1) for solutions u of $\mathscr{L}u = 0$ in $C^m(G)$ only. The estimates $(1_\varepsilon) = (1)$ with respect to $\Omega_\varepsilon = \{x \in \Omega \colon \operatorname{dist}(x, \Gamma) > \varepsilon\}$ instead of Ω hold with a constant c independent of small positive ε. Thus the estimate (1) as the limit of (1_ε), $\varepsilon \to 0$, also holds for solutions u of $\mathscr{L}u = 0$ in $C^{m-1}(G)$.

The construction of $\tilde{u}$ is reduced to local considerations in a routine way (cf. [2] or [18]). We assume that the pair $(\Omega, \mathscr{L})$ satisfies the condition

(A) *For each corner* k *there is a neighbourhood* V_k *of* k *such that* $V_k \cap G$ *is a relative neighbourhood of* k *in a closed sector* Q_k *with its vertex at* k *and the principal part* $\mathscr{L}_0$ *of* $\mathscr{L}$ *in* $V_k \cap G$ *is an operator* $\mathscr{L}_k$ *with constant coefficients.*

This condition is rather technical and we hope that it can be eliminated by more careful estimates. The pair $(Q_k, \mathscr{L}_k)$ is transformed into an operator $\mathscr{L}^k$ either in the first quadrant Q^+ or in the exterior Q^- of Q^+ under a suitable affine transformation of R^2.

2. Poisson kernels in the quadrant Q^+ or its exterior Q^- for a homogeneous linear properly elliptic partial differential operator $\mathscr{L}$ with constant coefficients. Let $\mathscr{P}_1, \ldots, \mathscr{P}_m$ and $\mathscr{Q}_1, \ldots, \mathscr{Q}_m$ denote the Poisson kernels for $\mathscr{L}$ in the half-planes $y > 0$ or $x > 0$, respectively, which were explicitly given in [2]. These kernels provide the solution u of the Dirichlet problem for $\mathscr{L}$ in the corresponding half-plane. If one considers the m-tuple $\mathbf{u}$ of derivatives $D_x^{k-1} D_y^{m-k} u$ instead of u, one is led to $m \times m$ matrices $\mathbf{P}$ and $\mathbf{Q}$ of integrable rational functions satisfying $\int \mathbf{P}(t)dt = \int \mathbf{Q}(t)dt = \mathbf{I}$ and to the ansatz

$$\mathbf{u}(x, y) = \frac{1}{y} \int_0^\infty \mathbf{P}\left(\frac{s-x}{y}\right)\varphi(s)ds + \frac{1}{x} \int_0^\infty \mathbf{Q}\left(\frac{t-y}{x}\right)\psi(t)dt \tag{6}$$

for the solution of the Dirichlet problem in Q^δ. The right-hand side is well defined outside of the coordinate axes, if $\varphi, \psi \in L_\infty(R_+)^m$. By continuity

$\mathbf{u}$ is also defined on the negative coordinate half-axes. The *multiple layer potential* (6) is the m-tuple of $(m-1)$st order derivatives of a solution u of $\mathcal{L}u=0$ in Q^+ as well as in Q^-. The boundary conditions $\mathbf{u}(x, \delta 0)=f(x)$ and $\mathbf{u}(\delta 0, y)=g(y)$ provide a system of $2m$ integral equations

$$(7_\delta)\qquad \begin{aligned} &\varphi(x)+\delta \int_0^\infty \mathbf{Q}(t)\psi(tx)dt=\delta f(x),\\ &\delta \int_0^\infty \mathbf{P}(s)\varphi(sy)ds+\psi(y)=\delta g(y). \end{aligned}$$

An equation

$$(7)\qquad \Phi(x)+\int_0^\infty \mathbf{M}(t)\Phi(tx)dt=\Psi(x)$$

in $L_\infty(R_+)^l$ with $\mathbf{M}\in L_1(R, dt)^{l^2}$ is the adjoint equation of the convolution equation $F+\mathbf{M}_1 * F=G$ in $L_1(R_+, dt/t)$ with $\mathbf{M}_1(t):=t\mathbf{M}(t)$. The Mellin transform

$$f\longrightarrow \hat{f}(s+it):=\int_0^\infty x^{s+it-1} f(x)dx$$

plays the same role with respect to the multiplicative group R_+ of positive real numbers as the Fourier transform does with respect to the additive group R. According to a theorem of N. Wiener the equation (7) is uniquely solvable in $L_\infty(R_+)^l$ if and only if

$$(8)\qquad \det[\mathbf{I}+\widehat{\mathbf{M}}_1(it)]\neq 0 \quad \text{for all real } t.$$

If this condition is fulfilled, then there is an $\mathbf{R}$ in $L_1(R_+, dt)^{l^2}$ such that the solution of (7) in $L_\infty(R_+)^l$ is given by

$$(9)\qquad \Phi(x)=\Psi(x)+\int_0^\infty \mathbf{R}(t)\Psi(tx)dt.$$

We have

Proposition 1. *If* $\mathbf{M}=\begin{bmatrix} 0 & \mathbf{Q} \\ \mathbf{P} & 0 \end{bmatrix}$ *satisfies the condition* (8), *then there is exactly one solution* $(\varphi^\delta, \psi^\delta)\in L_\infty(R_+)^{2m}$ *of* (7_δ) *for each* $(f, g)\in L_\infty(R_+)^{2m}$. *If* f *and* g *are bounded and continuous and satisfy* $f(0)=g(0)=0$, *then the multiple layer potential* $\mathbf{u}$ *with the density* $(\varphi^\delta, \psi^\delta)$ *may be continuously extended onto the boundary of* Q^δ *by* f *and* g.

In the preceding section we used a weak regularity property of $\tilde{u}$, namely, that $F\in C^m(R^2)$ implies $\tilde{u}\in W^m(\Omega)$. This property corresponds to a regularity property of the multiple layer potential. This property will be guaranteed in the following under additional assumptions. We set

$$\mathcal{F}_{1,\alpha}:=\{f\in CB(\bar{R}_+)\cap C^1(R_+): f(0)=0, \quad \sup_x x^\alpha |f'(x)|<\infty\}.$$

Multiple layer potentials with densities in $\mathcal{F}_{1,\alpha}^{2m}$ belong to $W^1_{p,loc}(Q^\delta)^m$ for all $p\in[1, 1/\alpha[$. Thus our aim is to solve the equations (7_δ) for $(f, g)\in\mathcal{F}_{1,\alpha}^{2m}$ in $\mathcal{F}_{1,\alpha}^{2m}$. It is easy to see that an operator $\mathfrak{M}\varphi(x)=\int\mathbf{M}(t)\varphi(tx)dt$ with a kernel $\mathbf{M}$ in $L_1(R_+, dt)$ maps $\mathcal{F}_{1,\alpha}$ into itself, if the function $t^{1-\alpha}\mathbf{M}(t)$ on R_+ is integrable with respect to the Lebesgue measure dt. We have

Proposition 2. *If there is an* $\alpha_0 \in]1/2, 1[$ *such that*

$$(10) \qquad \det\begin{pmatrix} \mathbf{I} & \widehat{\mathbf{Q}}_1(it+s) \\ \widehat{\mathbf{P}}_1(it+s) & \mathbf{I} \end{pmatrix} \neq 0 \quad \textit{for all real } t \textit{ and } 0 \leqq s < \alpha_0,$$

then the function $\mathbf{R}$ *in* $L_1(R_+, dt)^{4m^2}$ *from above satisfies* $\int t^\alpha \|\mathbf{R}(t)\| dt < \infty$ $(0 \leqq \alpha < \alpha_0)$.

Proof. According to both (10) and the structure of $\mathbf{P}$ and $\mathbf{Q}$ the functions

$$\widehat{\mathbf{R}}_1(it+s) := -\begin{pmatrix} \mathbf{I} & \widehat{\mathbf{Q}}_1 \\ \widehat{\mathbf{P}}_1 & \mathbf{I} \end{pmatrix}^{-1} \begin{pmatrix} 0 & \widehat{\mathbf{Q}}_1 \\ \widehat{\mathbf{P}}_1 & 0 \end{pmatrix} (it+s)$$

of t on R belong to the Schwartz space $S(R)^{4m^2}$ for all fixed $s \in [0, \alpha_0[$. Thus their inverse Mellin transforms are

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{\mathbf{R}}_1(it+s) x^{-it} dt =: \mathbf{R}_{1,s}(x) \in L_1(R_+, dt/t)^{4m^2}.$$

Since $\widehat{\mathbf{R}}_1$ is analytic in the strip $0 < s < \alpha_0$ and continuous in the strip $0 \leqq s < \alpha_0$, we have $\mathbf{R}_{1,s}(x) = x^s \mathbf{R}_{1,0}(x) \in L_1(R_+, dt/t)^{4m^2}$. This is just the assertion of the proposition.

Now we are ready to formulate the second condition on the original operator $\mathscr{L}$ on G, namely, the condition

(B) *There is an* $\alpha \in]1/2, 1[$ *such that*

$$\det\begin{pmatrix} \mathbf{I} & \widehat{\mathbf{Q}}_1^{\mathrm{k}}(\sigma) \\ \widehat{\mathbf{P}}_1^{\mathrm{k}}(\sigma) & \mathbf{I} \end{pmatrix} \neq 0$$

for all complex σ *in the strip* $0 \leqq \operatorname{Re} \sigma < \alpha$ *and for all corners* k *of* G, where $\mathbf{P}^{\mathrm{k}}$ and $\mathbf{Q}^{\mathrm{k}}$ are associated with $\mathscr{L}^{\mathrm{k}}$ mentioned at the end *of* the preceding section in the same way as $\mathbf{P}$ and $\mathbf{Q}$ are associated with $\mathscr{L}$.

Of course, one would like to know something about the condition (B). It should be not so hard to check the condition in any special case eventually by computational methods, but up to now we have no general results. As an example we mention

Proposition 3. *The condition* (8) *is fulfilled by the biharmonic operator* Δ^2 *in any sector.*

Proof. For Δ^2 in a sector with an angle ω the determinant in (8) has the value

$$\left(1 + \frac{t^2[\sin(\pi-\omega)]^2 - t^2[\operatorname{sh}(\pi-\omega)]^2}{[\operatorname{sh}(\pi t)]^2}\right)^2 - 4t^2[\sin(\pi-\omega)]^2 \frac{[\operatorname{ch}[(\pi-\omega)t]]^2}{[\operatorname{sh}(\pi t)]^2}.$$

This expression can be written as $(a+b)(a-b)$, and the second factor can be written as $(a_1+b_1)(a_1-b_1)$ again. All factors are positive for $t>0$. The limit of the expression in $t=0$ is also positive.

Besides of checking the condition (B) the question arises, whether this condition is natural or not. These questions are left open. In [4] Poisson kernels for Δ^2 in the quadrant were constructed by means of Neumann series. The corresponding multiple layer potentials possess the weak regularity property we mentioned above. Therefore the assertions of the following theorems hold in the special case that (A) is satisfied, that the angles at the corners are right angles and that the principal part $\mathscr{L}_0$ is Δ^2 near the corners

3. A-priori Estimates and Existence Theorems. The proofs of the following two theorems were outlined in the preceding sections.

Theorem 1. *Let $\mathscr{L}$ be a properly elliptic linear partial differential operator of order $2m>2$ on a bounded plane domain Ω with closure G such that the pair $(\Omega, \mathscr{L})$ satisfies the condition* (A) *in Sect. 1 and* (B) *in Sect. 2. If $\mathscr{L}$ is $\mathring{W}^m$ elliptic, then there are a positive constant c and a small positive number ε_0 such that for each $\varepsilon \in [0, \varepsilon_0]$ and for each solution $u \in C^{m-1}(G_\varepsilon) \cap C^{2m}(\Omega_\varepsilon)$ of the equation $\mathscr{L}u=0$ in $\Omega_\varepsilon=\{x \in \Omega : \operatorname{dist}(x, \Gamma)>\varepsilon\}$ the estimate*

$$\| u \|_{C^{m-1}(G_\varepsilon)} \leqq c \| u \|^{\Gamma_\varepsilon}_{m-1}$$

holds.

Theorem 2. *Under the same assumptions as in the first theorem but for uniformly strongly elliptic operators $\mathscr{L}$ there are $c, \varepsilon_0>0$ such that for each $\varepsilon \in [0, \varepsilon_0]$ and each solution $u \in C^{m-1}(G_\varepsilon) \cap C^{2m}(\Omega_\varepsilon)$ of $\mathscr{L}u=0$ in Ω_ε the estimate*

$$\| u \|_{C^{m-1}(G_\varepsilon)} \leqq c[\| u \|^{\Gamma_\varepsilon}_{m-1} + \| u \|_{C(G_\varepsilon)}]$$

holds.

It is known that the space $\mathscr{N}=\{u \in \mathring{W}^m(\Omega) : \mathscr{L}u=0 \text{ in } \Omega\}$ is a subspace of $\mathscr{N}_{\mathfrak{cl}} := \{u \in C^{m-1}(G) \cap C^{2m}(\Omega) : \mathscr{L}u=0 \text{ in } \Omega,\ D^\alpha u=0 \text{ on } \Gamma\ (|\alpha|<m)\}$. We are not aware whether $\mathscr{N}=\mathscr{N}_{\mathfrak{cl}}$ or not. From the preceding theorem one obtains easily the

Corollary. *Under the assumptions of Theorem 2 the space $\mathscr{N}_{\mathfrak{cl}}$ of classical solutions of the Dirichlet problem of $\mathscr{L}u=0$ in Ω with vanishing Dirichlet data has finite dimension.*

In general the second summand of the right-hand side of the Agmon-Miranda type estimate cannot be omitted. We have, however,

Theorem 3. *Let $(\Omega, \mathscr{L})$ satisfy the assumptions of Theorem 2. If $\mathscr{N}_{\mathfrak{cl}}=(0)$, then there is a constant c' such that for solutions $u \in C^{m-1}(G) \cap C^{2m}(\Omega)$ of $\mathscr{L}u=0$ in Ω the estimate*

$$\| u \|_{C^{m-1}(G)} \leqq c' \| u \|^{\Gamma}_{m-1}$$

holds. Such an estimate also holds in the case $\mathscr{N}_{\mathfrak{cl}} \neq (0)$ for solutions in any fixed complementary subspace to $\mathscr{N}_{\mathfrak{cl}}$.

Let $\mathscr{N}^*$ denote the analogue of $\mathscr{N}$ for the formally adjoint operator $\mathscr{L}^*$ of $\mathscr{L}$. Since $\mathscr{L}$ is uniformly strongly elliptic, the space $\mathscr{N}^*$ has a finite dimension q^*. Let $\mathscr{D}_{m-1}$ denote the space of Dirichlet data for $2m$-th order operators under the norm $\|\cdot\|^{\Gamma}_{m-1}$.

Theorem 4. *Under the assumptions of Theorem 2 there are $q_1^* \leqq q^*$ linearly independent continuous linear functionals Ψ_p on $\mathscr{D}_{m-1}$ such that for given $f \in \mathscr{D}_{m-1}$ there is a classical solution of the Dirichlet problem*

$$\mathscr{L}u=0 \text{ in } \Omega \text{ and } (D^\alpha u \mid \Gamma)_{|\alpha|<m}=f \tag{11}$$

if and only if $\Psi_p(f)=0$ $(1 \leqq p \leqq q_1^*)$.

Proof. Let $\lambda>0$ be a number such that $\mathscr{L}+\lambda$ is $\mathring{W}^m$ elliptic. Then there is a unique solution $T_\lambda f \in C^{m-1}(G) \cap W^m(\Omega)$ of the Dirichlet problem for $\mathscr{L}+\lambda$ and boundary values f induced by $F \in C^{m-1}(R^2) \cap W^m(\Omega)$. If $v \in \mathring{W}^m(\Omega)$ is a solution of (12) $\mathscr{L}v=\lambda T_\lambda f$, then it belongs to $C^{m-1}(G)$ (cf. [16]) and $u=v+T_\lambda f \in C^{m-1}(G) \cap W^m(\Omega)$ is a solution of (11). The equation (12), however, has a solution if and only if the conditions $(w, T_\lambda f)=0$ $(w \in \mathscr{N}^*)$ are fulfilled. The linear functionals $\Phi_w(f):=(w, T_\lambda f)$ for $w \in \mathscr{N}^*$ can be continuously extended onto $\mathscr{D}_{m-1}$. The linear space $\{\Phi_w : w \in \mathscr{N}^*\}$ has a finite dimension $q_0^* \leq q^*$. Since the f which are induced by $F \in C^{m-1}(R^2) \cap W^m(\Omega)$ lie dense in $\mathscr{D}_{m-1}$, we find a biorthogonal system $\{\Psi_1, \ldots, \Psi_{q_0^*}\}$ and $\{g_1, \ldots, g_{q_0^*}\}$ of functionals $\Psi_p=\Phi_{w_p}$ and Dirichlet data g_p induced by $G_p \in C^{m-1}(R^2) \cap W^m(\Omega)$.

Let $f \in \mathscr{D}_{m-1}$ satisfy $\Psi_p(f)=0$ $(1 \leq p \leq q_0^*)$. The boundary values f can be approximated by boundary values f_j which are induced by $F_j \in C^{m-1}(R^2) \cap W^m(\Omega)$ and fulfil the conditions $\Psi_p(f_j)=0$ $(1 \leq p \leq q_0^*)$. Therefore solutions u_j of the Dirichlet problem for $\mathscr{L}$ and data f_j can be found, as it was described above. The u_j may be chosen in a fixed complementary subspace to $\mathscr{N}_{\mathrm{cl}}$ such that the estimate of Theorem 3 holds. Thus the u_j converge to a solution of (11).

Up to now we have constructed solutions by means of Hilbert space methods. It might be possible that there are boundary values $f=\Sigma_p c_p g_p$ for which (11) is classically solvable, but the solution cannot be approximated by solutions in $W^m(\Omega)$ in the described way. In this case the set of solvability conditions is to reduce to $q_1^* \leq q_0^* \leq q^*$ suitable equations $\Psi_p(f)=0$.

Remark. In the case of second order equations the identity $q_1^*=q^*$ holds (cf. [3, 20]). This identity would also hold in our case, if classical or generalized solutions possess the slight regularity property which was proved for multiple layer potentials under the condition (B) in the preceding section.

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