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INTERPOLATION BY QUADRATIC AND CUBIC SPLINES IN L_p

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Summary. The approximation of differentiable functions and their derivatives by means of interpolated quadratic and cubic splines on equidistant set in L_p metric, $1\leq p<\infty$, is considered. The received estimations are expressed by the modulus of continuity in L_p of the corresponding derivatives of the approximated function.

In this paper we shall consider an error of approximation of functions by quadratic and cubic interpolated splines on finite interval $[a, b]$ in L_p-metric, $1\leq p<\infty$. The equidistant set of knots will be considered: $x_i=a+ih$, $i=0, 1, \ldots, n$, $\bar{x}_i=x_i-h/2$, $i=1, 2, \ldots, n$, $h=(b-a)/n$, $\bar{x}_0=a$, $\bar{x}_{n+1}=b$.

The function S_2 is called interpolated quadratic spline of the function f, if:

a) S_2 is a polynomial of degree ≤ 2 in $[\bar{x}_i, \bar{x}_{i+1}]$, $i=0,1,\ldots, n$;

(1) b) $S_2\in C^1[a, b]$;

c) $S_2(x_i)=f(x_i)$, $i=0, 1, \ldots, n$.

Also the function S_3 is called interpolation cubic spline of the function f, if:

a) S_3 is a polynomial of degree ≤ 3 in $[x_i, x_{i+1}]$, $i=0, 1, \ldots, n-1$;

(2) b) $S_3\in C^2[a, b]$;

c) $S_3(x_i)=f(x_i)$, $i=0, 1, \ldots, n$.

The general results for error of approximation of bounded, p-integrable function by means of quadratic and cubic interpolated splines in L_p-metric are given in [1]:

Theorem A. *If the $(b-a)$-periodic function f is bounded and $f\in L_p$, then*

$$\|f-S_k\|_{L_p}\leq c\tau_{k+1}(f; h)_{L_p},$$

where S_k, $k=2, 3$, are defined in (1) *and* (2).

The modulus $\tau_k(f; \delta)_{L_p}$ in above estimation is determined in the following way [2]: $\tau_k(f; \delta)_{L_p}=\|\omega_k(f, \cdot; \delta)\|_{L_p}$, where $\omega_k(f, x; \delta)=\sup\{|\Delta_h^k f(t)|, t, t+kh\in[x-k\delta/2, x+k\delta/2]\cap[a, b]\}$.

From Theorem A and from some of the properties of $\tau_k(f; \delta)_{L_p}$ — [2], [3], [4], $\tau_k(f; \delta)_{L_p}\leq c(k)\delta\omega_{k-1}(f'; \delta)_{L_p}$, $\tau_1(f; \delta)_{L_p}\leq\delta\|f'\|_{L_p}$, $\tau_1(f; \delta)_L\leq 2\delta \bigvee_0^1 f$-series of corollaries can be obtained.

Our purpose is to find an estimation for $\|f^{(i)}-S_k^{(i)}\|_{L_p}$, $i=0,1,\ldots,k$, $k=2,3$, when $f^{(k)}\in L_p$ (the condition that $f^{(k-1)}$ is an absolutely continuous function will be provided always when we require that $f^{(k)}\in L_p$).

Lemma 1. *Let* $x_i+\Sigma_{j=1,\,j\neq i}^{n}a_{ij}x_j=y_i$, $i=1,2,\ldots,n$, *be a system of linear equations and*

$$\sum_{j=1,\,j\neq i}^{n}|a_{ij}|\leq q<1,\qquad \sum_{i=1,\,i\neq j}^{n}|a_{ij}|\leq q<1.$$

Then the following estimations $\|x\|_{l_p}\leq(1-q)^{-1}\|y\|_{l_p}$ *hold, where* $\|x\|_{l_p}=\{\Sigma_{i=1}^{n}|x_i|^p\}^{1/p}$.

Lemma 2. *Let the* $(b-a)$*-periodic function* f *belong to* L_p. *If we denote by* f_h

$$f_h(x)=h^{-1}\int_0^h f(x+t)dt$$

then $\|f-f_h\|_{L_p}\leq\omega(f;\,h)_{L_p}$, $\|f_h'\|_{L_p}\leq h^{-1}\omega(f;\,h)_{L_p}$, $f_h'=(f_h)'$.

This lemma is proved in more general case in [5], [6].

Lemma 3. *Let the function* f *be* $(b-a)$*-periodic and* $f^{(i)}\in L_p$, $i=1,2$. *Then* $\|S_2^{(i)}\|_{L_p}\leq c_i\|f^{(i)}\|_{L_p}$, $i=1,2$, *where* $c_1=6$, $c_2=4$ *and* S_2 *is defined by* (2) *and* $S_2^{(k)}(a)=S_2^{(k)}(b)$, $k=1,2$.

Proof. Let us denote $S_2'(x_i)=m_i$, $S_2''(x_i)=M_i$, $x_{i+1/2}=x_i+h/2$, $x_{i-1/2}=x_i-h/2$. First we shall consider the case $f'\in L_p$. Then

$$\text{(3)}\quad \|S_2'\|_{L_p}=\Big(\sum_{i=1}^{n}\int_{x_{i-1/2}}^{x_{i+1/2}}|S'(x)|^p dx\Big)^{1/p}=\Big(\sum_{i=1}^{n}\int_{x_{i-1/2}}^{x_{i+1/2}}\Big|\int_{x_i}^{x}S''(t)dt+m_i\Big|^p dx\Big)^{1/p}$$

$$=\Big(\sum_{i=1}^{n}\int_{x_{i-1/2}}^{x_{i+1/2}}|M_i(x-x_i)+m_i|^p dx\Big)^{1/p}\leq\Big(\sum_{i=1}^{n}\int_{x_{i-1/2}}^{x_{i+1/2}}|M_i(x-x_i)|^p dx\Big)^{1/p}$$

$$+\Big(\sum_{i=1}^{n}\int_{x_{i-1/2}}^{x_{i+1/2}}|m_i|^p dx\Big)^{1/p}\leq\frac{h^{1+1/p}}{2}\Big(\sum_{i=1}^{n}|M_i|^p\Big)^{1/p}+h^{1/p}\Big(\sum_{i=1}^{n}|m_i|^p\Big)^{1/p}.$$

The numbers m_i, M_i satisfy the following linear equations [7, p. 42, p. 63]:

$$\text{(4)}\quad \frac{M_{i-1}}{8}+\frac{6}{8}M_i+\frac{M_{i+1}}{8}=\frac{\Delta^2 f_{i-1}}{h^2},\ i=1,2,\ldots,n,\ M_0=M_n,\ M_1=M_{n+1},$$

and

$$\text{(5)}\quad \frac{m_{i-1}}{6}+m_i+\frac{m_{i+1}}{6}=\frac{2}{3h}(f_{i+1}-f_{i-1}),\ i=1,2,\ldots,n,\ m_0=m_n,\ m_1=m_{n+1}.$$

From Lemma 1 and (4) follows

$$\text{(6)}\quad \Big(\sum_{i=1}^{n}|M_i|^p\Big)^{1/p}\leq\frac{4}{h^2}\Big(\sum_{i=1}^{n}|\Delta^2 f_{i-1}|^p\Big)^{1/p}=\frac{4}{h^2}\Big(\sum_{i=1}^{n}\Big|\int_{x_i}^{x_{i+1}}f'(t)dt-\int_{x_{i-1}}^{x_i}f'(t)dt\Big|^p\Big)^{1/p}$$

$$\leq\frac{4}{h^2}\Big(\sum_{i=1}^{n}\Big(\int_{x_{i-1}}^{x_{i+1}}|f'(t)|\,dt\Big)^p\Big)^{1/p}\leq\frac{4}{h^2}\Big(\sum_{i=1}^{n}\Big(\Big(\int_{x_{i-1}}^{x_{i+1}}|f'(t)|^p dt\Big)^{1/p}$$

$$\times\Big(\int_{x_{i-1}}^{x_{i+1}}dt\Big)^{1/q}\Big)^p\Big)^{1/p}=8h^{1/q-2}\|f'\|_{L_p},\quad \frac{1}{p}+\frac{1}{q}=1.$$

From Lemma 1 and (5) we find similarly

(7)
$$(\sum_{i=1}^{n} |m_i|^p)^{1/p} \leqq \frac{1}{h}(\sum_{i=1}^{n} |f_{i+1}-f_{i-1}|^p)^{1/p}$$

$$= \frac{1}{h}(\sum_{i=1}^{n}(\int_{x_{i-1}}^{x_{i+1}} |f'(t)|\, dt)^p)^{1/p} \leqq h^{1/q-1} \| f' \|_{L_p}, \quad \frac{1}{p}+\frac{1}{q}=1.$$

From (3), (6) and (7) follows $\| S_2' \|_{L_p} \leqq 6 \| f' \|_{L_p}$. Let now $f'' \in L_p$. Then

(8)
$$\| S_2'' \|_{L_p} = (\sum_{i=1}^{n} \int_{x_{i-1/2}}^{x_{i+1/2}} |M_i|^p dx)^{1/p} = h^{1/p}(\sum_{i=1}^{n} |M_i|^p)^{1/p}.$$

Using again Lemma 1 we receive

(9)
$$(\sum_{i=1}^{n} |M_i|^p)^{1/p} \leqq \frac{2}{h^2}(\sum_{i=1}^{n} |\Delta^2 f_{i-1}|^p)^{1/p} = \frac{2}{h^2}(\sum_{i=1}^{n} | \int_{x_{i-1}}^{x_i} \int_{v}^{v+h} f''(u) du dv |^p)^{1/p}$$

$$\leqq \frac{2}{h^2}(\sum_{i=1}^{n}(\int_{x_{i-1}}^{x_i} \int_{x_{i-1}}^{x_{i+1}} |f''(u)|\, du dv)^p)^{1/p} \leqq 4h^{1/q-1} \| f'' \|_{L_p}, \quad \frac{1}{p}+\frac{1}{q}=1,$$

and (8), (9) give us $\| S_2'' \|_{L_p} \leqq 4 \| f'' \|_{L_p}$.

Lemma 4. *Let the function f be $(b-a)$-periodic and $f^{(i)} \in L_p$, $i=1, 2, 3$. Then the following inequality*

$$\| S_3^{(i)} \|_{L_p} \leqq c_i \| f^{(i)} \|_{L_p}, \quad i=1, 2, 3,$$

holds, where $c_1=11$, $c_2=12$, $c_3=18$ and S_3 is defined by (2) and periodic conditions $S_3^{(k)}(a)=S_3^{(k)}(b)$, $k=1, 2$.

Proof. We shall prove only the case $f''' \in L_p$. The other two cases can be proved in a similar manner. Let us denote $S_3'(x_i)=m_i$, $S_3''(x_i)=M_i$, $i=0, 1, \dots, n$. Then

(10)
$$\| S_3''' \|_{L_p} = (\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} |S_3'''(t)|^p dt)^{1/p} = h^{1/p-1}(\sum_{i=1}^{n} |M_i - M_{i-1}|^p)^{1/p}.$$

The numbers M_i satisfy the following system of linear equations [7, p. 84]: $M_{i-1}+4M_i+M_{i+1}=6\Delta^2 f_{i-1}/h^2, i=1, 2, \dots, n, M_0=M_n, M_{-1}=M_{n-1}, M_1=M_{n+1}$, hence

(11) $(M_i-M_{i-1})+4(M_{i+1}-M_i)+(M_{i+2}-M_{i+1})=6\Delta^3 f_{i-1}/h^2, \; i=0, 1, \dots, n-1.$

From Lemma 1 and (11) we have

$$(\sum_{i=1}^{n} |M_i-M_{i-1}|^p)^{1/p} \leqq \frac{3}{h^2}(\sum_{i=1}^{n} |\Delta^3 f_{i-2}|^p)^{1/p}$$

$$= \frac{3}{h^2}(\sum_{i=1}^{n} | \int_{x_{i-2}}^{x_{i-1}} \int_{u}^{u+h} \int_{v}^{v+h} f'''(x) dx dv du |^p)^{1/p} \leqq \frac{3}{h^2}(\sum_{i=1}^{n}(\int_{x_{i-2}}^{x_{i-1}} \int_{x_{i-2}}^{x_i} \int_{x_{i-2}}^{x_{i+1}}$$

$$\times |f'''(x)|\, dx dv du)^p)^{1/p} \leqq 6(\sum_{i=1}^{n}(\int_{x_{i-2}}^{x_{i+1}} |f'''(x)|\, dx)^p)^{1/p} \leqq 18h^{1/q} \| f''' \|_{L_p}, \quad \frac{1}{p}+\frac{1}{q}=1.$$

The last inequality and (10) give $\| S_3''' \|_{L_p} \leq 18 \| f''' \|_{L_p}$.

Lemma 5. *Let f be $(b-a)$-periodic function and $f''' \in L_p$. Then the following inequality $\| f'' - S_2'' \|_{L_p} \leq 5h \| f'' \|_{L_p}$ holds, where S_2 is defined by* (2) *and the periodic conditions $S_2^{(k)}(a) = S_2^{(k)}(b)$, $k=1, 2$.*

The proof of this lemma will be omitted. In fact this lemma is proved in [7, p. 73] in case p-2. In general case the proof is the same replacing everywhere 2 by p and using Lemma 1.

Lemma 6. *Let f be $(b-a)$-periodic function and $f^{(\mathrm{IV})} \in L_p$. Then the inequality $\| f''' - S_3''' \|_{L_p} \leq 10\, h \| f^{(\mathrm{IV})} \|_{L_p}$ holds, where S_3 is determined by* (2) *and the periodic conditions $S_3^{(k)}(a) = S_3^{(k)}(b)$, $k=1, 2$.*

Proof. Denoting $S_3''(x_i) = M_i$ we have

$$(12) \qquad \| S_3''' - f''' \|_{L_p} = \Big(\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} | f'''(x) - f'''_{i-1/2} + f'''_{i-1/2} - \frac{M_i - M_{i-1}}{h} |^p dx \Big)^{1/p}$$

$$\leq \Big(\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} | \int_{x_{i-1/2}}^{x} f^{(\mathrm{IV})}(t) dt |^p dx \Big)^{1/p} + h^{1/p} \Big(\sum_{i=1}^{n} | f'''_{i-1/2} - \frac{M_i - M_{i-1}}{h} |^p \Big)^{1/p}$$

$$\leq \Big(\sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} \Big(\Big(\int_{x_{i-1}}^{x_i} | f^{(\mathrm{IV})}(t) |^p dt \Big)^{1/p} \Big(\int_{x_{i-1}}^{x_i} dt \Big)^{1/q} \Big)^p dx \Big)^{1/p} + h^{1/p} \Big(\sum_{i=1}^{n} | f'''_{i-1/2} - \frac{M_i - M_{i-1}}{h} |^p \Big)^{1/p}$$

$$\leq h \| f^{(\mathrm{IV})} \|_{L_p} + h^{1/p-1} \Big(\sum_{i=1}^{n} | h f'''_{i-1/2} - (M_i - M_{i-1}) |^p \Big)^{1/p}.$$

Using (11) and Lemma 1 we receive

$$(13) \quad \Big(\sum_{i=1}^{n} | h f'''_{i-1/2} - (M_i - M_{i-1}) |^p \Big)^{1/p} \leq \frac{1}{2} \Big(\sum_{i=1}^{n} | 6 \frac{\Delta^3 f_{i-2}}{h^2} - h f'''_{i-3/2} - 4h f'''_{i-1/2}$$

$$- h f'''_{i+1/2} |^p \Big)^{1/p} = \frac{1}{2} \Big(\sum_{i=1}^{n} | 6h f'''(\xi_i) - h f'''_{i-3/2} - 4h f'''_{i-1/2} - h f'''_{i+1/2} |^p \Big)^{1/p}$$

$$= \frac{h}{2} \Big(\sum_{i=1}^{n} | \int_{x_{i-3/2}}^{\xi_i} f^{(\mathrm{IV})}(t) dt + 4 \int_{x_{i-1/2}}^{\xi_i} f^{(\mathrm{IV})}(t) dt + \int_{x_{i+1/2}}^{\xi_i} f^{(\mathrm{VI})}(t) dt |^p \Big)^{1/p}$$

$$\leq 3h \Big(\sum_{i=1}^{n} \Big(\int_{x_{i-2}}^{x_{i+1}} | f^{(\mathrm{IV})}(t) | dt \Big)^p \Big)^{1/p} \leq 9 h^{1+1/q} \| f^{(\mathrm{IV})} \|_{L_p}, \quad \xi_i \in [x_{i-2}, x_{i+1}], \quad \frac{1}{p} + \frac{1}{q} = 1.$$

From (12) and (13) follows the lemma.

Theorem 1. *Let f be $(b-a)$-periodic function and $f'' \in L_p$, $i=1, 2$. Then the following inequality*

$$\| f^{(i)} - S_2^{(i)} \|_{L_p} \leq 10\, h^{2-i} \omega(f''; h)_{L_p}, \quad i=0, 1, 2,$$

holds, where S_2 is a periodic, quadratic interpolated spline, defined by (1) *and the periodic conditions $S_2^{(k)}(a) = S_2^{(k)}(b)$, $k=1, 2$.*

P r o o f. Denoting $f_h(x)=h^{-1}\int_0^h f(x+t)dt$ it is clear that $f_h''' \in L_p$, $f_h''(x) = h^{-1}\int_0^h f''(x+t)dt$ and

(14) $\|S_2''(f)-f''\|_{L_p} \leq \|f''-f_h''\|_{L_p} + \|f_h''-S_2''(f_h)\|_{L_p} + \|S_2''(f_h)-S_2''(f)\|_{L_p}.$

From Lemmas 2, 3, 5, we obtain

$$\|f''-f_h''\|_{L_p} \leq \omega(f'';\ h)_{L_p},\quad \|f_h''-S_2''(f_h)\|_{L_p} \leq 5h\|f_h'''\|_{L_p} \leq 5\omega(f'';\ h)_{L_p},$$

$$\|S_2''(f_h)-S_2''(f)\|_{L_p} = \|S_2''(f_h-f)\|_{L_p} \leq 4\|f_h''-f''\|_{L_p} \leq 4\omega(f'';\ h)_{L_p}.$$

The last inequalities and (14) give

(15) $$\|S_2''-f''\|_{L_p} \leq 10\omega(f'';\ h)_{L_p}.$$

Now using (15) and well-known inequality [7, p. 75]

(16) $$\|S_2^{(i)}-f^{(i)}\|_{L_p} \leq h\|S_2^{(i+1)}-f^{(i+1)}\|_{L_p},\ i=0, 1,$$

we prove the theorem.

T h e o r e m 2. *Let f be $(b-a)$-periodic function and $f' \in L_p$. Then*

$$\|f^{(i)}-S_2^{(i)}\|_{L_p} \leq 27h^{1-i}\omega(f';\ h)_{L_p},$$

where S_2 is a periodic, quadratic, interpolated spline, defined by (1), *and the periodic conditions $S_2^{(k)}(a)=S_2^{(k)}(b)$, $k=1, 2$,*

P r o o f. Similarly to the proof of the previous theorem

(17) $\|f'-S_2'\|_{L_p} \leq \|f'-f_h\|_{L_p} + \|S_2'(f)-S_2(f_h)\|_{L_p} + \|S_2'(f_h)-S_2'(f)\|_{L_p}$

and

(18) $$\|f'-f_h'\|_{L_p} \leq \omega(f';\ h)_{L_p},$$

(19) $\|S_2'(f_h)-S_2'(f)\|_{L_p} = \|S_2'(f_h-f)\|_{L_p} \leq 6\|f_h-f'\|_{L_p} \leq 6\omega(f';h)_{L_p}.$

From Theorem 1 we obtain

(20) $$\|S'(f_h)-f_h'\|_{L_p} \leq 10h\omega(f_h'';\ h)_{L_p}.$$

As

(21) $$\omega(f_h'';\ h)_{L_p} \leq \frac{2}{h}\omega(f';\ h)_{L_p},$$

finally we get from (17)—(21)

$$\|S'(f)-f'\|_{L_p} \leq 27\omega(f';\ h)_{L_p},$$

and for the completion of the theorem the inequality (16) must be used.

T h e o r e m 3. *Let the function f be $(b-a)$-periodic and $f''' \in L_p$. Then the following inequality*

$$\|S_3^{(i)}-f^{(i)}\|_{L_p} \leq 29h^{3-i}\omega(f''';\ h)_{L_p},\quad i=0, 1, 2, 3,$$

holds, where the periodic, interpolated cubic spline S_3 is determined by (2) *and the periodic conditions $S_3^{(k)}(a)=S_3^{(k)}(b)$, $k=1, 2$.*

Proof. As

$$(22)\quad \| S_3''' - f''' \|_{L_p} \le \| f''' - f_h''' \|_{L_p} + \| f_h''' - S_3'''(f_h) \|_{L_p} + \| S_3'''(f_h) - S_3'''(f) \|_{L_p},$$

using Lemmas 2, 4, 6 we obtain

$$(23)\quad \| f''' - f_h''' \|_{L_p} \le \omega(f'''; h)_{L_p},$$

$$(24)\quad \| f_h''' - S_3'''(f_h) \|_{L_p} \le 10h \| f_h^{(IV)} \|_{L_p} \le 10\omega(f'''; h)_{L_p},$$

$$(25)\quad \| S_3'''(f_h) - S_3'''(f) \|_{L_p} \le \| S_3'''(f_h - f) \|_{L_p} \le 18 \| f_h''' - f''' \|_{L_p} \le 18\omega(f'''; h)_{L_p}.$$

From (22)—(25) follows $\| S_3'''(f) - f''' \|_{L_p} \le 29\omega(f'''; h)_{L_p}$ and for the completion of the proof the inequality $\| S_3^{(i)} - f^{(i)} \|_{L_p} \le h \| S_3^{(i+1)} - f^{(i+1)} \|_{L_p}$, $i=0, 1, 2$, must be used.

Theorem 4. *Let the function f be $(b-a)$-periodic and $f'' \in L_p$. Then*

$$\| S_3^{(i)} - f^{(i)} \|_{L_p} \le 71 h^{2-i} \omega(f''; h)_{L_p}, \quad i=0, 1, 2,$$

where S_3 is a periodic cubic spline, determined by (2) and the periodic conditions $S_3^{(k)}(a) = S_3^{(k)}(b)$, $k=1, 2$.

Theorem 5. *Let the function f be $(b-a)$-periodic and $f' \in L_p$. Then*

$$\| S_3^{(i)} - f^{(i)} \|_{L_p} \le 154 h^{1-i} \omega(f'; h)_{L_p}$$

where S_3 is defined as in Theorem 4.

The proofs of the two last theorems may be done in a similar way as in Theorem 2, using Lemmas 2, 4, 6.

In the case when the function f is not periodic and the interpolated spline is determined either by the conditions $S_k'(a) = A$, $S_k'(b) = B$ or by $S_k''(a) = A$, $S_k''(b) = B$, $k=2, 3$ (quadratic, cubic) the form of the estimations in the Theorems 1—5 remains valid, adding to the right side an additional term of the form Ch^{m-i}, where $C = C(A, B)$, m, i—integer.

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