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THE INTERSECTION OF JAKIMOVSKI METHODS WITH RIESZ AND NÖRLUND METHODS

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Summary. In this paper the intersection of Jakimovski methods with Riesz and Nörlund methods is investigated. It is proved that an Jakimovski method (F, d) cannot be also a Riesz method and that an (F, d) method is also a Nörlund method if and only if it s given by a matrix $A=(\alpha_{n\nu})$, where $\alpha_{nn}=c$, $\alpha_{n-1,n}=1-c$, $\alpha_{n\nu}=0$ elsewhere with a constant $c \in C$, $c \neq 0$.

Introduction. Let $d:=\{d_k\}_{k=1}^{\infty}$ be a sequence of complex numbers with $d_k \neq -1$ for all $k \in N$. The Jakimovski method (F, d) is generated by the triangular matrix $A=(\alpha_{n\nu})_{\nu=0,\ldots,n;\, n=0,1,\ldots}$ where the elements $\alpha_{n\nu}$ are defined by the relation $\alpha_{00}=1$, $\Pi_{k=1}^{n}(z+d_k)/(1+d_k)=:\Sigma_{\nu=0}^{n}\alpha_{n\nu}z^{\nu}$. The special case (F, c), where c is a constant, $c \neq -1$ leads to the Euler method $(E, (1+c)^{-1})$. Relations between Jakimovski methods and Euler methods have been studied by Jakimovski [2] and Karamata [3]. It has been proved by Lorch and Newman [4], that the family of Jakimovski methods, that are also Hausdorff methods, is exactly the family of Euler methods (E, r), $r \neq 0$. The intersection of Jakimovski methods and Gronwall methods has been characterized by Luh [5] (see also Bustoz and Wright [1]). It is the object of this note to characterize those Jakimovski methods, which are also Riesz methods or Nörlund methods.

Let $p=\{p_n\}_{n=0}^{\infty}$ be a sequence of complex numbers with $P_n:=\Sigma_{\nu=0}^{n}p_\nu \neq 0$ for all $n \in N_0$. The Riesz-method (R, p) is given by the matrix $B=(\beta_{n\nu})$, where

$$\beta_{n\nu}=\begin{cases} p_\nu/P_n & \text{if } 0\leqq\nu\leqq n, \\ 0 & \text{if } \nu>n, \end{cases} \qquad (n=0, 1, \ldots)$$

and the Nörlund method (N, p) is given by the matrix $C=(\gamma_{n\nu})$

$$\gamma_{n\nu}=\begin{cases} p_{n-\nu}/P_n & \text{if } 0\leqq\nu\leqq n, \\ 0 & \text{if } \nu>n \end{cases} \qquad (n=0, 1, \ldots).$$

The Intersection of (F, d) Methods with (R, p) and (N, p) Methods. We first study the question if there are Riesz methods which are also Jakimovski methods.

Theorem 1. *There is no Riesz method* (R, p), *which is also an Jakimovski method* (F, d).

Proof. Obviously Jakimovski methods have the following properties:

$$(1) \qquad \alpha_{n0} = \prod_{k=1}^{n} \frac{d_k}{1+d_k} \quad (n \geq 1),$$

$$(2) \qquad \alpha_{nn} = \prod_{k=1}^{n} \frac{1}{1+d_k} \quad (n \geq 1),$$

$$(3) \qquad \alpha_{n,\,n-1} = \alpha_{nn} \sum_{k=1}^{n} d_k \quad (n \geq 1).$$

Assume that $A = (\alpha_{n\nu})$ is also (F, d) and (R, p).

1. For $n \geq 1$ we have

$$0 \neq \frac{p_0}{P_n} = \alpha_{n0} = \prod_{k=1}^{n} \frac{d_k}{1+d_k}$$

and therefore $d_k \neq 0$ for $k \geq 1$.

2. a) If $n \geq 1$,

$$p_n = p_0 \frac{p_n}{P_n} \frac{P_n}{p_0} = p_0 \frac{\alpha_{nn}}{\alpha_{n0}} = p_0 \prod_{k=1}^{n} \frac{1}{d_k} \neq 0,$$

from which we obtain $p_n^{-1} p_{n-1} = d_n$ for $n \geq 2$, and since $p_0/p_1 = d_1$ we have

$$4) \qquad p_n^{-1} p_{n-1} = d_n \text{ for all } n \geq 1.$$

b) For $n \geq 1$ it follows from (3), that $p_n^{-1} p_{n-1} = \Sigma_{k=1}^{n} d_k$. Hence (4) implies $p_n^{-1} p_{n-1} = d_n = \Sigma_{k=1}^{n} d_k$, which is a contradiction to 1.

We next characterize those Jakimovski methods, which are also Nörlund methods. Let c be a fixed complex number. The method (Z, c) is defined by the matrix $Z = (z_{n\nu})$, where

$$z_{00} = 1, \quad z_{n\nu} = \begin{cases} c & \text{if } \nu = n, \\ 1-c & \text{if } \nu = n-1 \ (n \geq 1), \\ 0 & \text{otherwise.} \end{cases}$$

We now prove the following result.

Theorem 2. *A summability method is an* (F, d) *method and also an* (N, p) *method, if and only if it is a* (Z, c) *method with* $c \neq 0$.

Proof. 1. Given is a method (Z, c) with $c \neq 0$.

a) If we choose $p = \{p_n\}_{n=0}^{\infty}$ with $p_0 = 1$, $p_1 = (1-c)/c$, $p_n = 0$ for $n \geq 2$, then we have $(N, p) = (Z, c)$.

b) If we choose $d = \{d_k\}_{k=1}^{\infty}$ with $d_1 = (1-c)/c$, $d_n = 0$ for $n \geq 2$, then from $\Pi_{k=1}^{n}(z+d_k)/(1+d_k) = (1-c+cz)z^{n-1}$ we conclude that $(F, d) = (Z, c)$.

2. Assume that $A = (\alpha_{n\nu})$ is also (F, d) and (N, p). Then $\alpha_{00} = 1$ and because $P_n \neq 0$ for all $n \in N_0$ we obtain from (3) for $n \geq 1$ $p_1/P_n = (p_0/P_n)\Sigma_{k=1}^{n} d_k$ or $p_1 = p_0 \Sigma_{k=1}^{n} d_k$. Therefore we have $p_0 \neq 0$, $p_1 = p_0 \cdot d_1$ and $d_k = 0$ for $k \geq 2$.

Furthermore from $\alpha_{n0}=p_n/P_n=\Pi_{k=1}^{n} d_k/(1+d_k)$ we obtain $p_n=0$ for $n\geqq 2$. Hence the matrix A has the elements

$$\alpha_{00}=1,\ \alpha_{n\nu}=\frac{p_{n-\nu}}{P_n}=\begin{cases} 1/(1+d_1) & \text{if } \nu=n, \\ d_1/(1+d_1) & \text{if } \nu=n-1 \quad (n\geqq 1), \\ 0 & \text{otherwise.}\end{cases}$$

If we define $c:=(1+d_1)^{-1}$, then $c\neq 0$ and $A=(Z, c)$.

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