КОНСТРУКТИВНАЯ ТЕОРИЯ ФУНКЦИЙ'81
София, 1983, с. 349—350

CONSTRUCTIVE FUNCTION THEORY'81
Sofia, 1983, p. 349—350

MONOTONE INTERPOLATION

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Summary. In this paper the existence of a monotone algebraic polynomial is shown, which interpolates the function x^α $(0<\alpha<1)$ in the points $x_i=-1+i/n$, $i=0, 1, \ldots, 2n$, and the exact order of its degree is found, when $n\to\infty$.

Let $0=x_0<x_1<\cdots<x_n=1$, $0=y_0<y_1<\cdots<y_n\leq 1$. The problem for the monotone interpolation can be formulated as follows: We seek an algebraic polynomial $P\in H_m$ (H_m is the set of algebraic polynomials of degree not greater than m), for which

(1) $$P(x_i)=y_i, \quad i=1, \ldots, n,$$

(2) $$P(x) \text{ is monotonely increasing for } x\in[0, 1],$$

(3) an estimate of the number m has to be found depending on some information already known concerning the knots x_i and the interpolation values y_i.

Wolibner [1], Kammerer [2] and Young [3] prove the existence of a polynomial P satisfying (1) and (2) without giving an estimate of its degree from above.

Let $A=\max\{\Delta y_i:\ 1\leq i\leq n\}=\max\{(y_i-y_{i-1}):\ 1\leq i\leq n\}$, $B=\min\{\Delta y_i:\ 1\leq i\leq n\}$, $C=\min\{\Delta x_i:\ 1\leq i\leq n\}$. The papers [4, 5, 6] supply some estimates of the degree of depending on the characteristics A, B, C. In [4] the following estimate of the degree of P is made: $m\leq c_1 nA/B$, where c_1 is an absolute constant. In [5] an exact estimate for m is found whenever $A/B\asymp n^\alpha$, $\alpha\geq 1$.

In [7] one can find the following estimate for the degree of the monotone interpolation polynomial P:

(4) $$m\leq c_2 C^{-1}\ln(A/B+e),$$

where c_2 is an absolute constant. The estimate (4) is exact in the sense that there exist points x_i, y_i, for which $m\geq c_3 C^{-1}\ln(A/B+e)$; (4) implies the results contained in [4] and [5].

Let $\varphi(x)=|x|^\alpha \operatorname{sign} x$, $x\in[-1, 1]$, $0<\alpha<1$. The following result of Sendov [8] is well-known in the theory of Hausdorff approximations: For every positive integer n there exists $R\in H_n$ such that

(5) $$r(\varphi;\ R) \leq c_4(\alpha)n^{-1},$$

where $r(\varphi;\ R)$ is the Hausdorff distance between φ and R.

The aim of the present paper is the following

Theorem 1. *If* $x_i = -1 + i/n$, $i = 0, 1, \ldots, 2n$, *then there exists* $Q \in H_m$, $Q'(x) \geq 0$ *for* $x \in [-1, 1]$, *such that*

(6) $$Q(x_i) = \varphi(x_i), \quad i = 0, 1, \ldots, 2n,$$

(7) $$m \leq c_5(\alpha)n.$$

Theorem 1 easily implies the result (5) from [8]. Note that the estimate (4) does not yield Theorem 1, since (4) gives only $m \leq c_6(\alpha)n \ln n$, but it does not supply the estimate (7). This comes to show that probably estimate (4) might be modified by using another characteristics for the points x_i, y_i as well; so that for the classes of functions, containing the function $\varphi(x)$, it should give better estimates than (4) itself concerning the degree of the monotone interpolation polynomial.

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Received on June 7, 1981