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НЕРЕШЕННЫЕ ЗАДАЧИ • UNSOLVED PROBLEMS

КРИВЕНКОВ, Ю.П. I. Сформулировать и доказать в необходимой форме условия наилучшего приближения для следующих задач.

Задача А. Определить $Z(x) \in L_p(\Omega)$, где $p \geq 1$, Ω – ограниченная область пространства E^n, из условия:

$$min\left\{\|Z-f\|_{L_p(\Omega)} : h(x, Z(x)) \dot{\geq} 0\right\},$$

в котором $f(x) \in L_p(\Omega)$; $h(x, Z(x)) \in L_q(\Omega)$, $q \geq 1$; знак $\dot{\geq}$ выражает необходимость специального конструирования для этой задачи отношения полуупорядоченности в $L_q(\Omega)$.

Задача В. Определить $Z(x) \in W_p^r(\Omega)$, где $r \geq 1$; $p \geq 1$, Ω –ограниченная область пространства E^n, из условия:

$$min\left\{\|Z-f\|_{W_p^r(\Omega)} : h(x, z(x), \nabla z(x)) \dot{\geq} 0\right\},$$

в котором $f(x) \in W_p^r(\Omega)$; $h(x, z, \nabla z) = (h_1(x, z, \nabla z), \ldots, h_n(x, z, \nabla z))$, $h_i(x, z(x), \nabla z(x)) \in L_q(\Omega)$, $q \geq \mathcal{I}$;
знак $\dot{\geq}$ выражает необходимость специального конструирования для этой задачи отношения полуупорядоченности в произведения n пространств: $L_q(\Omega) \times \ldots \times L_q(\Omega)$.

II. На основе идей квазидифференциального исчисления разработать метод нахождения полиномов наименее уклонявщихсе от заданной функции на отрезке по равномерной норме – прямым квазидифференцированием критерия:

$$\min_{A} \max_{x} |Pn(x) - f(x)| .$$

СТЕЧКИН, С.Б. Верно ли, что в любом бесконечномерном банаховом пространстве существует напустое замкнутое множество единственности, которое не является чебышевским множеством?

BAISHANSKI B.M. Is it true that

$$\min\{\|x^c-\sum_1^l a_j x^{c_j}\|_{L_\infty(0,1)} \mid a_j, c_j \text{ real}, |c_j - c| \geq 1\}$$
$$= \min\{\|x^c[1-xP(\log x)]\|_{L_\infty(0,1)} \mid P \text{ polynomials of degree } \leq l-1\}?$$

It is known (see the author's paper in this volume) that the equality above holds if $0 \leq c \leq 1$.

CHENEY E.W. For each natural number n, let $\lambda(n)$ denote the minimal Lebesque constant for polynomial interpolation of degree n. Thus

$$\lambda(n) = \inf_{0 \leq x_0 < \cdots < x_n \leq 1} \sup_{0 \leq t \leq 1} \sum_{i=0}^{n} \prod_{\substack{j=0 \\ j \neq i}}^{n} |t-x_j| / |t_i - x_j| .$$

Prove that $\lambda(1) < \lambda(2) < \cdots$.

Let $S = T = [0,1]$, and let π_n denote the space of all polynomials of degree at most n. Is the subspace

$$C(S) \otimes \pi_m(T) + \pi_n(S) \otimes C(T)$$

proximinal in $C(S \times T)$? . Expressing it otherwise, we ask whether the double infimum

$$\inf_{x_i \in C(S)} \inf_{y_i \in C(T)} \sup_{s \in S} \sup_{t \in T} \left| f(s,t) - \sum_{i=0}^{m} x_i(s) t^i - \sum_{i=0}^{n} y_i(t) s^i \right|$$

is <u>attained</u> for each $f \in C(S \times T)$.

REFERENCE. M.V.Golitschek and E.W.Cheney, "The best approximation of bivariate functions by separable functions". Contemporary Mathematics, vol.21 (1983), 125-136. (Amer.Math.Soc.)

Let G be an n-dimentional Chebyshev subspace in $C = C([0,1])$. For any set of n nodes $0 \leq x_1 < \cdots < x_n \leq 1$ there exists a basis $\{g_1, \ldots, g_n\}$ for G such that

$$g_i(x_j) = \delta_{ij}$$

The corresponding interpolation operator is $Lf = \sum_{i=1}^{n} f(x_i) g_i$. The Lebesque function for L is

$$\Lambda(x) = \sum_{i=1}^{n} |g_i(x)| .$$

Prove the Erdös - Bernstein conjecture for this family of operators. In other words, prove that $\|\Lambda\|$ is a minimum if and only if the nodes are so distributed that $\Lambda(x) = \|\Lambda\|$ for one point in each subinterval $(x_1, x_2), \ldots, (x_{n-1}, x_n)$.

REFERENCE. T.A.Kilgore and E.W.Cheney, "A theorem on interpolation in Haar subspaces", Aequationes Mathematica 14 (1976), 391-400. MR 53 # 13924.

4.Let U and V be proximinal subspaces in a Banach space. If U+V is closed, does it follow that U+V is proximinal?

CIESIELSKI,Z. Let

$$C(k,p) = \sup\{\int_{-1}^{1} fg\,dx : \|f\|_{L^p_{[-1,1]}} \le 1, \|g\|_{L^q_{[-1,1]}} \le 1, f,g \in P_k\},$$

where P_k is the set of the polynomials of degree at most k, $1/p + 1/q = 1$, $1 \le p \le \infty$.

Find "good" estimates for C (k,p).

REMARK. Hölder's inequality implies C (k,p) $\le$ 1. For p=2 C (k,p) = 1.

DUNHAM,C.B. Consider approximation on compact spaces X with elemets x by Chebyshev sets which are not suns (author, Canadian Math.Bull.) What types of sets X are possible? What sorts of properties can the set of points x at which connection fails have? In the only published examples X is an interval with the point x of failure of connection an end point. This problem was suggested by the talk of Stechkin.

FEICHTINGER.H.G. Consider the family $\mathcal{J}=\{f | f \in L^1(\mathbb{R}^m), f \ge 0, \int f(x)dx = 1\}$ and set $d(f) := \|f - f * f\|_1$ for $f \in \mathcal{J}$.

Are there (simple) conditions implying that one has $d(f) \le d(f*f)$ (e.g.,does the inequality hold for even,decreasing functions)? Actually, one has equality if (and only if?)f is the density function of the normal distribution. Results in this direction might be helpful in obtaining information concerning the number

$$d_0 := \inf\{d(f), f \in \mathcal{J}\}.$$

(which for simple reasons cannot be smaller than $\frac{1}{4}$).

SENDOV.Bl. Let A be the set of all sequences $a = \{a_i\}_{i=0}^{\infty}$ of real numbers for which $a_0 = -1, a_1 = 1$ and $|a_i| \le 1; i = 2,3,4,\ldots$

Let $w_3(a) = \sup\{|\Delta_h^3 a_p| : a \in A,\ h = 1,2,3,\ldots, p = 0,1,2,\ldots\}$,

where $\Delta_h^3 a_p = a_{p+3h} - 3a_{p+2h} + 3a_{p+h} - a_p$.

The sequence $a^* \in A$ is extremal if $w_3(a^*) \le w_3(a)$ for every $a \in A$.

Is there an extremal sequence $a^* \in A$ such that the sequence $(a_1^*, a_2^*, a_3^*, \ldots)$ is periodic?

TAMRAZOV.P.M. Let μ be any function of the modulus of continuity type ($\mu(x) > 0 \ \forall x > 0$, μ semiadditive).

Let G be an open set in the complex plane $\mathbb{C}$, $\overline{G}$ be its closure on the closed complex plane $\overline{\mathbb{C}}$, and ∂G be the boundary of G in $\mathbb{C}$.

Let $f : \overline{G} \longrightarrow \mathbb{C}$ be a continuous function, holomorphic in G and satisfying the condition

$$|f(z)-f(\zeta)| \leq \mu(|z-\zeta|) \ \forall z, \zeta \in \partial G \tag{1}$$

The following problems are of great interest.

P.1. Prove the existence of $C < +\infty$ nondepending on z, ζ and such that

$$|f(z)-f(\zeta)| \leq C\mu(|z-\zeta|) \ \forall z, \zeta \in G \cup \partial G \tag{2}$$

The least C in (2) will be denoted by $C(G, f, \mu)$.

P.2. If P.1 has the positive solution then find sup $C(G, f, \mu)$ over all G , f , μ .

P.3. Let G be a simply connected domain. Then it is known that $C(G, f, \mu) \leq 108$. Find sup $C(G, f, \mu)$ over all f, μ and simply connected domains G .

P.4. (A.A. Gončar). Find sup $C(U, f, \mu)$ over all f, μ for the unit disk U (it is known to be > 1.)

TIKHOMIROV, V. Let $B_p^N = \{x = (x_1, x_2, \ldots, x_N) : \sum_{i=1}^{N} |x_i|^p \leq 1\}$. Find $d_n(B_p^N, l_q^N)$ for $p < q$, $(p, q) \neq (1, 2)$. It is known that $d_n(B_1^N, l_2^N) = \sqrt{\frac{N-n}{N}}$.