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ON THE INTERPOLATION OF SOME CLASSES OF OPERATORS ACTING IN FAMILIES OF BANACH SPACES

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The interpolation of operator ideals on Banach couples is considered by many authors - J. Peetre, G. Sparr, S.G. Krein, J.I. Petunin, A. Person, A. Pietach, H. Triebel, D. Gaspar, V.I. Ovchinnikov, etc. In this paper we are going to study the complex method of interpolation in families of Banach spaces with respect to some classes of operators.

There are two independent ways of construction of interpolation spaces for families of Banach spaces [1], [2]. We shall consider spaces E_t forming an interpolation family of Banach spaces, i.e. a) there exists a Banach space U such that $k(t)E_t \overset{1}{\subset} U$ for a measurable function $k(t)$ ($t \in [0,2\pi)$) for which $\int \log^+ k(t)P(\theta,t)dt < \infty$ (here $|\theta| < 1$, $P(\theta,t)$ is the Poisson kernel); b) for every $a \in \bigcap_t E_t$, $\|a\|_{E_t}$ is a measurable function. The space $\beta = \left\{ a \in \bigcap E_t, \int \log^+ \|a\|_{E_t} P(\theta,t) dt < \infty \right\}$ is called the log-intersection space [1]. This the absolute integrability of $\log k(t)$ holds. Using the positive Nevalinna class $N^+(D)$, where D is the open unit disk, the spaces $E[\theta]$ and $E\{\theta\}$ are constructed in [1]. The space $E[\theta]$ is continuously imbedded in U but the space $E\{\theta\}$ may fail to be contained in U. On the other hand $E[\theta]$ and $E\{\theta\}$ coincide with equality of norms in many cases. If $k(t) \le M$ the sum $\sum E_t$ is defined; let $\Delta E_t = \left\{ a \in \cap E_t, \sup \|a\|_{E_t} < \infty \right\}$. Using vector valued holomorphic in D functions two interpolation spaces E_θ and $E_{\theta,U}$ are constructed in [2] without the requirement b). Then $\Delta E_t \overset{1}{\subset} E_\theta$, $E_{\theta,U} \overset{1}{\subset} \sum E_t$.

Let $h(t)$ be a positive function on $[0,2\pi)$ such that $\log h(t) \in L_1$. For every $a \in \sum k(t)E_t$ the generalized Peetre k-functional is defined as in [3]:

$$K(h(t),a,E_t) = \inf_{\substack{a=\sum a_j \\ a \in E_{t_j} \\ \sum \|a_j\|_{E_{t_j}} k(t_j) < \infty}} \sum h(t_j)k(t_j)\|a\|_{E_{t_j}}.$$

We say that $E \in \bar{K}_\theta(E_t)$ if the Banach space E is normally imbedded in $\sum k(t)E_t$ and for every function h(t) from above the inequality

$$K(h(t),a,E_t) \leq C \exp(1/2\pi \int \text{long } h(t)k(t).P(\theta,t)dt)\|a\|_E \qquad (1)$$

holds.

In [3] it is proved that if $E \in \bar{K}_\theta(E_t)$ then for every Banach space F and linear operator $S: \sum k(t)E_t \to F$, for which $\|Sa\|_F \leq M(t)\|a\|_{E_t}$ $(a \in E_t)$ with $\log M(t) \in L_1$ it holds that

$$\|S/_E\|_{E\to F} \leq C \exp(1/2\pi \int \log M(t).P(\theta,t)dt).$$

If the function h(t) is bounded with positive constants we define $J(h(t),a,E_t) = \sup h(t)\|a\|_{E_t}$ for $a \in \Delta E_t$.

We say that the Banach space E belongs to the class $J_\theta(E_t)$ if $\Delta E_t \overset{1}{\subset} E$ and $\|a\|_E \leq C \exp(1/2\pi \int \log h^{-1}(t).P(\theta,t)dt).J(h(t),a,E_t)$ for $a \in \Delta E_t$.

The spaces $E[\theta]$ and $E\{\theta\}$, E_θ and $E_{\theta,U}$ belong to the class $J_\theta(E_t)$. In the case $k(t) \leq M$ the spaces E_θ and $E_{\theta,U}$ satisfy the inequality (1) for all bounded by positive constants functions h(t) - [4]. In this case we say that the space belongs to the class $K_\theta(E_t)$

Proposition: If the spaces $E\{\theta\}$ and $E[\theta]$ coincide with equality of norms then $kE[\theta] \in \bar{K}_\theta(E_t)$, where $k = \exp(1/2\pi \int \log k(t).\, P(\theta,t)dt)$.

Proof: Let $m \leq h(t)$ and $\log h(t)$ is integrable. Then $h(t)k(t)\|a\|_{E_t} \geq m\|a\|_U$, i.e. $h(t)k(t)E_t \overset{1}{\subset} mU$ and the space $\sum h(t)k(t)E_t$ is defined. We consider a new interpolation family - instead of the containing space U we take the space $U^1 = \sum h(t)k(t)E_t$ and $k^1(t) = h(t)k(t)$. If $E[\theta] = E\{\theta\}$ the space $E[\theta]$ is the completion of β in the norm $\|\cdot\|_{E\{\theta\}}$ which does n't depend on the containing space. We use the imbedding of $E[\theta]$ in U^1 and the inequality from [1] :

$$\|a\|_{U^1} \leq \exp(1/2\pi \int \log h(t)k(t)\, P(\theta,t)dt)\; k^{-1}k\, \|a\|_{E[\theta]}.$$

If $h(t) \geq m$ we have $K(h(t),a,E_t) \leq \|a\|_{\sum k(t)h(t)E_t}$ and hence the space $kE[\theta]$ satisfies the desired inequality.

Let now h(t) be an arbitrary function for which $\log h(t) \in L_1$. The inequality holds for functions $h_n(t) = \max(1/n, h(t))$. But $K(h(t),a,E_t)$

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$\leq K(h_n(t),a,E_t)$ and the proper passage to the limit ends the proof, since if $h(t)=1$ then $\|a\|_{\sum k(t)E_t} \leq k\|a\|_E$ for $a \in E$, i.e. $kE \overset{1}{\subset} \sum k(t)E_t$, $E = E[\theta]$.

In [5], [6] there are some theorems about interpolation in families of Banach spaces with respect to (r,p)-absolutely summing and (r,p)-order summing operators. Recall that the class $P_{r,p}$ of (r,p)-absolutely summing operators, introduced by Mitjagin and Pelczinsky consists of operators $S : E \to F$ (here E,F are Banach spaces, $r \geqslant p$) such that $(\sum_1^n \|Sx_k\|_F^r)^{1/r} \leq \varrho W_p\{x_k\}$ for every finite set $x_1,\dots,x_n$ in E. If in the right side of the inequality is $\varrho W_p\{|x_k|\}$ (in the case when E is a Banach lattice or at least KN-lineal) we shall call the operator (r,p)-summing (with respect to the cone) and write $S \in P^+_{r,p}(E,F)$. The inf of ϱ in the inequalities are the norms $\Pi_{r,p}(S)$ and $\Pi^+_{r,p}(S)$ respectively. Let now F be a Banach lattice, E - Banach space. The linear operator S is called "correct" operator ($S \in \Pi(E,F)$) if the set SU_E is order-bounded in F (U_E is the unit ball of E). It is known that $\Pi(E,F)$ is a Banach space with norm $\|S\|_{\Pi(E,F)} = \inf\{\|e\|,\ e \geqslant SU_E\}$.

Let A(E,F) be an operator ideal or another class of operators. After Pietsch [7] we say that $S \in A^d(E,F)$ if $S' \in A(F',E')$.

Let E_t be an interpolation family and let there exist a Banach space V such that $k^{-1}(t)E_t \overset{1}{\subset} V$. Further let the condition b) for the family E'_t be fulfilled. Let J_{E_t} denote the canonical injection of E_t into $\sum k(t)E_t = U$.

Theorem 1: Let $r(e^{it})$ be measurable function on $[0,2\pi)$, $0 < p \leq r(e^{it}) < \infty$ and $r^{-1}(\theta) = 1/2\pi \int r^{-1}(e^{it})\, P(\theta,t)dt$ for $|\theta| < 1$. Let $E \in \bar{K}_\theta(E_t)$, F - arbitrary Banach space, $S \in L(U,F)$. If $S \in P^d_{r(e^{it}),p}(E_t,F)$ and $M(t) \geqslant \Pi_{r(e^{it}),p}(S')_{F' \to E'_t}$ such that $\log M(t) \in L_1$ then $S \in P^d_{r(\theta),p}(E,F)$ and

$$\Pi_{r(\theta),p}(S')_{F' \to E'} \leq C \exp(1/2\pi \int \log M(t), P(\theta,t)dt)$$

(here C is the constant from the inequality (1)).

Proof: Since $S \in L(U,F)$, then $SJ_{E_t} \in L(E_t,F)$, $SJ_E \in L(E,F)$. Let $b \in F'$ then $S'b \in U$, $(SJ_{E_t})'b \in E_t'$, $(SJ_E)'b \in E'$. Actually $(SJ_{E_t})'b = S'b$ as elements of E_t' and $(SJ_E)'b = S'b$ as elements of E'. For every $\varepsilon > 0$ there exists an element $a \in U_E$ such that $\|S'b\|_{E'} \le (1+\varepsilon)|\langle S'b,a\rangle| = (1+\varepsilon)|\langle b,Sa\rangle|$. Since $E \subset \sum k(t)E_t$ there exists a representation $a = \sum a_j$, $a_j \in E_{t_j}$ so that $\sum \|a_j\|_{E_{t_j}} k(t_j) < \infty$. The convergence of the last series gives that $S\sum a_j = \sum Sa_j = \sum SJ_{E_{t_j}} a_j$ as elements of F. Therefore $\|S'b\|_{E'} \le$ $(1+\varepsilon)|\langle b,\sum SJ_{E_{t_j}} a_j\rangle| \le (1+\varepsilon)\sum|\langle b,SJ_{E_{t_j}} a_j\rangle| = (1+\varepsilon)\sum|\langle (SJ_{E_{t_j}})' b,a_j\rangle| \le$ $(1+\varepsilon)\sum\|S'b\|_{E_{t_j}'}\|a_j\|_{E_{t_j}}$. Let $h(t) = k^{-1}(t)\|S'b\|_{E_t'}$. From the inequalities $k(t)\|S'b\|_V \le \|S'b\|_{E_t'} \le M(t)\|b\|_{F'}$ the absolute integrability of the function $\log\|S'b\|_{E_t'}$ holds. Since $E \in \bar{K}_\theta(E_t)$ we obtain

$$\|S'b\|_E \le (1+\varepsilon)K(h(t),a,E_t) \le (1+\varepsilon)C\exp(1/2\pi\int\log\|S'b\|_{E_t'}P(\theta,t)dt) \quad (2)$$

Let $b_1,\dots,b_n \in F$, $c(e^{it}) = \{c_k(e^{it})\}_1^n = \{\|S'b_k\|_{E_t'}^p\}_1^n$. By the generalized Hölder's inequality [5]

$$l_{r(\theta)/p}(c(\theta)) \le \exp(1/2\pi\int\log l_{r(e^{it})/p}(c(e^{it}))P(\theta,t)dt)$$

where

$$c(\theta) = \{\exp(1/2\pi\int\log c_k(e^{it})P(\theta,t)dt)\}_1^n$$

Having in mind the inequality (2) we get $(\sum_1^n\|S'b_k\|_{E'}^{r(\theta)})^{1/r(\theta)} \le$

$$(1+\varepsilon)C(l_{r(\theta)/p}(\exp(1/2\pi\int\log\|S'b_k\|_{E_t'}^p P(\theta,t)dt))^{1/p} \le$$

$$(1+\varepsilon)C\exp(1/2\pi\int\log l_{r(e^{it})/p}(\|S'b_k\|_{E_t}^p)P(\theta,t)dt))^{1/p} =$$

$$(1+\varepsilon)C\exp(1/2\pi\int\log(\sum_1^n\|S'b_k\|_{E_t}^{r(e^{it})})^{1/r(e^{it})}P(\theta,t)dt) \le$$

$$(1+\varepsilon)C\exp(1/2\pi\int\log M(t)P(\theta,t)dt)\,w_p\{b_k\}.$$

Hence $S' \in P_{r(\theta),p}(F',E')$ and the corresponding estimate for $\pi_{r(\theta),p}(S')$ is true, since ε is arbitrary positive number.

Let us note that a quite similar result holds for the operators of class $P^+_{r,p}$. From that result we shall get a corollary.

Another note: Let $k(t) \le M$, we may assume that $k(t) \equiv 1$. If there exists a bounded measurable function $M(t) \geqslant \Pi_{r(e^{it}),p}(S')$ then the condition $E \in \bar{K}_\theta(E_t)$ may be replaced by the condition $E \in K_\theta(E_t)$.

Lemma: Let E be normed space, F - Banach lattice whose norm is additive with respect to the positive summands. Let $S \in L(E,F)$. If $S' \in P^+_{1,1}(F',E')$ then $S \in \Pi(E,F)$ and $\|S\|_{\Pi(E,F)} \le \Pi^+_{1,1}(S')_{F' \to E'}$.

Before the proof we shall mention some facts about the decomposition on components. Let L be a component of F, $L^\circ = \{f \in F', f(x)=0 \text{ if } x \in L\}$. It is not difficult to prove that L° is a component of F': if $g \in L^\circ$ and $|f| \le |g|$, then $f \in L^\circ$; if $A \subset L^\circ$, $a = \sup A \in F'$ then $a \in L^\circ$. Further if P is a projector on L, then P' is a projector on $(L^d)^\circ$. Finally, let P_n be projectors on components F_n which form a decomposition of F, i.e. $J = P_1 + \ldots + P_n$, $P_i P_k = 0$ if $i \ne k$. Then $J' = P'_1 + \ldots + P'_n$ and $P'_i P'_k = (P_k P_i)' = 0$ if $i \ne k$, which together with the above mentioned gives that the components $(F^d_n)^\circ$ form a decomposition of F'.

Proof of the lemma: As F is a KB-space then $S \in \Pi(E,F)$ if $\|S\|_{\Pi(E,F)} = \|S\|^v_b = \sup \| \, |Sx_1| \vee \ldots \vee |Sx_k| \, \|_F < \infty$ chere sup is taken over all finite sets $d_1, \ldots, x_n \in U_E$, Let us denote by Pr_k the projector on the component of F on which $|Sx_k|$ is the max of all. We can say that Pr_k are disjoint without losing generality. Then $\| \max |Sx_k| \, \|_F = \| \sum Pr_k |Sx_k| \, \|_F = \sum \| Pr_k |Sx_k| \, \|_F = \sum \| Pr_k Sx_k \|_F$ since $|Pry| = Pr|y|$. Then there exist $f_k \in F'$, $\|f_k\|_{F'} \le 1$ such that $\sum \| Pr_k Sx_k \|_F = \sum | \langle Pr_k Sx_k, f_k \rangle |$. On the other hand, the last sum is equal to $\sum | \langle x_k, S' Pr'_k f_k \rangle |$ which does not exceed $\sum \| S\, Pr_k f_k \|_{F'} \le \Pi^+_{1,1}(S')$. $\sup_a \sum \langle |Pr_k f_k|, a \rangle = \Pi^+_{1,1}(S') \| \sum |Pr'_k f_k| \|_{F'}$. Since the norm of F is additive, then F' is a K-space of bounded elements and we have $|f| \le \|f\|_{F'} \cdot 1$ and as $\|f_k\|_{F'} \le 1$ then $|Pr'_k f_k| = Pr'_k |f_k| \le 1$. As we have said above $Pr_k f_k$ are disjoint and therefore $\sum_1^n |Pr'_k f_k| = |\sum_1^n Pr'_k f_k| \le 1$ and $\| \sum |Pr'_k f_k| \, \|_{F'} \le 1$. That gives the inequality $\| \max |Sx_k| \, \|_F \le \Pi^+_{1,1}(S')$ by $\|x_k\| \le 1$. Taking sup over all finite sets of $x_k \in U_E$ we obtain that S if a "correct" operator and $\|S\|_{\Pi(E,F)} \le \Pi^+_{1,1}(S')$.

Corollary: Let the family of Banach spaces E_t be as in theorem 1, let $S \in L(U,F)$, $E \in \bar{K}_\theta(E_t)$. Suppose either 1) F is a Banach lattice and reflexive Banach space, or 2) F is a Banach lattice with norm additive with respect to the positive summands. If $S \in \Pi(E_t,F)$ and there exists a function $M(t) \geq \|S\|_{\Pi(E_t,F)}$ such that $\log M(t) \in L_1$, then $S \in \Pi(E,F)$ and

$$\|S\|_{\Pi(E,F)} \leq C \exp(1/2\pi \int \log M(t)\, P(\theta,t)\,dt),$$

Proof: By lemma 5 [8] we have that the operator S' will be summing (with respect to the cone), i.e. $S' \in P^+_{1,1}(F',E'_t)$ and $\pi^+_{1,1}(S') \leq M(t)$. By the variant of theorem 1 for operators summing with respect to the cond we have that $S' \in P^+_{1,1}(F',E')$ and

$$\pi^+_{1,1}(S') \leq C \exp(1/2\pi \int \log M(t)\, P(\theta,t)\,dt).$$

If we have condition 1), by lemma 4 [8] $S'' \in \Pi(E'',F'')$, i.e. $S \in \Pi(E'',F)$ and hence $S: E \to F$ will be correct as well (SU_E will be order bounded in F). Let we have the condition 2), by the lemma we get $S \in \Pi(E,F)$. The inequality $\|S\|_{\Pi(E,F)} \leq \pi^+_{1,1}(S')$ ends the proof in both cases.

Again, if $k(t)$ and $M(t)$ are bounded the corollary holds for E belonging to the class $K_\theta(E_t)$.

Theorem 2: Let E be normed space, F_t - interpolation family of Banach spaces which are KB-spaces. Let $S : E \to \Delta F_t$, $S \in \Pi(E,F_t)$. Let the KB-space F belong to the class $J_\theta(F_t)$ and let there exist a function $M(t) \geq \|S\|_{\Pi(E,F_t)}$ such that $\log M(t) \in L_1$, then $S \in \Pi(E,F)$ and

$$\|S\|_{\Pi(E,F)} \leq C \exp(1/2\pi \int \log M(t)\, P(\theta,t)\,dt).$$

Proof: Let $x_1,\dots,x_n \in U_E$ and $y = |Sx_1| \vee \dots \vee |Sx_n|$. Since ΔF_t is also a Banach lattice we have $y \in \Delta F_t$. As $F \in J_\theta(F_t)$ it follows the inequality [3]: $\|y\|_F \leq C \exp(1/2\pi \int \log \|y\|_{F_t} P(\theta,t)\,dt) \leq C \exp(1/2\pi \int \log M(t)\, P(\theta,t)\,dt)$. Taking sup over all finite sets $x_1,\dots,x_n \in U_E$ we end the proof.

Theorem 3: Let $S \in L(U,F)$, let $\Delta k(t)E_t$ be dense in every Banach lattice E_t. Let the Banach lattice E belong to the class $K_\theta(E_t)$ and E_t', E' be KB-spaces. Let S be summing operator from E_t into F and there exist a function $M(t) \geq \pi^+_{1,1}(S)_{E_t \to F}$, $\log M(t) \in L_1$. Then S will be summing operator from E onto F and

$$\pi^+_{1,1}(S)_{E \to F} \leq C \exp(1/2 \int \log M(t)\, P(\theta,t)dt).$$

Proof: As $\Delta k(t)E_t$ is dense in every E_t it is dense in $\sum k(t)E_t$ and hence E_t is dense in $\sum k(t)E_t$. Therefore from the imbedding $k(t)E_t \overset{1}{\subset} \sum k(t)E_t$ it holds that $(\sum k(t)E_t)' \overset{1}{\subset} (k(t)E_t)' = k^{-1}(t)E_t'$. Since S acts from $U = \sum k(t)E_t$, then S' acts from F' into $(\sum k(t)E_t)' \overset{1}{\subset} \Delta k^{-1}(t)E_t'$. It is clear that if $b_1,\dots,b_n \in U_F$ then $S'b_k = (SJ_E)' b_k \in E'$ and hence $y = |S'b_1| \vee \dots \vee |S'b_n| \in E'$. From the above $y \in \Delta k^{-1}(t)E_t'$ too. For every $\varepsilon > 0$ there exists an element $a \in U_E$ such that $\|y\|_{E'} \leq (1+\varepsilon)|\langle a,y\rangle|$. Since $E \overset{1}{\subset} \sum k(t)E_t$ there exists a representation $a = \sum a_j$, $a_j \in E_{t_j}$ and $\sum \|a_j\|_{E_{t_j}} k(t_j) < \infty$. Since $\sum |\langle a_j,y\rangle| \leq \sum \|a_j\|_{E_{t_j}} \|y\|_{E_{t_j}} \leq$

$\|y\|_{\Delta k^{-1}(t)E_t} \sum \|a_j\|_{E_{t_j}} k(t_j)$, $\quad |\langle a,y\rangle| \leq \sum |\langle a_j,y\rangle|$ we obtain

$\|y\|_{E'} \leq (1+\varepsilon) \sum \|a_j\|_{E_{t_j}} k(t_j) \|y\|_{E'_{t_j}} k^{-1}(t_j)$. Let $h(t) = \|y\|_{E_t'} k^{-1}(t)$, since

$\|y\|_V \leq h(t) \leq \|y\|_{\Delta k^{-1}(t)E_t'}$ and $E \in K_\theta(E_t)$, then $\|y\|_{E'} \leq (1+\varepsilon)K(h(t),a,E_t) \leq$

$(1+\varepsilon)C \exp(1/2\pi \int \log\|y\|_{E_t'} P(\theta,t)dt) \leq (1+\varepsilon)C \exp(1/2\pi \int \log M(t)\, P(\theta,t)dt)$

The last inequality holds because $\|y\|_{E_t'} \leq \|S'\|_{\Pi(F',E_t')} \leq M(t)$.

Hence S' will be a correct operator from F into E and $\|S'\|_{\Pi(F',E')} \leq C \exp(1/2\pi \int \log M(t)\, P(\theta,t)dt)$ (ε is arbitrary positive number). Let now $x \in E$, let us consider $\|Sx\|$, $Sx \in F$. There exists an element $b \in F'$, $\|b\|_{F'} \leq 1$ such that $\|Sx\| = |\langle Sx,b\rangle| = |\langle x,S'b\rangle| \leq \langle |x|, |S'b|\rangle = \|S'\|^V_b \cdot \langle |x|, \frac{|S'b|}{\|S'\|^V_b}\rangle$. Since E' is a KB-space $|S'b| \leq |S'|$, where the element $|S'|$ is the abstract norm of S' ([10], th.8.6.2.) and

$\| \, |S'| \, \|_{E'} = \| S' \|^{\vee}_{b}$. Hence $\sum \| Sx_k \| \leq \| S' \|^{\vee}_{b} \sup_{\substack{a \geq 0 \\ \|a\|_{E'} \leq 1}} \sum \langle |x_k|, a \rangle$, i.e. S is a summing operator from E into F.

As $\| S' \|^{\vee}_{b} = \| S \|_{\Pi(F', E')}$ we have the proper inequality for $\Pi^{+}_{1,1}(S)_{E \to F}$.

Let us note that theorem 1 is a generalisation of theorem 7.2 from [9].

References

1. R.R.Coifman, M.Cwikel, R.Rochberg, Y.Sagher, G.Weiss. A theory of complex interpolation for families of Banach spaces. Advances in Math. 43 (1982) 203-209.
2. S.G.Krein, L.J.Nikolova. Holomorphic functions in a family of Banach spaces and interpolation. Soviet.Math.Dokl. Vol.21 (1980) o.1.
3. L.J.Nikolova. On the classes K_Θ and J_Θ in the case of interpolation in a family of Banach spaces (to appear).
4. L.J.Nikolova. On the interpolation of operators of weakened type. Compl.analysis and appl. Varna, 1985.
5. L.J.Nikolova. Interpolation of (r,p)-absolutely summing operators acting in a family of Banach spaces. Youth sc.fo diff.eqv.Varna '86
6. L.J.Nikolova. On the complex of interpolation for family of Banach lattices, some classes of operators. Compl.an., appl. Varna 1987.
7. A.Pietsch. Operator ideals. Berlin 1978.
8. V.L.Levin. About two classes of linear operators acting between Banach spaces and Banach lattices. Sib.Math.J.Vol.10. No.4 (1969), 903-909.
9. D.Gaspar. Interpolation and operator ideals on Banach spaces. S.L.O.H.A. Univ.Timisoara, No.2 (1983).
10. B.Z.Vulich. Introduction in the theory of partially ordered spaces. Moscow, 1961.

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