80 years of Blagovest Sendov – not so many

by Andrey Andreev

If you allow me to use mathematical terminology, I would characterize Blagovest Sendov as a "many-sided" person. The proof of this claim is easy. Leaving aside his public activities like:

1995–1997	Speaker of the Bulgarian Parliament (it is important to point out that Sendov has not been a member of any political party);
2003–2009	Bulgarian Ambassador in Japan;
1988–1991	President of the Bulgarian Academy of Sciences;
1973–1979	Rector of the Sofia University (the oldest and the biggest university in Bulgaria);
1980–1985	President of the International Association of the Universities;
1988–1993	President of the International Federation for Information Processing (IFIP);
1988–1992	Member of the Executive Committee and the Board of Directors of the International Foundation for the Survival and Development of Humanity

(this list is by no way complete), Sendov is a "many-sided" person also in Mathematics for his outstanding research in Numerical Methods, Zeros of Polynomials, Interval Analysis, Mathematical Education, Mathematical Biology, etc. However, Sendov's main love in Mathematics is Approximation Theory. As every love, it is hardly amenable to explanation. One can think that this is because his first scientific paper "On a class of regular-monotone functions", *Dokl. AN SSSR* **110** (1956), no. 1, 27-30, was presented by the world known specialist in Approximation Theory S. N. Bernstein, or because of the deep influence over him of prominent mathematicians like A. Kolmogorov, S. Nikolskii, S. Stechkin, D. Menshov and N. Bakhvalov during his specialization in Numerical Mathematics at Moscow State University, 1960–1961. Probably, it was one of the best mathematical environments in the world at that time, and it was a question of a talent and abilities for one to be lifelong inspired. Undoubtedly, Sendov owned such mathematical abilities, which helped him to broad the knowledge he has already gotten as a student at Sofia University from his teachers N. Obreshkov, L. Iliev, L. Chakalov, Y. Tagamlitski.

Working on a problem set by Kolmogorov about the entropy of the space of continuous functions, Sendov came to the conclusion that the Hausdorff distance is the natural metric in the space of bounded functions. So this metric, introduced in 1914 by Felix Hausdorff in his book “Grundzüge der Mengenlehre”, found its real application and a new branch in Approximation Theory was born, the so-called Hausdorff Approximation of Functions. Using this metric, Sendov jointly with V. Popov proved series of new results, which generalize almost all classical direct and converse theorems in uniform and L_p approximations. The main source of information about Hausdorff approximation of functions is Sendov's book “Hausdorff Approximations”, Kluwer Acad. Publ., 1990.

Keeping to the spirit of this Proceedings, it should be said that Sendov is “many-sided” person also in Approximation Theory. Without any claim for completeness, let me mention some of his significant scientific achievements.

Hausdorff Approximation. As it was said above, Sendov is the father of this branch in Approximation Theory. The first important Sendov's result is that every bounded function on a finite interval can be approximated by polynomials of degree n with a rate $\frac{\log n}{n}$. The idea to introduce a parameter in the original definition of the Hausdorff distance allowed many results on uniform approximation of functions to be obtained as particular cases of their Hausdorff analogues. An example is the famous Jackson theorem from 1912, which gives the rate of the best uniform polynomial approximation.

Parametric Approximation. In 1971 Sendov proved that the rate of parametric approximation of $|x|$ on $[-1,1]$ is $(3+2\sqrt{2})^{-n}$, which is better than $e^{-c\sqrt{n}}$, the famous D. Newman's result about rational approximation from 1979, although both methods make use of two algebraic polynomials.

Whitney Constant. In 1957 H. Whitney proved that for every natural number n there exists a constant W_n such that for every bounded function $f : [a,b] \to \mathbb{R}$ there exists a polynomial P_n of degree n for which

$$\|f - P_n\| \le W_n\, \omega_n(f; (b-a)/n).$$

The initial guess about the magnitude of W_n was $W_n \approx const.\, n^n$. Sendov posed the brave conjecture that $W_n = 1$, and succeeded to reduce W_n to 6. What a constant is this W_n which changes during the time as

$$const.n^n \to const.\, n \log n \to const.\, n \to const. \to 6 \to 2.$$

Sendov's result $W_n = 6$ from 1986 was improved in 1995 by Y. Kryakin to $W_n = 2$.

The Famous Sendov Conjecture. In 1959 Blagovest Sendov, being at that time an Assistant Professor of Academician Obreshkov, told him a simple conjecture: if a polynomial $f(z) = (z - z_1)\cdots(z - z_n)$, $n \ge 2$, has all its zeros $z_1, \ldots, z_n$ inside the closed unit disk $|z| \le 1$, then every zero z_i is at a distance at most 1 from at least one zero of the derivative f'. Sendov's conjecture is so attractive because it generalizes two classical theorems, which consider the relationship between the locations of the zeros of a polynomial f

and its derivative f': Rolle's theorem from 1691 and the Gauss-Lucas theorem. Although this conjecture has been an object of more than 130 papers so far, it still remains open, in the general case for $n > 8$.

The Averaged Moduli of Smoothness. The book "The Averaged Moduli of Smoothness: Applications in Numerical Methods and Approximation", B. Sendov and V. Popov, Wiley, 1988, reveals in details why this new characteristic of functions is a natural tool for the error estimation in many numerical methods which use discrete information – Bernstein operators, quadrature formulas, Runge-Kutta's methods, etc.

At the age of 80 Sendov deserves to be proud of all that was said above and also with being:

- author of more than 200 papers and more than 30 textbooks
- supervisor of 11 PhD students, among them such outstanding mathematicians as Vasil Popov and Borislav Bojanov. Probably the best expression here is the words from Louis Armstrong's song

  "...oh when the Saints, go marching in.

  I wanna be,

  be in that number..."
- "discoverer" of one of the fathers of the modern computer, John Atanassov, putting the name of this American scientist with Bulgarian roots along with the names of Blaise Pascal, Gottfried Leibniz, Charles Babbage, and John von Neuman.

At the end, it is not indiscretion for one to say that Sendov can be two times more proud with the fact that his first PhD student and later his co-author and close friend, Vasil Popov, became "a brilliant young Bulgarian mathematician" (D. Newman's "definition" after V. Popov developed totally new method for rational approximation, which completely solved a problem for the rate of approximation by rational functions) and in 1996 The Vasil A. Popov Prize was established for distinguished research accomplishments in Approximation Theory.

In respect of the "age of Mathematics", Blagovest Sendov's 80 years are less than arbitrary $\varepsilon > 0$. Let me join his friends and colleagues in wishing him good health and joy of life, and lot of energy for his work on the challenging mathematical problems.