Investigation of critical frequencies for inhomogeneous cracked magneto-electro-elastic media

Yonko D. Stoynov

Faculty of Applied Mathematics and Informatics, Technical University of Sofia.

Abstract: Critical frequencies of the dynamic anti – plane problem for functionally graded magneto – electro – elastic (MEE) media, containing an internal crack are considered. The crack is assumed magneto – electrically impermeable and the medium is exponentially inhomogeneous. The mathematical model consists of a system of partial differential equations and boundary conditions. A functional transformation of the generalized displacement vector is used to obtain a problem with constant coefficients. The fundamental solution is found by the Radon's transform. A non-hypersingular traction boundary integral equation method (BIEM) is used for the numerical solution. The effects of the inhomogeneity parameters, which may vary in different directions , and the external loading are studied. Numerical results for the extended crack opening displacements (COD) are given.

Keywords: magneto – electro – elastic medium, anti – plane crack, BIEM, COD.
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1.Introduction

MEE materials have drawn considerable interest in recent years, because of the wide application in smart systems. They also possess magnetoelectric property, that is not present in the piezoelectric or piezomagnetic constituent [1-4]. Works are mainly focused on the static case and few studies are devoted to dynamic problems. Zhou et al. [5] have investigated the dynamic behavior of two collinear interface cracks subjected to time harmonic shear waves. Feng et al. [6] considered the dynamic response of an interface impermeable crack subjected to antiplane mechanical and inplane electromagnetic impacts. Li [7] made an analysis of a cracked MEE medium under antiplane mechanical and inplane electromagnetic impacts in time domain. Another efficient method is BIEM. Rojah Diaz et al. [8] investigated dynamic crack interactions in MEE medium under inplane mechanical and electromagnetic loading using hypersingular traction BIE.

Lately, structures consisting of several layers have been used in many products. This type of structures accumulate stress between the layers, which can cause failures in products made of such materials. Functionally graded materials (FGM), that have been developed by the researchers, don't have this disadvantage, because their material properties are continuous. They can reduce stress under mechanical or thermal loads [9]. Another important application of these materials is in up-to-date system for vibration control and health monitoring [4].

In order to improve strength of the piezoelectric devices functionally graded piezoelectric materials also have been developed [10]. Rangelov et al. [11] investigated exponentially inhomogeneous piezoelectric finite domain with an anti-plane shear crack. The problem is solved numerically using BIEM.

FGM have been investigated by reducing the problem to singular integral equations using integral transforms and then solve them numerically. Feng and Su [12]

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considered functionally graded magnetoelectroelastic strip with a crack perpendicular to the boundary. Chue and Hsu [9] studied an internal crack perpendicular to the edge of a MEE half-plane. Jun Liang [13] investigated two parallel cracks in functionally graded MEE materials.

The aim of this study is to investigate cracked functionally graded MEE materials with exponentially varying properties, subjected to time harmonic load with a critical frequency. BIEM is used for the numerical solution.

2. Statement of the problem

Let's consider a linear MEE medium poled in Ox_3 axis of a rectangular coordinate system $Ox_1x_2x_3$, subjected to an external antiplane mechanical, and inplane electrical and magnetic time-harmonic load. We assume the medium is transversely isotropic. The governing equations in this case are:

$$\left|\begin{array}{l}(c_{44}(x)u_{3,i})_{,i}+(e_{15}(x)\varphi_{,i})_{,i}+(q_{15}(x)\psi_{,i})_{,i}+\rho(x)\omega^2u_3=0\\(e_{15}(x)u_{3,i})_{,i}-(\varepsilon_{11}(x)\varphi_{,i})_{,i}-(d_{11}(x)\psi_{,i})_{,i}=0\\(q_{15}(x)u_{3,i})_{,i}-(d_{11}(x)\varphi_{,i})_{,i}-(\mu_{11}(x)\psi_{,i})_{,i}=0\end{array}\right. \qquad (1)$$

where c_{44} is the elastic module, e_{15} is the piezoelectric coefficient, q_{15} is the piezomagnetic coefficient, ε_{11} is the dielectric permittivity, μ_{11} is the magnetic permeability, d_{11} is the magnetoelectric coefficient, ρ is the density, u_3 is the coordinate of the displacement vector along Ox_3 , φ and ψ are electric and magnetic potential respectively, ω is the frequency of the applied time-harmonic load, $x=(x_1,x_2)$. The body forces, electric charges and current densities are zero. Here comma denotes differentiation and we assume summation for repeated indexes.

A generalized tensor of elasticity $C_{iJKl}(x), i,l=1,2;\ J,K=3,4,5$ is defined as:

$$C_{i33l}(x)=\begin{cases}c_{44}(x), i=l\\0, i\neq l\end{cases}, C_{i34l}(x)=C_{i43l}(x)=\begin{cases}e_{15}(x), i=l\\0, i\neq l\end{cases},$$

$$C_{i35l}(x)=C_{i53l}(x)=\begin{cases}q_{15}(x), i=l\\0, i\neq l\end{cases}$$

$$C_{i44l}(x)=\begin{cases}-\varepsilon_{11}(x), i=l\\0, i\neq l\end{cases},\ C_{i45l}(x)=C_{i54l}(x)=\begin{cases}-d_{11}(x), i=l\\0, i\neq l\end{cases},\ C_{i55l}(x)=\begin{cases}-\mu_{11}(x), i=l\\0, i\neq l\end{cases}.$$

It is supposed that the material constants depend in one and the same manner on *x*: $C_{iJKl}(x)=C_{iJKl}h(x)$ and $\rho(x)=\rho h(x)$, $h(x)=e^{2<a,x>}$, $a=(a_1,a_2)$, $<.,.>$ is the scalar product.

Let us denote $u_J=(u_3,\varphi,\psi)$ and $\sigma_{iJ}=(\sigma_{i3},D_i,B_i)$ - a generalized displacement vector and a generalized stress tensor respectively. The small indices vary 1,2 and the large 3, 4,5. Then the system (1) can be written in the following more concise way:

$$(C_{iJKl}(x)u_{K,l})_{,l}+\rho_{JK}(x)\omega^2u_K=0 \qquad (2)$$

We have denoted $\rho_{JK}(x)=\begin{cases}\rho(x), J=K=3\\0, J,K=4\, or\, 5\end{cases}$. The boundary condition for (2) is:

$$t_J \Big|_{\Gamma} = 0 \tag{3}$$

Here the section of the crack and the plane Ox_1x_2 is $\Gamma = \Gamma^+ \cup \Gamma^-$ and it is a horizontal segment on the axis Ox_1 - the interval (*-c,c*), Γ^+ and Γ^- are the upper and lower bound of the crack, $t_J = \sigma_{iJ} n_i$ is the generalized traction and $n = (n_1, n_2)$ is the normal vector to the crack.

We will solve the boundary value problem (2),(3) transforming it into an equivalent integro – differential system of equations on the crack Γ and then solve this system numerically .To achieve this aim we have to find the fundamental solution of (2).

3.Fundamental solution.

Fundamental solution of (2) is the solution of the system:

$$\sigma^*_{iJM,i} + \rho_{JK}\omega^2 u^*_{KM} = -\delta_{JM}\delta(x,\xi) \tag{4}$$

δ_{JM} is the symbol of Kroneker, $\delta(x,\xi)$ is the Dirak's delta function, $\xi = (\xi_1, \xi_2)$ $\sigma^*_{iJM} = C_{iJKl}(x) u^*_{KM,l}$.

The solution of (4) is derived in [14] for different types of inhomogeneity functions and here we will present the solution for the considered exponential function for complitness of the explanation.

We will make the transformation $u^*_{KM} = h^{-1/2} U^*_{KM}$ in the equation (4) and get the following:

$$C_{iJKl} U^*_{KM,ii} + [\rho_{JK}\omega^2 - C_{iJKl} h^{-1/2}(h^{1/2})_{,ii}] U^*_{KM} = -h^{-1/2}(\xi)\delta_{JM}\delta(x,\xi) \tag{5}$$

Since $h(x) = e^{2(a_1x_1 + a_2x_2)}$, then $h^{-1/2}\Delta h^{1/2} = a_1^2 + a_2^2$, where Δ is the two dimentional Laplace operator. Let's denote $\sqrt{a_1^2 + a_2^2} = \nu$. We substitute in (5) and get:

$$C_{iJKl} U^*_{KM,ii} + [\rho_{JK}\omega^2 - C_{iJKl}\nu^2] U^*_{KM} = -h^{-1/2}(\xi)\delta_{JM}\delta(x,\xi) \tag{6}$$

The equation (6) has constant coefficients. The Radon's transform is applied to both sides of (6) see Zayed [15] and the following equation is obtained:

$$(C_{iJKl}\partial_s^2 + [\rho_{JK}\omega^2 - C_{iJKl}\nu^2])\hat{U}^*_{KM} = -h^{-1/2}(\xi)\delta_{JM}\delta(s - <\xi, m>) \tag{7}$$

$m = (m_1, m_2)$, $|m| = 1$. The solution of (7) depends on the sign of $\gamma = \dfrac{\rho\omega^2}{\tilde{a}} - \nu^2$, where $\tilde{a} = \tilde{c}_{44} + \dfrac{\tilde{e}_{15}^2}{\tilde{\varepsilon}_{11}}$, $\tilde{c}_{44} = c_{44} + \dfrac{(q_{15})^2}{\mu_{11}}, \tilde{e}_{15} = e_{15} - \dfrac{d_{11}q_{15}}{\mu_{11}}, \tilde{\varepsilon}_{11} = \varepsilon_{11} - \dfrac{(d_{11})^2}{\mu_{11}}$. We will study the case when the frequency is such that $\gamma = 0$ i. e. the critical frequency. The solutions of (7) have the form:

$$\hat{U}^*_{33} = -\frac{h^{-1/2}}{2\tilde{a}}|s-\tau|,\ \hat{U}^*_{34} = -\frac{h^{-1/2}\tilde{e}_{15}}{2\tilde{\varepsilon}_{11}\tilde{a}}|s-\tau|,\ \hat{U}^*_{35} = -\frac{h^{-1/2}}{2\mu_{11}\tilde{a}}(q_{15} - d_{11}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}})|s-\tau|$$

$$\hat{U}^*_{4J} = (g_1^J + 2\frac{g_2^J}{\nu^2})\frac{e^{\nu|s-\tau|}}{2\nu} - \frac{g_2^J}{\nu^2}|s-\tau|,\ J = 3,4,\ \hat{U}^*_{45} = (b_1 + 2\frac{b_2}{\nu^2})\frac{e^{\nu|s-\tau|}}{2\nu} - \frac{b_2}{\nu^2}|s-\tau|$$

$$\hat{U}^*_{53} = (r_1 + 2\frac{r_2}{\nu^2})\frac{e^{\nu|s-\tau|}}{2\nu} - \frac{r_2}{\nu^2}|s-\tau|,\ \hat{U}^*_{54} = (p_1 + 2\frac{p_2}{\nu^2})\frac{e^{\nu|s-\tau|}}{2\nu} - \frac{p_2}{\nu^2}|s-\tau|,$$

$$\hat{U}^*_{55} = (z_1 + 2\frac{z_2}{\nu^2})\frac{e^{\nu|s-\tau|}}{2\nu} - \frac{z_2}{\nu^2}|s-\tau|$$

where

$$\tau =< \xi, m > \quad g_1^3 = \frac{\tilde{c}_{44}}{\tilde{e}_{15}}\frac{h^{-1/2}}{\tilde{a}} - \frac{h^{-1/2}}{\tilde{e}_{15}} \quad g_2^3 = -\frac{\tilde{\gamma}_{33}}{\tilde{e}_{15}}\frac{h^{-1/2}}{2\tilde{a}} \quad g_1^4 = \tilde{c}_{44}\frac{h^{-1/2}}{\tilde{\varepsilon}_{11}\tilde{a}}, g_2^4 = \tilde{\gamma}_{33}\frac{h^{-1/2}}{2\tilde{\varepsilon}_{11}\tilde{a}}$$

$$b_1 = \tilde{c}_{44}\frac{h^{-1/2}}{\tilde{e}_{15}\mu_{11}\tilde{a}}(q_{15} - d_{11}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}) - h^{-1/2}\frac{q_{15}}{\tilde{e}_{15}\mu_{11}}),\ b_2 = \tilde{\gamma}_{33}\frac{h^{-1/2}}{2\tilde{e}_{15}\mu_{11}\tilde{a}}(q_{15} - d_{11}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}})$$

$$r_1 = -\frac{q_{15}}{\mu_{11}}\frac{h^{-1/2}}{\tilde{a}} - \frac{d_{11}}{\mu_{11}}g_1^3,\ r_2 = \nu^2\frac{q_{15}}{\mu_{11}}\frac{h^{-1/2}}{2\tilde{a}} - \frac{d_{11}}{\mu_{11}}g_2^3,\ p_1 = -(\frac{q_{15}}{\mu_{11}}\frac{h^{-1/2}\tilde{e}_{15}}{\tilde{\varepsilon}_{11}\tilde{a}} + \frac{d_{11}}{\mu_{11}}g_1^4)$$

$$p_2 = \nu^2\frac{q_{15}}{\mu_{11}}\frac{h^{-1/2}\tilde{e}_{15}}{2\tilde{\varepsilon}_{11}\tilde{a}} - \frac{d_{11}}{\mu_{11}}g_2^4,\ z_1 = -(\frac{q_{15}h^{-1/2}}{\mu_{11}^2\tilde{a}}(q_{15} - d_{11}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}) + \frac{d_{11}}{\mu_{11}}b_1) + \frac{h^{-1/2}}{\mu_{11}},$$

$$z_2 = \nu^2\frac{q_{15}h^{-1/2}}{2\mu_{11}^2\tilde{a}}(q_{15} - d_{11}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}) - \frac{d_{11}}{\mu_{11}}b_2.$$

The inverse transform of Radon is applied in two stages. In the first stage we find $K(\hat{U}^*_{JK}(s)) = \int_{-\infty}^{+\infty}\frac{\partial_\sigma(\hat{U}^*_{JK}(\sigma))}{s-\sigma}d\sigma$, and then $U^*_{JK} = \frac{1}{4\pi^2}\int_{|m|=1} K(\hat{U}^*_{JK})\Big|_{s=<x,m>} dm$

The following lemmas take place.

Lemma 1 For the distribution $K(|s-\tau|)$ it is fulfilled in the sense of generalized functions that $K(|s-\tau|) = \int_{-\infty}^{+\infty}\frac{\partial_\sigma(|\sigma-\tau|)}{s-\sigma}d\sigma = 2\ln|s-\tau|$ for $s \neq \tau$.

Lemma 2 The following identity is fulfilled in the sense of generalized functions

$$K(e^{\nu|s-\tau|}) = \nu\{\cosh(\nu\beta) + 2[chi(\nu\beta)\cosh(\nu\beta) - shi(\nu\beta)\sinh(\nu\beta)]\}|_{\beta=|s-\tau|}$$

for $s \neq \tau$. Here $chi(z) = \gamma + \ln z + \int_0^z \frac{\cosh u - 1}{u}du$ and $shi(z) = \int_0^z \frac{\sinh u}{u}du$ are hyperbolic cosine and sine integral, γ is the Euler-Mascheroni's constant, see Bateman, Erdely [16]. We notice that fundamental solutions after the Radon transform are linear combinations of $|s-\tau|$ and $e^{\nu|s-\tau|}$ with known coefficients and $K(\hat{U}^*_{JK})$ can be found for $J,K = 3,4,5$. The next theorem follows from lemmas 1 and 2 and the inverse Radon transform.

Theorem1 The fundamental solution of (2) is

$$u^*_{KM} = \frac{h^{-1/2}}{4\pi^2}\int_{|m|=1} K(\hat{U}^*_{KM})\Big|_{s=<x,m>} dm.$$

4.Incident plane wave

Incident plane wave is the solution $v = \begin{pmatrix} v_3 \\ v_4 \\ v_5 \end{pmatrix}$ of the system (2). First we make the transformation $v_K = h^{-1/2}(x)V_K$ and then seek the solution of the new system in the form $V=\begin{pmatrix}V_3\\V_4\\V_5\end{pmatrix}$, where $V_J = p_J e^{ik<x,\xi>}$, $x=(x_1,x_2)$, $\xi=(\xi_1,\xi_2)$, $J=3,4,5$, p_J and k are constants. After making the necessary calculations we get that k has to satisfy the identity $k^2+\nu^2=\dfrac{\rho\omega^2(\varepsilon_{11}\mu_{11}-d_{11}^2)}{\det M}$, where $M=\begin{pmatrix}c_{44} & e_{15} & q_{15}\\ e_{15} & -\varepsilon_{11} & -d_{11}\\ q_{15} & -d_{11} & -\mu_{11}\end{pmatrix}$ and also it can be shown that $\dfrac{\varepsilon_{11}\mu_{11}-d_{11}^2}{\det M}=\dfrac{1}{\tilde{a}}$ which means that for the critical frequency $k=0$. Finally we get the following solution (after normalization $p_3=1$):

$$v=\begin{pmatrix}v_3\\v_4\\v_5\end{pmatrix}=h^{-1/2}(x)\begin{pmatrix}V_3\\V_4\\V_5\end{pmatrix}=h^{-1/2}(x)\begin{pmatrix}1\\ \dfrac{\mu_{11}e_{15}-d_{11}q_{15}}{\varepsilon_{11}\mu_{11}-d_{11}^2}\\ \dfrac{q_{15}\varepsilon_{11}-d_{11}e_{15}}{\varepsilon_{11}\mu_{11}-d_{11}^2}\end{pmatrix} \tag{8}$$

From (8) using the formula $T_J=C_{iJMl}(x)v_{M,l}n_i$ we get the traction

$$T=\begin{pmatrix}T_3\\T_4\\T_5\end{pmatrix}=-a_2\begin{pmatrix}1\\0\\0\end{pmatrix}\frac{\det M}{\varepsilon_{11}\mu_{11}-d_{11}^2}e^{a_1x_1+a_2x_2}.$$

5.Boundary integral equation

Following Wang and Zahng [17] for the piezoelectric case and Stoynov and Rangelov [18] for the magnetoelectroelastic homogeneous case we obtain the BIE:

$$t_J^{in}=-C_{iJKl}(x)n_i(x)\int_\Gamma[(\sigma^*_{\eta PK}(x,y,\omega)\Delta u_{P,\eta}(y,\omega)-\rho_{QP}\omega^2u^*_{QK}(x,y,\omega)\Delta u_P(y,\omega))\delta_{\lambda l}$$
$$-\sigma^*_{\lambda PK}(x,y,\omega)\Delta u_{P,l}(y,\omega)]n_\lambda(y)d\Gamma(y) \tag{9}$$

u^*_{QK} and σ^*_{iPK} are found in **3.**, $t_J^{in}(x,\omega)=T_J$ is found in **4**. and $\Delta u_J = u_J\Big|_{\Gamma^+}-u_J\Big|_{\Gamma^-}$ are the unknown COD. We reduce (9) to a system of linear equations and solve it numerically. Then we can find the traction field in every point $x\in R^2\setminus\Gamma$.

6. Numerical realization

The numerical solution scheme follows the procedure developed in Rangelov et al. [11] and Stoynov and Rangelov [18]. A FORTRAN program is created and the numerical results are obtained using PC – Core 2 Duo CPU E8500, 3.16GHz and 2.53GHz, 3GB RAM. In the examples the length of the crack is $c = 5mm$. The composite MEE material $BaTiO_3 / CoFe_2O_4$ has the following constants:

$\rho = 6490 kg/m^3$, $e_{15} = 5.8 C.m^{-2}$, $q_{15} = 275 N.A^{-1}.m^{-1}$, $\varepsilon_{11} = 56.4x10^{-10} C^2.N^{-1}.m^{-2}$, $c_{44} = 44x10^9 N.m^{-2}$, $d_{11} = 0.005x10^{-9} N.s.V^{-1}.C^{-1}$, $\mu_{11} = -297x10^{-6} N.s^2.C^{-2}$. The inhomogeneity parameters are presented in the following form: $a_1 = \frac{\beta}{2c}\cos(\alpha)$, $a_2 = \frac{\beta}{2c}\sin(\alpha)$. The critical frequency is $\omega = \nu\sqrt{\frac{\tilde{a}}{\rho}}$.

6.1 Validation test

The results are validated with the static homogeneous problem, [19]. The comparison is given in Fig.1(in all figures $x = x_1.10^{-3}$):

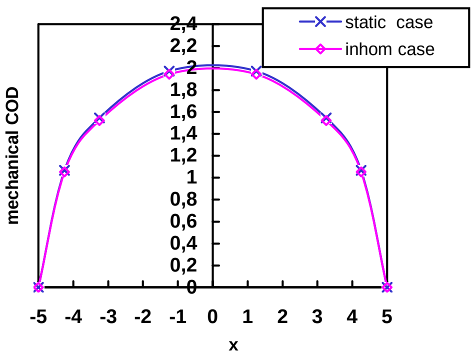
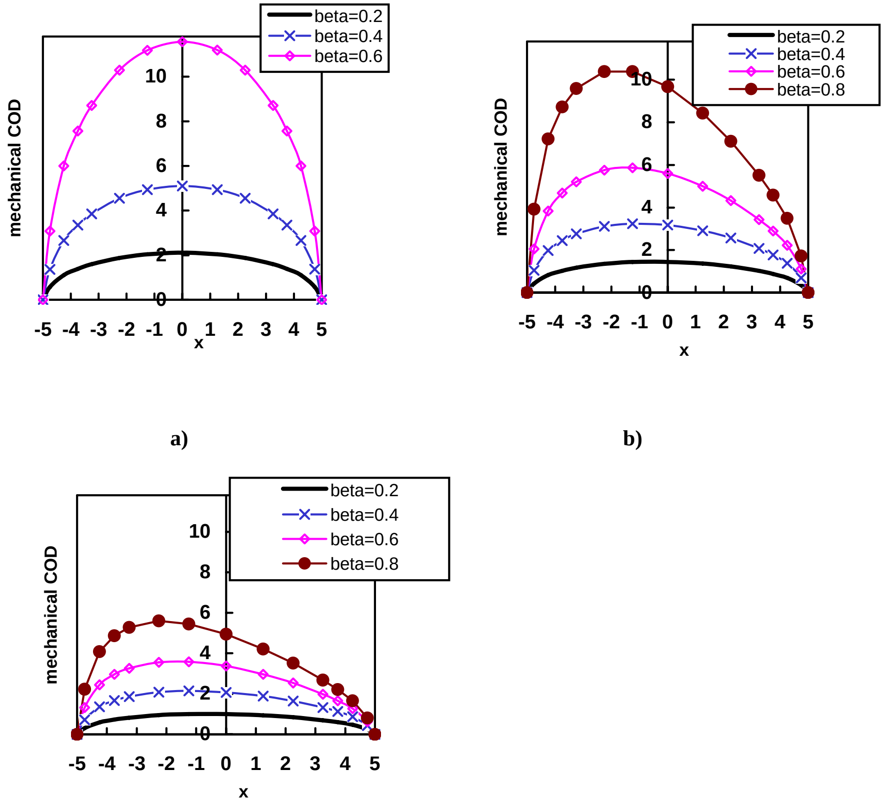


FIGURE 1. The magnitude of the mechanical COD at different points on the crack. The mechanical COD is multiplied by 10^{13}.

We can see close coincidence of the results. The error is within 5%.

6.2 Parametric studies

Numerical results for COD are given for fixed $\alpha = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}$. In Fig .2 a) $\alpha = \frac{\pi}{2}$ is fixed and β is increasing. We see the value of COD is also increasing. Fig.2 a)-c) show that the maximum values for CODs decrease when α changes from $\alpha = \frac{\pi}{2}$ to $\alpha = \frac{\pi}{6}$ and these values are achieved for negative values of *x*. The graphs are not symmetrical, but biased to the left. In all Figures 2 a)-c) mechanical CODs are multiplied by 10^{-1}.



a) b)

c)

FIGURE 2. The magnitude of the mechanical COD at different points on the crack for fixed $\alpha = \dfrac{\pi}{6}$

7. Conclusions

Critical frequencies for the dynamic time-harmonic problems in cracked MEE media is considered here. The medium is exponentially inhomogeneous and the crack is electromagnetically impermeable. Fundamental solutions are presented in a closed form. BIEM is used for the numerical computations. A FORTRAN program code in Fortran is developed based on the BIEM. The results reveal the sensitivity of the COD to the parameters of the inhomogeneity function.

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