A Novel CNN Based Image Denoising Model

Angela Slavova
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Sofia 1113, Bulgaria
e-mail: *slavova@math.bas.bg*
Victoria Rashkova
Department of Informatics
Rousse University
Rouse 7017, Bulgaria
e-mail: *viktoriqi₇8@abv.bg*

Abstract- **The nonlinear diffusion PDE have been broadly applied in image processing. In this paper we propose a convection - diffusion filter by adding a convection term in the modified diffusion equation as a physical interpretation for removing the noise. We study the dynamics of such equations by the discretization of this convection - diffusion model via Cellular Nonlinear Networks (CNN). Numerical experiments show that our method is reasonably better in removing noise.**

Index terms- **convection-diffusion equations, Cellular Neural Networks (CNN), algorithms, filters, image processing**

I. Introduction

Recently many mathematical models for image processing have been widely applied in computer visualization. The nonlinear diffusion partial differential equation have been broadly applied in image processing since the first model was introduced in 1987 [4]. Through the time evolution, the diffusion can effectively remove the noise as well as having edge enhancement simultaneously. Since then various nonlinear diffusion filters have been widely proposed in implementing the image denoising / enhancement, edge detection and flow filed visualization . The common feature for nonlinear diffusion model is that the diffusion coefficient is small as the gradient of image is large. However the diffusion coefficient is a function of the convolution of the Gaussian kernel and solution such that this requires an extra cost in computing the nonlinear diffusion coefficient. In the numerical experiments we find that when using the nonlinear diffusion model in denoising the noise is not quite good [5]. Hence we propose a convection- diffusion filter by adding a convection term in the modified diffusion equation as a physical interpretation for removing the noise. The aim of this paper is to focus on the noise removal algorithm for extracting the target information (image) precisely. The main idea of our model algorithm is to diffuse the noise by following the convection direction during time evolution. To prevent the numerical layer in the discontinuities on the relative coarse grids we use the Cellular Neural Network (CNN) approach [1,2,6].

Here we propose a popular nonlinear convection- diffusion model for image demising as well as image compression. We shall modify nonlinear isotropic equation [4] and we shall construct both convection and diffusion terms based on the gradient of image intensity; thus the direction of image smoothing is normalized to the gradient of image intensity. Consider our filter: the convection- diffusion problem (CD) with manipulation in both diffusion and convection terms for controlling the smoothing process:

$$\begin{cases} \frac{\partial u}{\partial t} - \varepsilon(|\nabla u|)\Delta u + \\ +\beta(|\nabla u|).\nabla u = 0 \text{ in } \Omega \times I \\ \frac{\partial u}{\partial n} = 0 \text{ on } \partial\Omega \times I \\ u(x,0) = u_0(x) \text{ on } \Omega \end{cases} \quad (1)$$

where the diffusion coefficient is denoted by $\varepsilon(|\nabla u|) \equiv \frac{1}{1+|\nabla u|^2}$ and the convection vector $\beta(|\nabla u|) \equiv \gamma\frac{\nabla u^\perp}{|\nabla u|_\epsilon}$) for positive constant γ (the

size of convection vector) with the Evans- Spruck regularization [6] $|\nabla u|_\epsilon \equiv \sqrt{u_x^2 + u_y^2 + \epsilon}$ for $0 < \varepsilon \ll 1$ to avoid the singularity. Since the diffusion coefficient ε is inhibited as the gradient of image intensity is big, CD preserves the edges of image and protects the brightness of the image simultaneously.

It is well known that if the diffusion coefficient ε is sufficiently small in comparison with the quantity $|\beta|h$ where the solution is discontinuous then the Galerkin finite element scheme leads to severe oscillation [5]. The streamline diffusion finite element method resolves the oscillation problem; however it causes some artifacts (overshooting/ down shooting) around the edge of discontinuities. To avoid above phenomena we have proposed a CNN approach.

In this paper we shall study the following convection-diffusion problem:

$$\begin{cases} \frac{\partial u}{\partial t} = b\nabla u + cu, & \text{in} \quad D \equiv (0,1)^2 \\ u = 0 & \text{on} \quad \partial D, \end{cases} \quad (2)$$

where $b(x,t) \geq \beta$, $c(x,t) \geq 0$ and $c_0^2(x,t) \equiv (c - b_x/2) \geq \gamma$, $x, t \in \bar{D}$, β and γ are some positive constants. It is known [5] that for this assumptions there exists an unique solution of the CD problem (2).

In the next section we shall construct the CNN algorithm for studying the dynamics of the CD equation (2). Section 2 deals also with CNN model of the convection-diffusion equation and its dynamics. Two examples are given in order to demonstrate the simulation results.

II.   CNN APPROACH FOR STUDYING THE DYNAMICS OF THE CONVECTION-DIFFUSION MODEL

Partial Differential equation (PDE) based models have recently shown to be useful for multiscale analysis [8,9]. They can be classified in three main categories: a) linear isotropic filters; b) nonlinear isotropic filters; c) nonlinear anisotropic filters. The main disadvantage of PDE based techniques is that from a computational point of view they are rather time-consuming and therefore they are not suitable for real-time image processing. For this purpose we shall apply CNN algorithm for studying the CD model (2).

It is known that some autonomous CNNs represent an excellent approximation to nonlinear partial differential equations (PDEs). The intrinsic space distributed topology makes the CNN able to produce real-time solutions of nonlinear PDEs. Consider the following well-known PDE, generally referred to us in the literature as a reaction-diffusion equation:

$$\frac{\partial u}{\partial t} = f(u) + D\nabla^2 u,$$

where $u \in \mathbf{R}^N$, $f \in \mathbf{R}^N$, D is a matrix with the diffusion coefficients, and $\nabla^2 u$ is the Laplacian operator in $\mathbf{R}^2$. There are several ways to approximate the Laplacian operator in discrete space by a CNN synaptic law with an appropriate A-template.

In our case the CNN model of CD equation (2) is:

$$\dot{u}_j(t) = b * A_1 * u_j + c * u_j, 1 \leq j \leq N, \quad (3)$$

where $A_1 = (1, -2, 1)$ is one dimensional discretize Laplacian CNN template. The convolution operator $*$ is defined below:

Definition 1 *For any cloning template A which defines the dynamic rule of the cell circuit, we define the convolution operator $*$ by the formula*

$$A * z_{ij} = \sum_{C(k,l) \in N_r(i,j)} A(k-i, l-j)z_{kl},$$

where $A(m,n)$ denotes the entry in the pth row and rth column of the cloning template, $p = -1, 0, 1$, and $r = -1, 0, 1$, respectively.

In this section we will introduce an approximative method for studying the dynamics of CNN model (3), based on a special Fourier transform. The idea of using Fourier expansion for finding the solutions of PDEs is well known in physics. It is used to predict what spatial frequences or modes will dominate in nonlinear PDEs. In CNN literature this approach, has been developed for analyzing the dynamics of CNNs with symmetric templates [1].

We shall investigate the dynamic behavior of CNN model (3) by use of Harmonic Balance Method well known in control theory and in the study of electronic oscillators [3] as describing function method. The method is based on the fact that all cells in CNN are identical [1], and therefore by introducing a suitable double transform, the network can be reduced to a scalar Lur's scheme [3].

We shall present the algorithm briefly:

1. Apply the double Fourier transform:

$$F(s,z) = \sum_{k=-\infty}^{k=\infty} z^{-k} \int_{-\infty}^{\infty} f_k(t) exp(-st) dt,$$

to the CNN equation (3).

2. Find the transform function $H(s,z) =$, where $s = i\omega_0$, $z = exp(i\Omega_0$, $i = \sqrt{-1}$, ω_0 is a temporal frequency, Ω_0 is a spatial frequency.

3. Look for possible solutions of (3) in the form:

$$u_j = U_{m_0} sin(\omega_0 t + j\Omega_0)$$

4. The amplitude U_{m_0}, the temporal frequency ω_0 and the spatial frequency Ω_0 are unknowns to be determined.

5. Our CNN model (3) is a finite circular array of N cells we have finite set of spatial frequencies:

$$\Omega_0 = \frac{2\pi k}{N}, 0 \le k \le N-1. \quad (4)$$

Based on the above considerations the following proposition hold:

Proposition 1
CNN model (3) of the convection-diffusion problem (2), consisting of circular array of N cells, has state solution $u_j(t)$ with a finite set of spatial frequencies $\Omega_0 = 2\pi k/N$, $0 \le k \le N-1$.

We obtain the following simulation results for different values of the parameters β and γ:
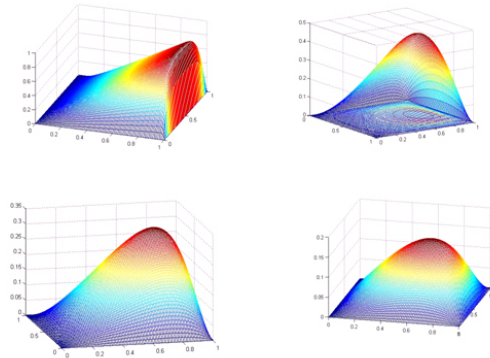

Fig.1. Simulations of the CNN algorithm for CD problem (2).

III.   EXAMPLES

Example 1. Consider the following singularly perturbed boundary value problem

$$-\varepsilon\Delta u + bu_x + cu = 0, \quad in \quad \Omega \equiv (0,1)^2 \quad (5)$$

$$u = 0, \quad on \quad \partial\Omega.$$

In order to construct a robust numerical method for the considered problem, it is of key interest to have information on a behavior of the solution. The state equation of the CNN model of (5) is:

$$-\varepsilon A_1 * u_i + b * A_1 * u_i + c * u_i = 0 \quad (6)$$

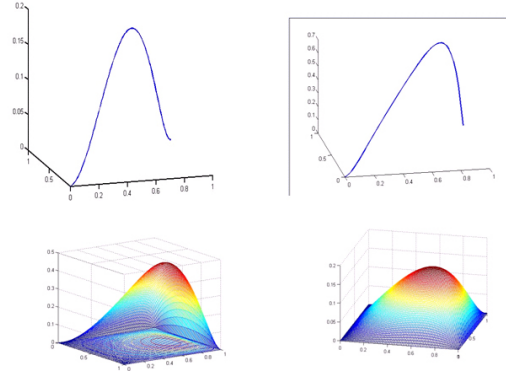Applying CNN algorithm we obtain simulation results shown on Fig.2.

Fig.2. Simulation of the CNN algorithm for the problem (5).

Example 2. We consider the following example of (2) on the layer-adapted mesh:

$$-\varepsilon\Delta u + (1+x^3)u_x + (1+xy)u = 0, in\Omega \quad (7)$$

$$u = 0, on\partial\Omega$$

CNN model of the above system is:

$$-\varepsilon A_1 * u_i + (1+x^3)A_1 * u_i + (1+xy)u_i = 0 \quad (8)$$

$$1 \le i \le N$$

We obtain simulation results of (8) on Fig.3.

Remark 1. We consider a CNN programmable realization allowing the calculation of all necessary processing steps in real time. The network
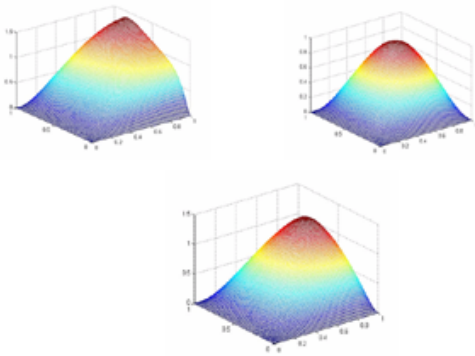
Fig.3. Simulation of the CNN algorithm for the problem (7).

parameter values of CNN models, are determined in a supervised optimization process.During the optimization process the mean square error is minimized using Powell method and Simulated Annealing [7]. The results are obtained by the CNN simulation system MATCNN applying 4th order Runge-Kutta integration.

IV.   ACKNOWLEDGMENTS

V.   CONCLUSION

In this paper a convection-diffusion equation for processing image denoising is studied. In order to construct a robust numerical method we apply CNN approach. By means of harmonic balance method we prove that the solutions of the CNN model exist and they have a finite set of spatial frequencies $\Omega_0 = 2\pi k/N$, $0 \leq k \leq N - 1$. Computational results are presented for two test problems with known solutions in order to verify the obtained theoretical results.

REFERENCES

[1] Chua L.O., Yang L., Cellular Neural Network: Theory and Applications, *IEEE Trans. CAS*, vol. 35, pp. 1257-1290, Oct. 1988.

[2] Chua L.O., Hasler M., Moschytz G.S., Neirynsk J., Autonomous cellular neural networks: a unified paradigm for pattern formation and active wave propagation, *IEEE Trans. CAS-I*, vol. 42, N 10, pp. 559-577, Oct. 1995.

[3] Mees A.I., *Dynamics of Feedback Systems*, London, England:Wiley, 1981.

[4] P. Perona, J. Malik, Scale space and edge detection using anisotropic diffusion. In: Proceedings of IEEE Computer Soc. Workshop and Computer Vision, 1987, pp. 16-22.

[5] Y. Shih, H. C. Elman, Modified streamline diffusion schemes for convection - diffusion problems, Computational Methods in Applied Mechanics and Engineering, 1998.

[6] A.Slavova,Cellular Neural Networks: Dynamics and Modelling, Kluwer Academic Publishers, 2003.

[7] R. Tetzlaff, F. Gollas, Modeling complex systems by reaction-diffusion Cellular Nonlinear Networks with polynomial Weight-Functions,Proc. IEEE CNNA2005, 2005.

[8] J.Weickert, A review of nonlinear diffusion filtering, Lecture Notes in Computer Science, Springer, Berlin, vol. 1252, pp. 3-28, 1997.

[9] J. Weickert, B. Benhamouda,Nonlinear diffusion scale-spaces: From the continuous to the discrete setting, in Proc. ICAOS96: Images, Wavelets and PDEs, vol.219, M.O. Berger et al., Eds. New York: Springer, pp. 111-118, 1996.