Institute of Mathematics and Informatics
Bulgarian Academy of Sciences

International Conference
Mathematics Days in Sofia

July 7–10, 2014, Sofia, Bulgaria

Abstracts

Sofia, 2014

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Content

Preface

The International Conference "Mathematics Days in Sofia" held in Sofia, Bulgaria, in the period July 7-th – July 10-th, 2014 is organized by the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences.
The purpose of this event is to present the current state of the art in all areas of mathematics and computer science and its applications. One of the main goals of the Conference is to bring together Bulgarian mathematicians and computer scientists working all over the world. In this way, will expect to enrich the relations between the mathematicians working in Bulgaria and the Bulgarian mathematical diaspora.
Our idea is to organize such conferences every 2–3 years and to make them traditional scientific forums. Similar events were organized by the Institute of Mathematics and Informatics in 2007 celebrating the 60-th anniversary of the Institute and about 10 years ago by the Faculty of Mathematics and Informatics of the University of Sofia.
The meeting includes the following sections:

- Algebra, Logic, and Combinatorics;
- Analysis, Geometry, and Topology;
- Differential Equations and Mathematical Physics;
- Mathematical Modelling (including Approximation Theory, Numerical Mathematics, Probability and Statistics, Operations Research, and Biomathematics);
- Mathematical Aspects of Computer Science;

and the following mini-symposia:

- Advanced Analytical and Numerical Techniques for Applications;
- Algebraic Methods in Quantum Field Theory;
- Approximation Theory and Special Functions (2nd series);

- Geometry Days in Sofia;
- Transform Methods and Special Functions'14;
- Variational Analysis.

The Conference consists of 50-minutes invited plenary talks, 45-minutes invited talks for sections and mini-symposia and 30-minutes communications. The plenary and the invited section speakers are established Bulgarian and foreign scientists working in international recognized mathematical centers. More than 300 scientists from more than 40 countries will participate at the Conference. They will deliver more than 200 invited talks and communications.

Vesselin Drensky
Chairman
of the Programme Committee

Julian Revalski
Chairman
of the Organising Committee

Plenary Talks

Cohomologically noetherian modules and rings

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Keywords: Projective resolution, rate of growth, noetherian ring, cohomology products, center of triangulated category

A starting point for the homological study of algebraic structures, such as modules or rings, is to "resolve" them by using constructions that involve only standard building blocks. Such resolutions are not uniquely defined, and it is of significant interest to work with "smaller" ones. In order to do this, one needs estimates for the sizes of resolutions, as well as necessary and/or sufficient conditions for the existence of resolutions of prescribed sizes. Various approaches and results, both old and new, will be discussed in the talk. It will be directed to a general mathematical audience.

Optimization problems with stochastic order constraints

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Keywords: stochastic dominance, increasing-convex order, risk quantification, duality, numerical methods of optimization

Stochastic orders formalize preferences among random outcomes and are widely used in statistics and economics. We focus on stochastic optimization problems involving stochastic-order relations as constraints that relate performance functionals, depending on our decisions, to benchmark random outcomes. We discuss the relation of univariate and multivariate stochastic orders to utility functions, conditional value at risk, as well as general coherent measures of risk. Necessary and sufficient conditions of optimality and duality theory for problems with stochastic order constraints involve expected utility theory, dual (rank-dependent) utility theory, and coherent measures of risk, providing a link between various approaches for risk-averse optimization. Some attention is paid to the numerical treatment of the problems. Several applications will be outlined.

Almost complex structures on quaternionic Kaehler manifolds

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Keywords: quaternionic Kaehler manifolds

We show that compact quaternionic-Kaehler manifolds of positive type admit no globally defined almost complex structure, even in the stable sense, except for

the complex Grassmannians $Gr_2(\mathbb{C}^{n+2})$

Acknowledgements. This is joint work with Andrei Moroianu and Uwe Semmelmann.

Kac-Moody algebras, Riemann-Hilbert problems and new soliton equations

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Keywords: Kac-Moody algebras, Riemann-Hilbert problems, soliton equations

We start with a Riemann-Hilbert Problem (RHP) with canonical normalization whose sewing functions depend on several additional variables. Using Zakharov-Shabat theorem we are able to construct a family of ordinary differential operators for which the solution of the RHP is a common fundamental analytic solution [1–3]. This family of operators obviously commutes. Thus we are able to construct new classes of integrable nonlinear evolution equations.

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The Lichnerowicz-Obata sphere theorems on a quaternionic contact manifold

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Keywords: qc-manifold, Biquard connection, sub-Laplacian, Bochner-Weitzenböck formula

Lichnerowcz [5] showed that on a compact Riemannian manifold of dimension n for which the Ricci curvature is greater than or equal to that of the unit n-dimensional sphere $S^n(1)$, the first eigenvalue λ_1 of the Laplace operator is greater than or equal to the first eigenvalue of the sphere, $\lambda_1 \geq n$. Subsequently Obata [6] proved that equality is achieved if and only if the Riemannian manifold is isometric (metric gauge equivalent) to $S^n(1)$ observing that the trace-free part of the Riemannian Hessian of an eigenfunction f with eigenvalue $\lambda = n$ vanishes.

We prove quaternionic contact (qc)-versions of the Lichnerowicz's and Obata's sphere theorems. Qc-geometry is a special sub-Rimannian geometry of dimension $4n + 3$ with the local flat model the quaternionic Heisenberg group and the 3-Sasakian round sphere. Qc-geometry also plays a fundamental róle in investigating the best constant and extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg groups ([1]).

On a $(4n+3)$-dimensional compact qc manifold we find a Lichnerowicz's type positivity condition in terms of the Ricci tensor and the torsion of the Biquard connection and prove that this implies the first nonzero eigenvalue λ_1 of the sub-Laplacian is greater than or equal to the first eigenvalue of the 3-Sasakian round unit sphere, $\lambda_1 \geq 4n$ ([2]). We also show that the equality is achieved if and only if the qc manifold is qc homothetic (qc gauge equivalent) to the standard 3-Sasakian unit sphere ([3]) provided the dimension is bigger than 7.

The work relies on considerations in [1]. To prove the inequality, we write on a qc manifold a Bochner-Weitzenböck-type formula for the sub-Hessian in terms of the curvature and torsion of the Biquard connection. We observe that in the equality case the trace-free part of the sub-Hessian of an extremal eigenfunction vanishes and show that this condition forces the torsion of the Biquard

connection to be zero which leads the Riemannian Hessian is trace-free achieving M is isometric to the unit sphere due to the Obata theorem. In order to prove the qc-equivalence part we show that the qc-conformal curvature vanishes, which gives the local qc conformal equivalence with the 3-Sasakian sphere [4], and then prove a Liouville-type result leading the existence of a global qc-conformal map between M and the 3-Sasakian sphere which turns out to be qc-homothetic.

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Introduction to tropical mathematics

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Keywords: tropical algebra and geometry

I will introduce basic notions of tropical algebra and geometry. A tropical semiring is endowed with operations of addition and multiplication (no subtrac-

tion!) satisfying the usual rules of associativity, commutativity and distributivity, plus the property saying that the addition is idempotent.

The main example of a tropical semiring is the set of real numbers with the addition given by the maximum, and the multiplication given by the usual sum. There are tropical analogs of algebraic subvarieties of a coordinate space. These tropical varieties are piecewise-linear closed subsets of the euclidean space. Many analogs of classical results from algebraic geometry hold in the tropical setting.

I will explain the relation between tropical geometry and algebraic geometry over nonarchimedean fields. Also I'll speak about applications of tropical geometry to Mirror Symmetry, and to potential counterexamples to the Hodge conjecture.

Finite ring geometries and linear codes over rings

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Keywords: chain rings, projective Hjelmslev geometries, arcs, R-analogs of designs, blocking sets

Linear codes over finite chain rings have received a great deal of attention since the discovery that the families of Kerdock and Preparata codes are images of $\mathbb{Z}_4$-linear codes. This triggered the research on linear codes over special classes of rings, notably finite chain rings. In the last 15 years, a geometric theory of the linear codes over finite chain rings was developed. The main idea is to link the chain ring codes to multisets of points in certain ring geometries called projective Hjelmslev geometries.

Given a finite chain ring R, the projective Hjelmslev geometry $\mathrm{PHG}({}_RR^n)$ is defined as the incidence structure $(\mathcal{P}, \mathcal{L}, I)$, where $\mathcal{P}$ is the set of all free rank 1 submodules of ${}_RR^n$, $\mathcal{L}$ is the set of all free rank 2 submodules of ${}_RR^n$, and the incidence $I \subset \mathcal{P} \times \mathcal{L}$ is given by set-theoretical inclusion. For an in-depth introduction to the coordinate ring geometries $\mathrm{PHG}({}_RR^n)$, we refer to [1, 2] and the references there. There exists an one-to-one correspondence between the isomorphism classes of linear codes of full length over a finite chain ring

R, and the classes of projectively equivalent multisets of points in $\mathrm{PHG}({}_RR^n)$. This correspondence suggests the detailed investigation of some special point configurations in $\mathrm{PHG}({}_RR^n)$ such as arcs, blocking sets, caps, spreads etc. The talk is focused on some recent advances in the following problems:

(1) the problem of determining the cardinality of an oval in $\mathrm{PHG}({}_RR^3)$, where R is a chain ring of nilpotency index 2;

(2) the problem of finding the largest size of a (k, n)-arc (n fixed) in projective Hjelmslev planes over a ring of nilpotency index 2;

(3) the largest arc problem with respect to the Heise homogeneous weight in planes over chain rings of nilpotency index 2; a partial analogue of Bonisoli's theorem on constant weight codes (arcs);

(4) the problem of characterizing the minimal blocking sets in projective and affine Hjelmslev planes, in particular, the construction of Redei-type blocking sets in projective Hjelmslev planes;

(5) the problem of finding necessary and sufficient conditions for the existence of λ-spreads in $\mathrm{PHG}({}_RR^n)$ for a given shape $\lambda = (\lambda_1, \ldots, \lambda_n)$; more generally, constructions of non-trivial R-analogs of designs in projective Hjelmslev spaces.

Acknowledgements. This research has been supported by the Science Research Fund of Sofia University under Contract No. 37/28.04.2014.

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Character varieties and nested Hilbert schemes

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Keywords: character varieties, parabolic Higgs bundles

I will describe a framework for computing the topology of character varieties of punctured Riemann surfaces. Using the spectral correspondence on root stacks and an appropriate version of geometric engineering we identify the refined parabolic stable pair theory of certain stacky Calabi-Yau threefolds with a generating series for enumerative invariants of nested Hilbert schemes on surfaces. Combined with a wall-crossing formula for parabolic ADHM sheaves this formalism gives an effective algorithm for computing Poincare polynomials of distinguished symplectic leaves in the moduli of tamely ramified parabolic Higgs bundles and of the corresponding character variety. Through a field theory limit this verifies a recent conjecture of Hausel, Letellier, and Rodriguez-Villegas. This is a joint work with W.-y.Chuang, E.Diaconescu, and R.Donagi.

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Estimates for the resolvent and spectral gaps for non self-adjoint operators

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Keywords: eigenvalues free regions, scattering resonances, estimates for the cut-off resolvent of the Laplacian

In mathematical physics and dynamical systems in many problems appear non self-adjoint operators P without spectrum in a strip $\mathcal{C} = \{z \in \mathbb{C} : a < \operatorname{Re} z < b\}$ called **spectral gap**. The estimates of the norm of the resolvent $\|(P-z)^{-1}\|$ for $z \in \mathcal{C}$ play an important role in the investigation of the local decay of the energy, the analysis of the scattering resonances and the analytic continuation of the dynamical zeta function in $\mathcal{C}$. We will discuss results and open problems concerning the estimate in $\mathcal{C}$ of the inverse $\dfrac{1}{f(z)}$ of a holomorphic function $f(z)$ without zeros in $\mathcal{C}$, the cut-off resolvent $\|\varphi(-\Delta_D - z)^{-1}\varphi\|$ of the Dirichlet Laplacian Δ_D for trapping perturbations and the Ruelle transfer operator for dynamical systems related to contact Anosov flows and billiard flow.

Besov and Triebel-Lizorkin spaces associated with non-negative self-adjoint operators and applications

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Keywords: Besov spaces, Triebel-Lizorkin spaces, frames, nonlinear approximation

Classical and nonclassical Besov and Triebel-Lizorkin spaces are developed in the general setting of a metric measure space with the doubling property and in presence of a non-negative self-adjoint operator whose heat kernel has Gaussian

localization and the Markov property. A natural effective realization of this setting appears in the general framework of strictly local regular Dirichlet spaces with a complete intrinsic metric, where it suffices to only verify the local Poincaré inequality and the global doubling condition on the measure. Sub-exponentially localized and compactly supported frames are constructed and utilized for decomposition of Besov and Triebel-Lizorkin spaces. These decompositions are also applied for Littlewood-Paley decomposition of Hardy spaces and nonlinear n-term approximation. In particular, our theory allows to develop Besov and Triebel-Lizorkin spaces and their frame characterization on the sphere, cube, ball, and simplex with weights as well as in the context of Lie groups, Riemannian manifolds, and other nonclassical settings.

Big data, high-performance computing, and MapReduce algorithms on grids

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Keywords: Hadoop, MapReduce, local aggregation, partitioning, relaxation, life simulation

We discuss the emergence of data-intensive computing and then explore the applicability of business-oriented big-data frameworks, such as Hadoop MapReduce, to traditional scientific computing processes. In particular, we investigate the suitability of MapReduce for simulation of grid-based models by developing message-passing MapReduce algorithms and empirically evaluating their performance on the Amazon's Elastic MapReduce cloud. We outline MapReduce challenges (such as insufficient speed) and opportunities (such as fault-tolerance and unlimited data size) in scientific computing.

Unique factorization domains, cluster algebras, and maximal green sequences

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Keywords: Cluster algebras, double Bruhat cells, maximal green sequences.

Cluster Algebras were defined by Fomin and Zelevinsky in 2000. They have rich combinatorial structure that was linked to many areas of pure mathematics and mathematical physics, such as representation theory, Teichmüller theory, Poisson and algebraic geometry.

In this talk, I will make a general introduction to the subject and describe a new algebraic approach based on noncommutative unique factorization domains and their Poisson counterparts. It can be applied both to classical and quantum cluster algebras, and does not rely on specific combinatorics as the previous approaches. With its help we settle several open problems in the area, ranging from an algebraic conjecture on cluster structures on coordinate rings (works of Berenstein, Fomin and Zelevinsky) to a combinatorial problem on maximal green sequences (due to Keller). Parts of the talk are joint work with Ken Goodearl (University of California, Santa Barbara).

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Domain decomposition methods for multiphysics applications

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Keywords: domain decomposition, mortar finite element, mltiphysics, multiscale

We discuss a domain decomposition computational framework for multiphysics problems. The simulation domain is decomposed into a union of subdomains, each one associated with a physical, mathematical, and numerical model. The equations are discretized locally on a fine scale, while physically meaningful interface conditions are imposed weakly on a coarse scale using mortar finite elements. We address the stability and accuracy of the numerical methods, as well as efficient algorithms for solving the resulting coupled algebraic system on parallel computers. Applications to coupled surface and groundwater flows, flows in fractured deformable reservoirs, and arterial flows are presented.

Section: Algebra, Logic, and Combinatorics

Invited Talks

On the Noether number

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Keywords: polynomial invariants, finite groups, degree bounds

Given a group G of linear transformations of a finite dimensional vector space V, a *polynomial invariant* is a polynomial function on V constant along the G-orbits. The polynomial invariants constitute a graded subalgebra in the algebra of polynomial functions on V. A classical theorem of E. Noether asserts that for a finite group G of linear transformations of a finite dimensional complex vector space V, the algebra of polynomial invariants on V is generated by homogeneous elements of degree at most $|G|$. Motivated by this result the *Noether number* $\beta(G)$ of G is defined as the supremum of the degree of an indecomposable polynomial invariant as V ranges over all finite dimensional complex G-modules (by an *indecomposable invariant* we mean an invariant not contained in the algebra generated by lower degree invariants). With this notation Noether's Theorem mentioned above asserts $\beta(G) \leq |G|$. In the talk we shall survey improvements of this inequality, focusing mainly on joint results with Kálmán Cziszter.

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Sieving very thin sets of primes

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Keywords: Prime chains, sieve methods

We discuss a new method to bound the number of primes in certain very thin sets. For each prime p, only 1 or 2 residue classes modulo p are omitted, and so the traditional small sieve produces poor bounds. The sets S under consideration have the property that if $p \in S$ and $q|(p-1)$, then $q \in S$. We prove that either S contains all primes or $\#\{p \in S : p \le x\} = O(x^{1-c})$ for some positive c. We descibe applications of such prime sets to Carmichael's conjecture, iterates of arithmetic functions and recent work of the speaker, Sergei Konyagin and Florian Luca on groups with perfect order subsets.

Acknowledgements. Work supported by National Science Foundation Grants DMS-0901339 and DMS-1201442.

The world of enumeration reducibility

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Keywords: enumeration reducibility, degree structures, Turing degrees, definability

The main focus in degree theory, established as one of the core areas in Computability Theory, is to understand a mathematical structure, which arises as a formal way of classifying the computational strength of an object. The most studied examples of such structures are that of the Turing degrees, $\mathcal{D}_T$, based on

the notion of Turing reducibility, as well as its local substructures, of the Turing degrees reducible to the first jump of the least degree, $\mathcal{D}_T$ ($\leq 0'_T$), and of the computably enumerable degrees, $\mathcal{R}$. In investigating such a mathematical structure among the main question that we ask is: how complex is this structure. The complexity of a structure can be inspected from many different aspects: how rich is it algebraically; how complicated is its theory; what sets are definable in it; does it have nontrivial automorphisms. The question about definability, in particular, is interrelated with all of the other questions, and can be seen as a key to understanding the natural concepts that are approximated by the corresponding mathematical formalism.

One approach for understanding a structure, often used in mathematics, is to place this structure in a richer context, a context which would reveal new hidden relationships. The most promising candidate for such a larger context for $\mathcal{D}_T$ is the structure of the enumeration degrees, introduced by Friedberg and Rogers in 1959. The Turing degrees can be embedded in the enumeration degrees via a natural embedding. Hence $\mathcal{D}_T$ can be regarded as a substructure of $\mathcal{D}_e$, which is isomorphic to An important question, which immediately arises in this context, first set by Rogers in 1967, is whether $\mathcal{D}_T$ is first order definable in $\mathcal{D}_e$. After several partial solution, this problem was finally solved by Cai, Ganchev, Lempp, Miller and Soskova.

In this talk we will give an overview of all definability results based on the notion of Kalimullin pairs that have led to the final solution of the long standing problem of the definability of the Turing degrees in the context of the Enumeration degrees.

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EPW sextics and Hilbert squares of K3 surfaces

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Keywords: hyperkähler varieties, K3 surfaces

We prove that the Hilbert square $S^{[2]}$ of a general primitively polarized $K3$ surface S of degree $d = 2(4n^2 + 8n + 5), n \geq 1$ is birational to a double Eisenbud-Popescu-Walter sextic of discriminant $-2d$. This is an analog of the result of Hassett (2000): The Hilbert square of a $K3$ surface of degree $d = 2(n^2 + n + 1), n \geq 2$ is isomorphic to the Fano family of lines of a special cubic fourfold of discriminant d. Our result answers positively the following conjecture of O'Grady in the case when r is even: On the Hilbert square of a general K3 surface of genus $r^2+2, r \geq 1$ there is an antisympectic involution. The case $r = 2$ has been studied in detail by O'Grady.

Acknowledgements. Research partially supported by MTM2012-32670 of MICINN, by the Research fund for international professors of the Seoul National University, and by the Autonoma University of Madrid mobility research program.

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On certain Diophantine problems with primes and almost primes

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Keywords: Diophantine problems, prime numbers

We consider some classical Diophantine equations and inequalities and show that in certain cases they have solutions in prime or almost prime variables.

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Communications

Volume gradients and homology in towers of residually-free groups

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Keywords: volume gradient, ℓ^2 Betti numbers, residually-free groups

We study the asymptotic growth of homology groups and the cellular volume of classifying spaces as one passes to normal subgroups $G_n < G$ of increasing finite index in a fixed finitely generated group G, assuming $\bigcap_n G_n = 1$. We focus in particular on finitely presented residually free groups, calculating their ℓ_2 Betti numbers, rank gradient and asymptotic deficiency.

If G is a limit group and K is any field, then for all $j \geq 1$ the limit of $\dim H_j(G_n, K)/[G, G_n]$ as $n \to \infty$ exists and is zero except for $j = 1$, where it equals $-\chi(G)$. We prove a homotopical version of this theorem in which the dimension of $\dim H_j(G_n, K)$ is replaced by the minimal number of j-cells in a $K(G_n, 1)$; this includes a calculation of the rank gradient and the asymptotic deficiency of G. Both the homological and homotopical versions are special cases of general results about the fundamental groups of graphs of *slow* groups.

We prove that if a residually free group G is of type FP_m but not of type FP_∞, then there exists an exhausting filtration by normal subgroups of finite index G_n so that $\lim_n \dim H_j(G_n, K)/[G : G_n] = 0$ for $j \leq m$. If G is of type FP_∞, then the limit exists in all dimensions and we calculate it.

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On index-exponent relations and Brauer p-dimensions of finitely-generated field extensions

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Keywords: Brauer group, (Schur) index-exponent relations, Brauer/absolute Brauer p-dimension, finitely-generated extension

Let E be a field of absolute Brauer dimension $\mathrm{abrd}(E)$, and F/E a transcendental finitely-generated extension. The present research shows that the Brauer dimension $\mathrm{Brd}(F)$ is infinite, if $\mathrm{abrd}(E) = \infty$. When the absolute Brauer p-dimension $\mathrm{abrd}_p(E)$ is infinite, for some prime number p, it proves that for each pair (n, m) of integers with $n \geq m > 0$, there is a central division F-algebra of Schur index p^n and exponent p^m. When $\mathrm{abrd}_p(E) < \infty$, lower bounds on the Brauer p-dimension $\mathrm{Brd}_p(F)$ are obtained in several frequently used special cases. Finally, it is proved that if q is prime or $q = 0$, then there exist characteristic q fields $E_{q,k}\colon\ k \in \mathbb{N}$, such that $\mathrm{Brd}(E_{q,k}) = k$ and $\mathrm{abrd}_p(E_{q,k}) = \infty$, for every prime $p > \max\{2, q\}$. Moreover, when $q > 0$, $E_{q,k}$ can be chosen so that $[E_{q,k}\colon E_{q,k}^q] = \infty$. These results solve negatively a problem posed in Section 4 of the survey, presented in [1].

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Comparison between the two definitions of AI

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Keywords: artificial intelligence, AI definition

Two different definitions of the Artificial Intelligence concept have been proposed in papers [1] and [2]. The first definition is informal. It says that any program that is cleverer than a human being, is acknowledged as Artificial Intelligence. The second definition is formal because it avoids reference to the concept of 'human being'. The readers of papers [1] and [2] might be left with the impression that both definitions are equivalent and the definition in [2] is simply a formal version of that in [1]. This talk will compare both definitions of Artificial Intelligence and, hopefully, will bring a better understanding of the concept.

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SL_2-invariants of free metabelian Lie algebras

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Keywords: free metabelian Lie algebras, classical invariant theory, noncommutative invariant theory

Let KX_d be a vector space with basis $X_d = \{x_1, \dots, x_d\}$ over a field K of characteristic 0. One of the main topics of classical invariant theory is the study of the algebra of invariants $K[X_d]^{SL_2(K)}$, where KX_d is a module of the special linear group $SL_2(K)$ isomorphic to a direct sum $V_{k_1} \oplus \cdots \oplus V_{k_r}$ and V_k is the $SL_2(K)$-module of binary forms of degree k. Noncommutative invariant theory deals with the algebra of invariants $F_d(\mathfrak{V})^G$ of the group $G < GL_d(K)$ acting on the relatively free algebra $F_d(\mathfrak{V})$ of a variety of K-algebras $\mathfrak{V}$. When the group G is finite and $\mathfrak{V}$ is a variety of associative algebras (including the case of the variety of all associative algebras) the picture is quite clear, see e.g. [6]. Not too much is known when $\mathfrak{V}$ is a variety of Lie algebras, see [1, 3]. The invariants $F_d(\mathfrak{V})^{UT_2(K)}$ of the unitriangular group $UT_2(K)$ were considered in [5, 4]. Recently the special case $F_d(\mathfrak{A}^2)^{UT_2(K)}$, where $\mathfrak{V} = \mathfrak{A}^2$ is the variety of metabelian Lie algebras was studied in detail in [2].

We consider the algebra $F_d(\mathfrak{A}^2)^{SL_2(K)}$ of $SL_2(K)$-invariants of $F_d(\mathfrak{A}^2)$ and describe the cases when this algebra is finitely generated. This happens if and only if $KX_d \cong V_1 \oplus V_0 \oplus \cdots \oplus V_0$ or $KX_d \cong V_2$ as an $SL_2(K)$-module (and in the trivial case $KX_d \cong V_0 \oplus \cdots \oplus V_0$). For small d we give a list of generators even when $F_d(\mathfrak{A}^2)^{SL_2(K)}$ is not finitely generated.

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Associative algebras and Lie algebras defined by Lyndon words

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Keywords: Lyndon words, Lie algebras, associative algebras, Artin-Shelter regular algebras, growth of algebras

Assume that $X = \{x_1, \cdots, x_g\}$ is a finite alphabet and $\mathbf{k}$ is a field. We study the class $\mathfrak{C}(X, W)$ of associative graded $\mathbf{k}$-algebras A generated by X and with a fixed obstructions set W consisting of Lyndon words in the alphabet X. Important examples are the monomial algebras $A = \mathbf{k}\langle X\rangle/(W)$, where W is an antichain of Lyndon words of arbitrary cardinality and the enveloping algebra

$U\mathfrak{g}$ of any X-generated Lie $\mathbf{k}$-algebra $\mathfrak{g}$. We prove that all algebras A in $\mathfrak{C}(X, W)$ share the same Poincaré-Birkhoff-Witt type $\mathbf{k}$-basis built out of the so called *Lyndon atoms* N (determined uniquely by W) but, in general, N may be infinite. We prove that A has polynomial growth if and only if the set of Lyndon atoms N is finite. In this case A has a $\mathbf{k}$-basis $\mathfrak{N} = \{l_1^{\alpha_1} l_2^{\alpha_2} \cdots l_d^{\alpha_d} \mid \alpha_i \geq 0, 1 \leq i \leq d\}$, where $N = \{l_1, \cdots, l_d\}$. Surprisingly, in the case when A has polynomial growth its global dimension does not depend on the shape of its defining relations but only on the set of obstructions W. We prove that if A has polynomial growth of degree d then A has global dimension d and is standard finitely presented, with $d - 1 \leq |W| \leq d(d-1)/2$. We study when the set of standard bracketings $[W] = \{[w] \mid w \in W\}$ is a Gröbner-Shirshov Lie basis. We use our general results to classify the Artin-Schelter regular algebras A generated by two elements, with defining relations $[W]$ and global dimension ≤ 7. We give an extremal class of monomial algebras, the Fibonacci-Lyndon algebras, F_n, with global dimension n and polynomial growth, and show that the algebra F_6 of global dimension 6 cannot be deformed, keeping the multigrading, to an Artin-Schelter regular algebra.

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Directed metric dimension of oriented complete bipartite graphs

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Keywords: directed distance, directed metric dimension, oriented graph, oriented complete bipartite graph

Let D be an oriented graph. Let $S = \{s_1, s_2, \ldots, s_k\}$ be an ordered subset of $V(D)$ and v be a vertex of D. A representation of v with respect to S is a k-vector $r(v|S) = (d(v, s_1), d(v, s_2), \ldots, d(v, s_k))$, where $d(x, y)$ denotes the directed distance of x to y. If all vertices have representations and if two distinct vertices have distinct representation then S is a resolving set of D. The minimum cardinality of a resolving set in D is the directed metric dimension of D, $dim(D)$. Necessary and sufficient conditions of 1-dimensional oriented graphs are known and in this talk we discuss some sufficient conditions for 2-dimensional oriented graphs. We shall conclude by presenting metric dimension of some oriented complete bipartite graphs.

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Tangent codes

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Keywords: Zariski tangent bundle, zeta polynomial of a code, minimum distance of a code, covering radius of a code, Hamming isometry of a code

Let $\overline{\mathbb{F}_q}$ be the algebraic closure of a finite field $\mathbb{F}_q$ and $X \subset \overline{\mathbb{F}_q}^n$ be an affine variety, defined over $\mathbb{F}_q$. We study the typical behavior of an $\mathbb{F}_{q^m}$-Zariski tangent

space $T_a(X, \mathbb{F}_{q^m})$, $a \in X(\mathbb{F}_{q^m}) := X \cap \mathbb{F}_{q^m}^n$, $m \in \mathbb{N}$, from the point of view of the error correcting codes, by the means of global geometric properties of X.

Duursma obtains an explicit relation between the homogeneous weight enumerator of a Reed-Solomon or a Goppa code and the zeta function of the supporting projective curve. Based on this correspondence, he introduces the notions of a zeta polynomial $P_C(t)$ and a zeta function $\frac{P_C(t)}{(1-t)(1-qt)}$ of an abstract linear code $C \subset \mathbb{F}_q^n$. He says that C satisfies the Riemann Hypothesis Analogue if $P_C(t) = P_C(0)\prod_{i=1}^{\sigma+1}(1-\omega_i t)$ for $\omega_i \in \mathbb{C}$ with $|\omega_i| = \sqrt{q}$ for all $1 \leq i \leq \deg P_C$.

For any $k \in \mathbb{N}$ and $\sigma \in \mathbb{Z}$ with $0 \leq \sigma \leq k$ let us suppose that the polynomial $f \in \mathbb{F}_q[x_{\sigma+1}, \ldots x_{k+1}]$ has $\frac{\partial f}{\partial x_j} \not\equiv 0$ for $\forall \sigma + 1 \leq j \leq k+1$ and consider the hypersurface $V_{k+1-\sigma}(f) = \{a \in \overline{\mathbb{F}_q}^{k+1-\sigma} \mid f(a) = 0\}$. We say that the hypersurface $\overline{\mathbb{F}_q}^{\sigma} \times V_{k+1-\sigma}(f) \subset \overline{\mathbb{F}_q}^{k+1}$, defined over $\mathbb{F}_q$ has splitting index σ.

Proposition 1. *If $T_a(\overline{\mathbb{F}_q}^{\sigma} \times V_{k+1-\sigma}(f), \mathbb{F}_q)$ with $1 \leq \sigma \leq k$, $a \in \mathbb{F}_q^{\sigma} \times V_{k+1-\sigma}(f)(\mathbb{F}_q)$ satisfies the Riemann Hypothesis Analogue then*

$$|q - (k+1-\sigma)| \leq (\sigma+1)\sqrt{q} \quad \text{and} \quad \left|q\frac{k+\sigma-1}{k} + \frac{k+1-3\sigma}{\sigma}\right| \leq (\sigma+1)\sqrt{q}.$$

In particular, $T_b(\overline{\mathbb{F}_q}^{\sigma} \times V_{k+1-\sigma}(f), \mathbb{F}_{q^m})$ with $b \in \mathbb{F}_{q^m}^{\sigma} \times V_{k+1-\sigma}(f)(\mathbb{F}_{q^m})$ are not subject to the Riemann Hypothesis Analogue for sufficiently large $m \in \mathbb{N}$.

Let $X \subset \overline{\mathbb{F}_q}^n$ be an affine variety and $\Pi_i : X \to \Pi_i(X) \subseteq \overline{\mathbb{F}_q}^{n-d}$ be the puncture of X at the coordinates, labeled by $i = (i_1, \ldots, i_d)$ for some $1 \leq i_1 < \ldots < i_d \leq n$. We say that X is d-generic if for any d-tuple of indices $i = (i_1, \ldots, i_d)$ the puncture $\Pi_i : X \to \Pi_i(X)$ if a finite morphism

Proposition 2. *Let $X \subset \overline{\mathbb{F}_q}^n$ be an affine variety, defined over $\mathbb{F}_q$, $k = \dim X$ and $d \in \mathbb{N}$, $d \leq n - k$.*

(i) If the degree of X is relatively prime to $p = \mathrm{char}(\mathbb{F}_q)$ then there exist $N_0(X), \ldots, N_{k-1}(X) \in \mathbb{N}$, $\delta(X, d) \in \mathbb{N}$, such that for sufficiently large $m \in \mathbb{N}$ there exist at least $\deg X[q^{mk} + \sum_{i=0}^{k-1}(-1)^{k-i}N_i(X)q^{mi}] - \delta(X,d)q^{m(k-1)}$ points $a \in X(\mathbb{F}_{q^m})$ with minimum distance $d(T_a(X, \mathbb{F}_{q^m}) > d$.

(ii) If $d(T_a(X, \mathbb{F}_{q^m}) > d$ and $\Pi_i(a) \in \Pi_i(X)^{\mathrm{smooth}}$ for all $m \geq m_o$ and for all $i = (i_1, \ldots, i_d)$, $1 \leq i_1 < \ldots < i_d \leq n$ then X is d-generic.

We say that $x^{\tau} := x_{\tau_1} \ldots x_{\tau_k}$, $1 \leq \tau_1 < \ldots < \tau_k \leq n$ is a transcendence basis monomial of an affine variety $X \subset \overline{\mathbb{F}_q}^n$ if $\dim X = k$ and $\overline{x_{\tau_1}}, \ldots, \overline{x_{\tau_k}} \in \overline{\mathbb{F}_q}[X] = \overline{\mathbb{F}_q}[\overline{x_1}, \ldots, \overline{x_n}] = \overline{\mathbb{F}_q}[x_1, \ldots, x_n]/I(X, \overline{\mathbb{F}_q})$ is a maximal subset of $\overline{x_1}, \ldots, \overline{x_n} \in \overline{\mathbb{F}_q}[X]$, which is transcendent over $\overline{\mathbb{F}_q}$.

Proposition 3. *Let $x^{\tau(j)} := x_{\tau(j)_1} \dots x_{\tau(j)_k}$, $1 \leq j \leq s$ be all the transcendence basis monomials of an affine variety $X \subset \overline{\mathbb{F}_q}^n$ of $dim X = k$. Then:*

(i) X is $(n - 1 - \dim V(x^{\tau(1)}, \dots, x^{\tau(s)}))$-generic;

(ii) the covering radius of $T_a(X, \mathbb{F}_{q^m})$, $a \in X(\mathbb{F}_{q^m})$ is $\dim V\left(\frac{x_1 \dots x_n}{x^{\tau(1)}}, \dots, \frac{x_1 \dots x_n}{x^{\tau(s)}}\right)$ for sufficiently large $m \in \mathbb{N}$.

Proposition 4. *Let $X \subset \overline{\mathbb{F}_q}^n$ be an affine variety, defined over $\mathbb{F}_q$, $\psi_1, \dots, \psi_n \in \mathbb{F}_q[x_1, \dots, x_n] \setminus \{0\}$, $\sigma \in \mathrm{Sym}(n)$ and $p = \mathrm{char}(\mathbb{F}_q)$. Then*

$$\Psi = (x_{\sigma(1)}\psi_{\sigma(1)}(x_1^p, \dots, x_n^p), \dots, x_{\sigma(n)}\psi_{\sigma(n)}(x_1^p, \dots, x_n^p) : X \longrightarrow \Psi(X)$$

is an $\mathbb{F}_q$-isomorphism of affine varieties, inducing Hamming isometries $(d\Psi)_a : T_a(X, \mathbb{F}_{q^m}) \to T_{\Psi(a)}(\Psi(X), \mathbb{F})_{q^m})$ for $\forall a \in X(\mathbb{F}_{q^m}) \setminus V(\psi_1 \dots \psi_n)$, $\forall m \in \mathbb{N}$.

Proposition 5. *Let $X \subset \overline{\mathbb{F}_q}^n$ be an affine variety, defined over $\mathbb{F}_q$ and $\mathcal{I} \to S$ be a family of Hamming isometries $\mathcal{I}(a) = (\mathcal{I}(a)_{ij})_{i,j=1}^n \in GL(n, \mathbb{F}_q)$, $\mathcal{I}(a) : T_a(X, \mathbb{F}_q) \to \mathcal{I}(a)(T_a(X, \mathbb{F}_q))$ for $\forall a \in S \subseteq X(\mathbb{F}_q)$. Denote $L^{\alpha}_{\mathbb{F}_q} := \prod_{\beta \in \mathbb{F}_q \setminus \{\alpha\}} \frac{t-\beta}{\alpha-\beta}$ and consider $\varphi_i = \sum_{a \in S} \left[\sum_{j=1}^n \mathcal{I}(a)_{ij}(x_j - x_j^q) \right] L^{a_1}_{\mathbb{F}_q}(x_1^p) \dots L^{a_n}_{\mathbb{F}_q}(x_n^p) \in \mathbb{F}_q[x_1, \dots, x_n]$. Then $\varphi = (\varphi_1, \dots, \varphi_n) : X \to \varphi(X) \subset \overline{\mathbb{F}_q}^n$ is an $\mathbb{F}_q$-morphism of affine varieties with $(d\varphi)_{\Phi_p^{-1}(a)} = \mathcal{I}(a)$ for $\forall a \in S$.*

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Accurate eigenvalues of singular totally nonnegative matrices

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Keywords: eigenvalues, totally nonnegative matrices, high relative accuracy

In 2005 Fallat and Gekhtman fully characterized the Jordan canonical form of the irreducible totally nonnegative matrices. In particular, all nonzero eigenvalues are simple and the possible Jordan structures of the zero eigenvalues are well understood and described. Starting with the bidiagonal decomposition of these matrices, we present an algorithm for computing all the eigenvalues, including the Jordan blocks, to high relative accuracy in what we believe is the first example of Jordan structure being computed accurately in the presence of roundoff errors.

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Group actions on Lie algebras and their identities

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Keywords: G-identities, group action, Lie algebras, T-ideals

Let G be a finite group acting faithfully on the 3-dimensional simple Lie algebra sl_2 over the field of the complex numbers. A classical result of F. Klein (1884) states that then G must be one of the following groups: C_n, the cyclic group of order n, D_n, the dihedral group of order $2n$, A_4, S_4 and A_5. Here S_n and A_n are the symmetric and the alternate groups on n letters, respectively.

It is of importance to describe the G-identities of sl_2, that is the identities with the corresponding group action.

The analogous problem for the associative algebra of the 2×2 matrices was studied and solved by Berele (2004).

Here we describe in each of the above cases for G, a basis of the G-identities of sl_2. We note that the methods we use rely on the description of the graded identities of sl_2.

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Noether's problem for finite p-groups

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Keywords: Noether's problem, the rationality problem, metacyclic p-groups

Let K be a field and G be a finite group. Let G act on the rational function field $K(x(g) : g \in G)$ by K automorphisms defined by $g \cdot x(h) = x(gh)$ for any $g, h \in G$. Denote by $K(G)$ the fixed field $K(x(g) : g \in G)^G$. *Noether's problem* then asks whether $K(G)$ is rational (= purely transcendental) over K. It is related to the inverse Galois problem, to the existence of generic G-Galois extensions over k, and to the existence of versal G-torsors over k-rational field extensions. So far just a handful of results about Noether's problem are obtained when the groups are nonabelian. For example, Kang proves in [1] the rationality of $K(G)$ over K, if G is any metacyclic p-group and K is any field containing enough roots of unity. In this report, we give a positive answer to Noether's problem if G is any central group extension of the general metacyclic p-group, provided that K is infinite and it contains sufficient roots of unity. We also discuss several results of the first author about Noether's problem for abelian extensions of cyclic p-groups (see [2,3]). The key idea to prove our results is to find a faithful G-subspace W of the regular representation space $\bigoplus_{g \in G} K \cdot x(g)$ and to show that W^G is rational over K. The subspace W is obtained as an induced representation from some properly chosen abelian subgroup H. The construction of a faithful G-subspace W requires a full knowledge of the generators and relations in G. Since such a classification is lacking, we prove several group theoretical results that give the needed information.

Acknowledgements. This work is partially supported by a project No. RD/13.03.2014 of Shumen University.

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On the covering number of small symmetric groups and some sporadic simple groups

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Keywords: group theory, combinatorics, *GAP*, linear programming

We say that a group G has a finite covering if G is a set theoretical union of finitely many proper subgroups. The minimal number of subgroups needed for such a covering is called the covering number of G denoted by $\sigma(G)$.

Let S_n be the symmetric group on n letters. For odd n Maroti determined $\sigma(S_n)$ with the exception of $n = 9$, and gave estimates for n even showing that $\sigma(S_n) \leq 2n-2$. Using *GAP* calculations, as well as incidence matrices and linear programming, we show that $\sigma(S_8) = 64, \sigma(S_{10}) = 221, \sigma(S_{12}) = 761$.

We also show that Maroti's result for odd n holds without exception proving that $\sigma(S_9) = 256$.

We establish in addition that the Mathieu group M_12 has covering number 208, and improve the estimate for the Janko group J_1 given by P.E. Holmes.

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Grinberg-Kazhdan-Drinfeld theorem

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Keywords: arc space, jet scheme, formal neighborhood, Grinberg-Kazhdan-Drinfeld theorem

The theorem represents the formal neighborhood of a closed arc over a scheme of finite type. Proved initially over the complex numbers by Grinberg and Kazhdan, Drinfeld gave then a purely algebraic proof over arbitrary fields. In this talk we discuss briefly the idea of his proof, some examples, and propose a geometrically meaningful equivalent of the theorem in the case of morphisms.

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Identities in matrix algebras with Grassmann entries

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Keywords: identities in algebras, Grassmann algebra, matrix algebras with Grassmann entries

The talk presents the latest results of the author concerning the existence of polynomial identities in matrix algebras with Grassmann entries (elements of the infinite dimensional Grassmann algebra G).

In 2002 Uzi Vishne gave in [1] the explicit form of two multilinear polynomials of degree 8 and showed that they are identities of minimal degree for the matrix algebra $M_2(G)$. Since then there are no published contributions to the topic even for $n = 2$.

The talk concerns some concrete subalgebras of $M_n(G)$ for any $n \geq 2$ for which identities of minimal degree are given.

The identities of block-diagonal matrices with building blocks the elements of the considered algebras are discussed as well.

A conjecture is made for the type of the identities in $M_n(G)$ for any $n \geq 2$ and this conjecture is confirmed in the case $n = 2$ for the identities pointed by Vishne.

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Degree spectra of sequences of structures

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Keywords: degree spectra, definability in a structure, Turing degrees

There is a close parallel between classical computability and the effective definability on abstract structures. For example, the Σ^0_{n+1} sets correspond to the sets definable by means of computable infinitary Σ_{n+1} formulae on a structure $\mathfrak{A}$. In his last paper [3], Soskov gives an analogue for abstract structures of Ash's reducibilities between sets of natural numbers and sequences of sets of natural numbers. He shows that for every sequence of structures $\vec{\mathfrak{A}}$, there exists a structure $\mathfrak{M}$ such that the sequences that are ω-enumeration reducible to $\vec{\mathfrak{A}}$ coincide with the c.e. in $\mathfrak{M}$ sequences. He generalizes the method of Marker's extensions for a sequence of structures. Soskov demonstrates that for any sequence of structures its Marker's extension codes the elements of the sequence so that the n-th structure of the sequence appears positively at the n-th level of the definability hierarchy. The results provide a general method given a sequence of structures to construct a structure with n-th jump spectrum contained in the spectrum of the n-th member of the sequence. As an application a structure with spectrum consisting of the Turing degrees which are non-low$_n$ for all $n < \omega$ is obtained. Soskov shows also an embedding of the ω-enumeration degrees into the Muchnik degrees generated by spectra of structures.

We apply these results and generalize the notion of degree spectrum with respect to an infinite sequence of structures $\vec{\mathfrak{A}}$ in two ways as Joint spectra of $\vec{\mathfrak{A}}$ and Relative spectra of $\vec{\mathfrak{A}}$. We study the set of all lower bounds of the generalized notions in terms of enumeration and ω-enumeration reducibility.

Acknowledgements. This work was partially supported by the Sofia university Science Fund and FMI, SU.

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Pairs of elements of orders 2 and 3 in the Ree groups ${}^2G_2(3^n)$

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Keywords: (2,3)-generated group, Hurwitz group

We classify all pairs of elements x and y with $|x| = 2$ and $|y| = 3$ in the group $G = {}^2G_2(3^n)$ (odd $n > 1$) such that either $|xy| = 7$ or $G = \langle x, y\rangle$.

Acknowledgements. This work was supported by the European Social Fund through the Human Resource Development Operational Programme under contract BG051PO001-3.3.06-0052 (2012/2014).

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Classification of the locally divergent orbits on reductive homogeneous spaces and applications

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Keywords: algebraic groups, lattice, homogeneous space

We describe the closures of locally divergent orbits under the action of maximal tori on G/Γ where G is a product of $r \geq 2$ reductive algebraic groups over local fields and Γ is an irreducible lattice in G. We prove that if $r = 2$ then the closure of every locally divergent but not closed orbit is a union of finitely many orbits, in particular, it is not homogeneous. This result disprove a conjecture of Margulis. On the other hand, if $r > 2$ we prove that the conjecture is "generally" true. We provide a number-theoretical application of the latter result. The particular case when G is a direct product of several copies of $SL(2)$ has been treated in [1].

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Determinant functors and K theory

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Keywords: determinant functors, Waldhausen K-theory

I will explain work, in collaboration with F. Muro [1], on a small algebraic model for the truncation (to dimensions 0 and 1) of Waldhausen K-theory, and how this can lead to an explicit calculation of the K_1 group [2], and to applications to universal determinant functors as introduced by Deligne. Finally, I will explain a unified framework for determinant functors [3], that settles conjectures of Maltsiniotis and of Neeman, and a question of Grothendieck to Knudsen.

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Section: Analysis, Geometry, and Topology

Invited Talks

From locally conformally Kähler to bi-Hermitian structures

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Keywords: complex manifolds, symplectic manifolds, Poisson structures, Kähler geometry

R. Goto has proved that one can deform any Kähler metric on a compact complex manifold endowed with a holomorphic Poisson structure into a family of generalized Kähler structures. I will discuss a framework in which this result can be suitably extended to the non-Kähler case, and thus present new existence results for bi-Hermitian metrics on certain classes of compact complex surfaces with odd first Betti number.

Acknowledgements. This is a joint work with Michael Bailey and Georges Dloussky.

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Explicit constructions of RIP matrices and related problems

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Keywords: restricted isometry property, RID matrices, sumsets estimates

We give a new explicit construction of $n \times N$ matrices satisfying the Restricted Isometry Property (RIP). Namely, for some $\epsilon > 0$, large N and any n satisfying $N^{1-\epsilon} \leq n \leq N$, we construct RIP matrices of order $k \geq n^{1/2+\epsilon}$ and constant $\delta = n^{-\epsilon}$. This overcomes the natural barrier $k = O(n^{1/2})$ for proofs based on small coherence, which are used in all previous explicit constructions of RIP matrices. Key ingredients in our proof are new estimates for sumsets in product sets and for exponential sums with the products of sets possessing special additive structure. The paper was published in Duke Math. J. in 2011. Our construction is still the only one which breaks the $n^{1/2}$ barrier.

Complex surfaces and indefinite metrics

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Keywords: compact complex surfaces, para-hypercomplex structures

We study geometric structures on compact complex surfaces that are related to split quaternions and lead to existence of anti-self-dual metrics. These structures, called para hypercomplex, para hyperhermitian and para hyperkaehler (or hypersymplectic), are analogs of the hypercomplex, hyperhermitian and hyperkaehler structures in the definite case. Every compact complex surface admitting a para-hyperhermitian structure has vanishing first Chern class and, unlike the definite case, many of these surfaces carry infinite dimensional families of such structures. In the talk the relation among the different structures together with several examples and some open problems will be considered.

Acknowledgements. The second named author is partially supported by a grant from Simons Foundation (#246184)

Kobayashi's conjecture for K3 surfaces and for hyperkähler manifolds

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Keywords: Kobayashi pseudometric, K3 surface, hyperkähler manifold, ergodic complex structures

The Kobayashi pseudometric on a complex manifold M is the maximal pseudometric such that any holomorphic map from the Poincare disk to M is distance-decreasing. Kobayashi has conjectured that this pseudometric vanishes on Calabi-Yau manifolds. Using ergodicity of complex structures we prove Kobayashi's conjecture for any hyperkaehler manifold that admits a deformation with a Lagrangian fibration, if its Picard rank is not maximal. We shall discuss the proof of Kobayashi's conjecture for K3 surfaces and for certain hyperkaehler manifolds. These results are joint with S. Lu and M. Verbitsky.

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From R. Thom to M. Gromov: A new approach to the *h*-principle

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Keywords: singular foliations, Haefliger's structures, Γ_n-structures, open manifolds, immersion, Smale's eversion of the sphere

Already in 1973, W. Thurston showed that the *jiggling lemma* [4] short-circuits the h-principle of M. Gromov for proving that the homotopy fibre of $B\Gamma_n \xrightarrow{\pi} BGL(n)$ is n-connected. Here $B\Gamma_n$ denotes the classifying space of the Haefliger's structures (or singular foliations) [1] and $BGL(n)$ denotes the classifying space of the n-linear bundles; the map π classifies the normal bundle to the considered singular foliation.

Thanks to a lemma by R. Thom [3], this approach may be enhanced. This gives a new approach to the very classical immersion theory and to geometric structures (symplectic, contact or complex) on open manifold.

Acknowledgements. Joint work with Gaël Meigniez, (Université Bretragne-Sud, France), supported by ERC Geodycon.

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Comparison of invariant functions

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Keywords: convex, $\mathbb{C}$-convex and strongly pseudoconvex domain, Carathéodory and Bergman distances, Lempert function

It follows by the Lempert theorem that the Carathéodory distance and the Lempert function coincide on any convex domain and any bounded C^2-smooth $\mathbb{C}$-convex domain in $\mathbb{C}^n$. The aim of the talk is to discuss weaker versions of the Lempert theorem on $\mathbb{C}$-convex domains, containing no complex lines, as well as on strongly pseudoconvex domains. More precisely, based on [1–4], we shall compare (up to additive constants) the Carathéodory distance, Bergman distance and the Lempert function on certain families of domains.

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Trace formulae for perturbed Landau Hamiltonians

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Keywords: Landau Hamiltonians, high-energy spectral asymptotics, Radon transforms

The Landau Hamiltonian describes the energy of a two-dimensional quantum particle subject to a constant magnetic field. Its spectrum consists of eigenvalues of infinite multiplicity, called Landau levels. I will consider the Landau operator perturbed by electric potentials which decay at infinity. The spectrum of the perturbed operator consists of eigenvalue clusters which accumulate at the Landau levels. First, I will discuss the effect of high-energy shrinking of these clusters. Moreover, I will provide an explicit description of the asymptotic eigenvalue density in terms of appropriate Radon transforms of the perturbing potential. The talk will be based on the articles [1, 2].

Acknowledgements. The partial support of the Chilean Science Foundation *Fondecyt* under Grant 1130591 is gratefully acknowledged.

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Local properties of homogeneous ANR compacta

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Keywords: cohomological dimension, cohomology groups, homogeneous compact, n-manifold

The study of homogeneous compact metric absolute neighborhood retracts (ANR) is inspired by the Bing-Borsuk conjecture that any such a space of dimension n is an Euclidean n-manifold (by the way, this conjecture implies the Poincaré conjecture). In the present talk we discuss some local cohomological properties of ANR-compacta.

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Communications

On closed "minimal" k-imitations of closed convex sets in $\mathbb{R}^n$

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Keywords: convex set, k-imitation

If $k \in \mathbb{N}$ then two subsets A and B of $\mathbb{R}^n$, $1 \leq k < n$, are called k-imitations of each other if a k-plane meets A if and only if it meets B. Given a close convex subset B of $\mathbb{R}^n$, $n \geq 2$, and $1 \leq k < n$ we discuss how one can construct "minimal" closed k-imitations C of B, that is, closed k-imitations C of B that are minimal with respect to dimension.

About well-posedness in vector optimization

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Keywords: vector optimization, perfect pseudometric, effective perturbation, minimizing sequence

Fix a Banach reticulate lattice E. A function $\rho : X \times X \longrightarrow \mathbb{E}$ is called a *pseudometric* on a space X if: $\rho(x, x) = 0$, $\rho(x, y) = \rho(y, x)$ and $\rho(x, z) \leq \rho(x, y)+$

$\rho(y,z)$ for all $x,y,z \in X$. A pseudometric ρ is called a *perfect pseudometric* on a space X if it is continuous, any ρ-Cauchy sequence $\{x_n\}$ has an accumulation point in X and the sets $\{y \in X : \rho(x,y) = 0\}$ are compact.

We put $E^\infty = E \cup \{\infty\}$ and consider that $x \leq \infty$ for each $x \in E^\infty$.

The analogous of Ekeland's variational principle there exists for perfect pseudometric ρ and any proper function. Denote by $Ef(f,X,E)$ the set of all minimal elements of the function f in X.

If $y \in E$, then the sequence $\{x_n \in X : n \in \mathbb{N}\}$ is called a *y-minimizing sequence* of the problem $\min(f,X,E)$ if $\lim_{n\to\infty} f(x_n) = y$ and $y \leq f(x_{n+1}) \leq f(x_n)$ for each $n \in \mathbb{N}$. The sequence $\{x_n \in X : n \in \mathbb{N}\}$ is called an *almost y-minimizing sequence* of $\min(f,X,E)$ if $\lim_{n\to\infty} f(x_n) = y$ and $y \leq f(x_n)$ for each $n \in \mathbb{N}$. The minimization problem $\min(f,X,E)$ is said to be *well-posed at* (respectively *weakly well-posed at*) $y \in E$ if $y \in \mathrm{Inf}(f,X,E)$ and any y-minimizing (respectively, pseudo y-minimizing) sequence $\{x_n \in X : n \in \mathbb{N}\}$ is convergent in X.

Denote by $C^*(X,E)$ the Banach lattice of all mappings $g \in C(X,E)$ with the bounded image $g(X)$ and the norm $\|g\| = \sup\{\|g(x)\| : x \in X\}$, and by $C^o(X,E)$ the Banach lattice of mappings $g \in C(X,E)$ with relatively compact image $f(X)$.

For $f : X \longrightarrow \mathbb{E}^\infty$ we define the *continuous perturbations compact perturbations*, *bounded perturbations*) $f+g$. The perturbation $f+g$ is called an *x-effective perturbation*, $x \in X$, of the mapping f if there exists $y \in Ef(f+g,X,E)$ such that $f(y) \leq f(x)$. We put $C^*_{ef}(X,E,f) = \{g \in C^*(X,E) : f+g$ *is a continuous effective perturbation of* $f\}$ and $C^o_{ef}(X,E,f) = \{g \in C^o(X,E) : f+g$ *is a continuous effective perturbation of* $f\}$. For each point $b \in \mathrm{dom}(f) = f^{-1}(X)$ there exist many compact b-effective perturbation. The minimization problem $\min(f,X,E)$ is said to be *well-posed at $y \in E$* if $y \in \mathrm{Inf}(f,X,E)$ and any y-minimizing sequence $\{x_n \in X : n \in \mathbb{N}\}$ is convergent in X. We shall say that $y \in X$ is a *strict effective solution* of the minimization problem $\min(f,X,E)$ if there exists an open subset U of the space X such that $y \in Ef(f,X,E) \cap U$ and $f(y) \leq f(x)$ for each $x \in U \cap \mathrm{dom}(f)$. Denote by $Ef_s(f,X,E)$ the set of all strict effective solutions of the minimization problem $\min(f,X,E)$. For a mapping $f : X \longrightarrow E^\infty$ we put $C_{sef}(X,E,f) = \{g \in C^*(X,E) : f+g$ *is a continuous strict effective perturbation of* $f\}$.

Theorem 1. *Let $f : X \longrightarrow E$ be a continuous bounded from below mapping. Then the set $C_{sef}(X,E,f)$ is dense in the space $C^*(X,E)$.*

Theorem 2. *Let $f : X \longrightarrow E$ be a continuous bounded from below mapping.*

If the space X is first-countable, then the set $C_{wp}(f, X, E)$ is dense in the space $C^*(X, E)$.

Hardy–type inequalities with weights

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Keywords: Hardy inequality, weights, sharp estimates

Hardy-type inequality with weights is derived in abstract form. Particular cases are presented to demonstrate the applicability of the method and to show generalizations of existing results. Sharpness of inequalities is proved and the results are illustrated with several examples.

Continuity properties in different hypertopologies for non-additive set multifunctions

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Keywords: continuity property, regularity, non-additive set multifunction, hypertopology

Various continuity properties (including regularity), their properties, relationships and applications for non-additive set multifunctions in different hypertopologies are discussed.

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Some fixed point theorems in *p*-quasi-cone metric spaces

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Keywords: quasi-cone metric space, p-quasi-cone metric space, Hausdorff p–quasi–cone metric space, contractive mapping, ordered Banach space.

Huang and Zhang [1] have introduced the concept of cone metric spaces replacing the set of real numbers by an ordered Banach space. They have proved some theorems about fixed points in cone metric spaces. Abdeljaward and Karapinar [2] and Sonmez [3] have given a definition of quasi–cone metric spaces. Shaddad and Noorani [4] have proved some fixed point theorems in quasi–cone metric spaces.

In this paper, we have introduced the concept of p-quasi-cone metric space for $p \geq 1$ which is a generalization of quasi-metric spaces where $p = 1$. Also we have proved some new fixed point theorems in p-quasi-metric spaces using contractive conditions which generalize the results of Shaddad and Noorani [4].

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Tabulation of prime knots by arc index

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Keywords: prime knot, arc presentation, arc index, grid diagram

Every knot can be presented on the union of finitely many half planes which have a common boundary line, so that each half plane contains a single arc of the knot. Such a presentation is called an arc presentation of the knot. The arc index of a knot is the minimal number of half planes needed in its arc presentations [2].

A grid diagram of a knot is a knot diagram constructed by finitely many vertical line segments and the same number of horizontal line segments such that at each crossing a vertical segment crosses over a horizontal segment. A grid diagram with n vertical segments is easily converted to an arc presentation on n half planes, and vice versa. Grid diagrams are useful in several ways. A slight modification of a grid diagram gives a front projection of its Legendrian

imbedding. Grid diagrams are used to compute Heegaard Floer homology and Khovanov homology.

Various authors have determind the arc index of prime knots up to 11 arcs [1, 3–7]. In this work, we've tabulated prime knots of arc index twelve up to 16 crossings. This is achieved by generating grid diagrams of twelve arcs which contain all prime knots of arc index twelve. Among the prime knots with arc index twelve, the torus knot of type (5,7) has the largest crossing number 28 and it is the only one with this crossing number.

Crossings	3–9	10	11	12	13	14	15	16
Prime knots with arc index 12	0	123	0	627	1412	3180	6216	7955

Acknowledgements. This talk will be similar to the one that the first author presented in Oberwolfach last year.

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International Conference Mathematics Days in Sofia
July 7–10, 2014, Sofia, Bulgaria

Monogenic, hypermonogenic and holomorphic Cliffordian functions

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Keywords: Clifford analysis, monogenic, hypermonogenic and holomorphic Cliffordian functions, special functions

The aim of this talk is to make an overview on three generalizations to higher dimensions of the function's theory of one complex variable. The first one concerns the so-called monogenic functions which were introduced by F. Brackx, R. Delanghe and F. Sommen (1982), the second is the theory of hyper-monogenic functions developed by H. Leutwiler (1992), and the last one studies the theory of holomorphic Cliffordian functions due to G. Laville and the author (1998). A careful analysis of the connections between those three classes of functions argues for the holomorphic Cliffordian ones.This is a set which is endowed with many function theoretical tools that are also offered for complex holomorphic functions. Basically, they were introduced in order to contain the functions $x \mapsto x^n (n \in \mathbf{N}$, x being a paravector) and to be stable under any directional derivation. Consequently, they form a class of functions containing the two others.

The basic notions in this three theories will be given. Some applications will be evoked, especially those of the possible construction of high dimensions analogues of elliptic functions and also of Clifford valued automorphic forms.

Gabor frames and quasiasymptotics of distributions

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Keywords: Gabor fames, distributions, quasisymptotics

We show the convergence of the Gabor frame series on the space of test functions $\mathcal{S}(\mathbb{R})$ and its dual space, the space of tempered distributions $\mathcal{S}'(\mathbb{R})$, and give characterization of boundedness in $\mathcal{S}'(\mathbb{R})$ in terms of Gabor frame coefficients. Our results are then applied to study local and non-local asymptotic properties of Schwartz distributions via Gabor frame expansions.

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International Conference Mathematics Days in Sofia
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Some Tauberian conditions for discrete power series methods of summability

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Keywords: Tauberian conditions, power series methods, discrete power series methods

In this paper we give some Tauberian conditions in order to obtain convergence of a series from its discrete power series summability.

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On regularly weighted generated sequences

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Keywords: Tauberian theorems, Tauberian conditions, Regularly weighted generated sequences, Slowly oscillating sequences, Slowly decreasing sequences

We introduce a new concept which is called a regularly weighted generated sequence for sequences of real numbers. Moreover, we obtain some Tauberian conditions in terms of regularly weighted generated sequences for the power series method of summability, and generalize some classical Tauberian theorems given by Tietz [Acta Sci. Math. 54 (1990), Nos 3–4, 355–365].

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Fixed point theorems for quasicontractions in complete dislocated and b-dislocated metric spaces

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Keywords: dislocated metric, b-dislocated metric, quasicontraction, common fixed point

In this paper, we prove some unique fixed point results for quasicontractions in the setting of complete dislocated and b-dislocated metric spaces. Our theorems involve one and two selfmappings and extend and generalize some several known results of literature in a class of spaces that is effectively larger then metric spaces.

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Section: Differential Equations and Mathematical Physics

Invited Talks

Resonances in scattering by two magnetic fields at large separation and a complex scaling method

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Keywords: Aharonov-Bohm effect, resonances, complex scaling method

We study the quantum resonances in magnetic scattering in two dimensions. The scattering system consists of two obstacles by which the magnetic fields are completely shielded. The trajectories trapped between the two obstacles are shown to generate the resonances near the positive real axis, when the distance between the obstacles goes to infinity. The location is described in terms of the backward amplitudes for scattering by each obstacle. A difficulty arises from the fact that even if the supports of the magnetic fields are largely separated from each other, the corresponding vector potentials are not expected to be well separated. To overcome this, we make use of a gauge transformation and develop a new type of complex scaling method. We can cover the scattering by two solenoids at large separation as a special case. The obtained result heavily depends on the magnetic fluxes of the solenoids. This indicates that the Aharonov-Bohm effect influences the location of resonances.

Acknowledgements. This is joint work with Hideo Tamura from Okayama University in Japan.

Alien limit cycles

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Keywords: 16th Hilbert problem

We give a short historical introduction to the problem of limit cycles (the second part of the 16th Hilbert problem). Some recent developments, related to the existence of alien limit cycles and bifurcations of limit cycles from polycycles are discussed.

On cubic 1-D Schrödinger equation with rapidly decreasing potential

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Keywords: Schrödinger equation, asymptotic profile, potentials

We consider the Cauchy problem for the nonlinear Schrödinger equation with cubic nonlinearity

$$\left| \begin{array}{l} iu_t + \partial_x^2 - W(x)u = \pm u|u|^2, \quad (t,x) \in \mathbb{R} \times \mathbb{R} \\ (u)|_{t=0} = u_0, \end{array} \right. \tag{1}$$

where $W : \mathbb{R} \to \mathbb{R}$ is a real-valued, $W \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R})$ and W is decaying sufficiently rapidly at infinity, namely following [1] we require

$$\int_{\mathbb{R}} \langle x \rangle^3 |W(x)| dx < \infty. \tag{2}$$

We shall assume that the point spectrum of $H = -\partial_x^2 + W(x)$ is empty, i.e.,

$$Hf - zf = 0, f \in L^2(\mathbb{R}), z \in \mathbb{C} \Longrightarrow f = 0. \tag{3}$$

Finally, we shall assume that W is generic, that is the transmission coefficient $T(\tau)$ satisfies

$$T(0) = 0. \tag{4}$$

The main result asserts that for $s \in (1/2, 1)$ one can find constants $C > 0$ and $\delta > 0$ so that whenever

$$u_0 \in H^s(\mathbb{R}) \cap L^1(\mathbb{R}) : \|u_0\|_{L^1} + \|u_0\|_{H^s} \leq \delta,$$

the unique global solution $u \in C([0, \infty), H^s) \cap L^\infty_{t,x}$ to the Cauchy problem (1) exists and moreover it satisfies

$$\sup_{t>0} \sqrt{t} \|u(t, \cdot)\|_{L^\infty} \leq C \left(\|u_0\|_{L^1} + \|u_0\|_{H^s} \right). \tag{5}$$

Asymptotic profile of the solution is presented too. The results extend the previous results for the supercritical case (power nonlinearity $u|u|^{p-1}$ with $p > 3$ and rapidly decaying potential) obtained in [1] and the case of free Laplace operator (i.e., $W = 0$) obtained in [2].

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Non-integrability of the Painlevé equations as Hamiltonian systems

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Keywords: Painlevé equations, Hamiltonian systems, Morales-Ruiz-Ramis-Simó theory, differential Galois theory

In this talk I will present non-integrable results (obtained jointly with O. Christov and E. Horozov) for the second, fourth, fifth and sixth Painlevé equations from the point of view of the Hamiltonian dynamics.

Shallow water waves with asymptotic conditions at infinity

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Keywords: non-linear equations, shallow water waves

We consider the initial value problem for a family of shallow water equations on the line $\mathbb{R}$ with various asymptotic conditions.

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Asymptotics of the number of the transmission eigenvalues

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Keywords: interior transmission eigenvalues, Weyl formula with remainder, eigenvalue-free regions

This is a joint work with Vesselin Petkov. Let $\Omega \subset \mathbf{R}^d$, $d \geq 2$, be a bounded, connected domain with a C^∞ smooth boundary $\Gamma = \partial\Omega$. A complex number $\lambda \in \mathbf{C}$, $\lambda \neq 0$, will be said to be a transmission eigenvalue if the following problem has a non-trivial solution:

$$\left\{ \begin{array}{ll} (\nabla c_1(x)\nabla + \lambda n_1(x))\, u_1 = 0 & \text{in} \quad \Omega, \\ (\nabla c_2(x)\nabla + \lambda n_2(x))\, u_2 = 0 & \text{in} \quad \Omega, \\ u_1 = u_2, \;\; c_1\partial_\nu u_1 = c_2\partial_\nu u_2 & \text{on} \quad \Gamma, \end{array} \right. \tag{1}$$

where ν denotes the exterior Euclidean unit normal to Γ, $c_j, n_j \in C^\infty(\overline{\Omega})$, $j = 1, 2$, are strictly positive real-valued functions. The purpose is to study the asymptotic behavior of the counting function $N(r) = \sharp\{\lambda - \text{transmission eigenvalue} : |\lambda| \leq r^2\}$, $r > 1$, under the condition

$$c_1(x)n_1(x) \neq c_2(x)n_2(x), \quad \forall x \in \Gamma. \tag{2}$$

Our main result is the following

Theorem 1. *Assume* (2) *fulfilled. Assume also either the condition*

$$c_1(x) = c_2(x), \quad \forall x \in \Gamma, \tag{3}$$

or the condition

$$c_1(x) \neq c_2(x), \quad \forall x \in \Gamma. \tag{4}$$

Then, the transmission eigenvalues form a discrete set in $\mathbf{C}$ *and we have the following asymptotics*

$$N(r) = (\tau_1 + \tau_2)r^d + \mathcal{O}_\varepsilon(r^{d-\kappa+\varepsilon}), \quad r \to +\infty, \tag{5}$$

for every $0 < \varepsilon \ll 1$*, where*

$$\tau_j = \frac{\omega_d}{(2\pi)^d} \int_\Omega \left(\frac{n_j(x)}{c_j(x)} \right)^{d/2} dx,$$

ω_d *being the volume of the unit ball in* $\mathbf{R}^d$*, and* $\kappa = \frac{1}{2}$ *if* (3) *holds,* $\kappa = \frac{2}{5}$ *if* (4) *holds. Moreover, if in addition to* (4) *we assume either the condition*

$$\frac{n_1(x)}{c_1(x)} \neq \frac{n_2(x)}{c_2(x)}, \quad \forall x \in \Gamma, \tag{6}$$

or the condition

$$\frac{n_1(x)}{c_1(x)} = \frac{n_2(x)}{c_2(x)}, \quad \forall x \in \Gamma, \tag{7}$$

then (5) *holds with* $\kappa = \frac{1}{2}$.

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Communications

Berezin transform of two arguments

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Keywords: Bergman kernel, harmonic functions, holomorphic functions, Berezin transform

We describe the asymptotic behavior of the Berezin transform of two arguments which extends the standard notion of the Berezin transform on function spaces with a reproducing kernel (the Bergman spaces). The asymptotic analysis we give is in the setting of the holomorphic and the harmonic Fock spaces, respectively.

Bi-characteristic curves of body and surface waves and application in geophysics

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Keywords: Bi-characteristic curves, seismic modelling

New approach to 3D modelling of elastic piecewise homogeneous media is introdused. The method is develop for geophisical surveys and is based on the general principle of tomography with Earthquake as a source of the signal and receiver stations on the surface. The wave propagation in solid media is described by a system of three strongly coupled hyperbolic equations with piece – wise constant

coefficients. The characteristic set and bi-characteristic curves of this system are computed in a homogeneous half-space with free boundary and the formulae of reflection and diffraction of the bi-characteristics on the internal boundaries of the media. Applications of the characteristic set and bi-characteristic curves for the inverse problem in geophysics and Earth modelling are given.

Acknowledgements. The research was supported by CEI Research Fellowship Programme (CERES).

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Protter problem for 3-D Keldish type equations

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Keywords: Keldish type equations, boundary value problems, quasi-regular solutions, uniqueness of solution

About sixty years ago M. Protter formulated new boundary value problems for elliptic-hyperbolic equations of Tricomi type as 3-D analogues of the classical two-dimensional Guderley-Morawetz transonic problem. When the equations are considered only in the hyperbolic-parabolic part of the original domain, the Protter problems are 3-D analogues of Darboux or Cauchy-Goursat plane problems. Then the boundary data is prescribed on the parabolic boundary and on a portion of the hyperbolic boundary of the domain. In contrast to 2-D case, the Protter problems for weakly hyperbolic equations are strongly ill-posed. In the present paper similar statement of Protter problem for equations of Keldish type is given. In that case the data is prescribed only on one characteristic and analogically to the elliptic case no data is specified on the parabolic boundary. It is shown that the Protter problem for Keldish type equations is not correctly set and sufficient conditions for uniqueness of quasi-regular solution are found.

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On symmetries and conservation laws of the Majda–Biello system

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Keywords: evolution equations, symmetries, conservation laws, Hamiltonian operators

In 2003, A. J. Majda and J. A. Biello derived and studied the so-called reduced equations for equatorial baroclinic-barotropic waves

$$\begin{aligned} u_t &= du_{xxx} - vu_x - uv_x \\ v_t &= v_{xxx} - uu_x, \end{aligned}$$

to which we refer as to the Majda–Biello system. The equations in question describe the nonlinear interaction of long-wavelength equatorial Rossby waves and barotropic Rossby waves with a significant midlatitude projection in the presence of suitable horizontally and vertically sheared zonal mean flows.

We present a Hamiltonian structure for Majda–Biello system and describe all generalized symmetries and conservation laws for the latter. It turns out that there are only three symmetries corresponding to x-translations, t- translations and to a scaling of t, x, u and v, and four conservation laws, one of which is associated to the conservation of energy, the second conserved quantity is just the Hamiltonian functional and the other two are Casimir functionals of the Hamiltonian operator admitted by our system. Our result provides *inter alia* a rigorous proof of the fact that the Majda–Biello system have just the conservation laws mentioned in the paper by Majda and Biello.

On a free boundary variant of a conjecture of De Giorgi

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Keywords: free boundary problem, De Giorgi's conjecture, minimal graph, monotone minimizer, barriers

Drawing out a fascinating connection between Bernstein's problem, on the one hand, and the study of global, bounded and monotone solutions to the semilinear elliptic equation $\Delta u = u^3 - u$ in $\mathbb{R}^n$, on the other, a famous conjecture of De Giorgi states that the level sets of such solutions are hyperplanes, at least in dimension $n \leq 8$. The conjecture was verified for $n \leq 8$ by Savin. Recently, Del Pino, Kowalczyk and Wei constructed a counterexample in dimension $n = 9$, using an intricate fixed point argument. In this talk I would like to discuss the construction of such a counterexample in an appealing free boundary variant of De Giorgi's conjecture. Our approach uses only the elementary means of the method of barriers.

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Nonlinear curl-curl problem

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Keywords: variational methods, strongly indefinite functional, concentration compactness, Kerr-type nonlinear medium, unified field theory for classical electrodynamics

The question of the existence of localized in space solutions of the Maxwell's equations in nonlinear dispersive media, has engaged the interest of researchers both in mathematics and in physics. In a Kerr-type nonlinear medium, solving the Maxwell's equations for a monochromatic fields with a real valued complex amplitudes, simplifies to solving the semilinear elliptic equation

$$\nabla\times\nabla\times E + V(x)E = \Gamma(x)|E|^2E, \tag{1}$$

for a vector field $E : \mathbb{R}^3 \to \mathbb{R}^3$, where $\nabla\times$ denotes the *curl* operator, V and Γ belong to $L^\infty(\mathbb{R}^3, \mathbb{R})$.

A unitarian field theory for classical electrodynamics, introduced by Benci and Fortunato in 2004, considers a semilinear perturbation of the Maxwell's equations (SME). The nonlinearity yields the existence of a finite-energy solitary waves of the SME, by which the particles are described. In the magnetostatic case, the SME reduce to the semilinear elliptic equation

$$\nabla\times\nabla\times A = f'(A), \tag{2}$$

where f' is the gradient of a smooth function $f : \mathbb{R}^3 \to \mathbb{R}$, and $A : \mathbb{R}^3 \to \mathbb{R}^3$ is the gauge potential related to the magnetic field.

Both problems imply specific conditions for their characterizing parameters, V and Γ and f, with regard to the physical situations they describe. However, from a more general mathematical perspective equations (1) and (2) share common difficulties.

Existence of nontrivial finite-energy solutions of a slightly generalized version of equation (1)

$$\nabla\times\nabla\times u + V(x)u = \Gamma(x)|u|^{p-1}u,$$

is proven for three types of coefficients V and Γ, and an exponent $p \in (1, 5)$. The approach is taken from a variational calculus perspective. Applying a special

cylindrically-like symmetric ansatz, the initial vector-fields variational problem is restricted to a scalar-functions one. Further, the latter variational problem is reduced to a minimization problem on a constraint manifold. By using concentration compactness techniques and tools from the nonlinear functional analysis the existence of minimizers is attained. The reduced by the ansatz dimensionality is suitable for a numerical treatment. As a starting point, the method of steepest descent for finding local minimizers is presented, as well as an outlook for more advanced and comprehensive investigations.

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Curved A_∞-algebras and simplicial gauge theory

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Keywords: gauge theory, A_∞-algebra

The notion of curved differential graded (dg) algebra encodes the main algebraic properties of connections and curvatures. When transferring curved dg algebra structures via chain contractions, one obtains more general homotopy associative algebraic structures known as curved A_∞-algebras.

We will propose a definition of abstract algebraic gauge theory based on curved A_∞-algebras and show that classical gauge theory and also gauge theories arising from non-commutative manifolds provide examples of this definition.

As a final example, we will present a construction of gauge theory on the simplicial matrix-valued cochains on a triangulated manifold.

Time permitting, possible applications to constructive quantum Yang-Mills theory will be discussed as well.

This is a joint project based on our previous work [1].

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Supercritical and critical cases for 2D and 3D BVP for quasilinear equations of mixed elliptic-hyperbolic type

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Keywords: elliptic-hyperbolic semilinear equations, critical Sobolev exponents, nonexistence of nontrivial solutions

This talk is based on our joint work with D. Lupo, K. Payne (Italy) and L. Dechevski (Norway), already published, or in progress.

Starting from the ground-breaking paper of Pohožaev (1965), it is well known that the homogeneous Dirichlet problem for semilinear elliptic equations such as $\Delta u + u|u|^{p-2} = 0$ in Ω – a bounded subset of $\mathbb{R}^n$, with $n \geq 3$, permits only the

trivial solution $u \equiv 0$ if the domain is star-shaped, the solution is sufficiently regular, and $p > 2^*(n) := 2n/(n-2)$. The latter quantity is the critical exponent in the Sobolev embedding of $H_0^1(\Omega)$ into $L^p(\Omega)$ for $p \leq 2^*(n)$, which fails to be compact at the critical exponent. In dimension $n = 2$, the critical nonlinearity is of exponential type, but it has been shown that the nonexistence principle in supercritical case also holds for certain two dimensional problems of mixed elliptic-hyperbolic type for the mixed elliptic-hyperbolic Gellersted operator L, and is also valid for a large class of such problems even in higher dimensions. In dimension 2, such operators have a long-standing connection with transonic fluid flow. All such operators are invariant with respect to a certain anisotropic dilation which defines a suitable notion of star-shapedness by using the flow of the vector field which is the infinitesimal generator of the invariance. In all cases, for the operator L the critical exponent phenomenon is of pure power type $u|u|^{p-2}$, where p agrees with a critical Sobolev exponent ($p = 2^*(n, m)$) in the embedding of a suitably weighted version of $H_0^1(\Omega)$ into $L^p(\Omega)$. As usual, in BVP for such mixed elliptic-hyperbolic Gellersted operator L, the boundary data's part Σ is a proper subset or all of $\partial\Omega$. The set Γ on which no data is prescribed is a piece of a characteristic surface. The lack of a boundary condition on Γ complicates the control of the corresponding boundary integral in the Pohožaev argument, but if Γ is characteristic and tangential to the dilation flow, a sharp Hardy-Sobolev inequality ensures that the contribution along Γ has the "right" sign, that suffices for completion of the estimate.

In all $2D$ and multimensional cases the results are like

Theorem. *Let $\Omega \subset \mathbf{R}^2$ be a Guderley-Morawetz domain (or Tricomi domain, or Tricomi-Frankl domain). Under some restrictions on the boundary (which are fixed in each case), if the nonlinearity is $F(u) = u^p$, if $p > 2^*(2, m) = 2(m+4)/m$ (the critical Sobolev exponent), then for solutions $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ it follows $u \equiv 0$. The same is true also at the critical case $p = 2^*(1, m)$ if we suppose in addition that some part of the boundary is strongly star-like surface at its noncharacteristic points.*

The multidimensional cases are given for some Protter-Morawetz problems, which linear variant is strongly over determined in a sense of classical solvability.

Two type of Open Questions:

1. Some nonexistence results for generalized solutions also?
2. Existence results of nontrivial solutions in the subcritical cases?

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Semi-Fredholm solvability of the $(3+1)-D$ Protter problems for the wave equation

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Keywords: Wave equation, boundary value problems, generalized solution, semi-Fredholm solvability, asymptotic expansion

Boundary value problems introduced by M. H. Protter for the nonhomogeneous wave equation are studied in $(3+1)-D$ domain, bounded by two characteristic cones and a non-characteristic ball. They could be considered as multidimensional analogues of the Darboux problem in the plane – homogeneous data are prescribed on one of the cones, and on the noncharacteristic part of the boundary. However, unlike its planar analogue, the Protter problem is not well posed. In fact, in the framework of classical solvability the problem is not Fredholm, because it has an infinite-dimensional cokernel. Alternatively, it is known that the unique generalized solution of a Protter problem may have a strong power-type singularity at the vertex O of the boundary light cone. This singularity is isolated at the point O and does not propagate along the cone. When the right-hand side function is a harmonic polynomial we find the exact asymptotic expansion of the generalized solution in negative powers of the distance to the singular point O. In the general case of smooth right-hand side function, we discuss the semi-Fredholm solvability of Protter problem. We give necessary and sufficient conditions for the existence of bounded solution and find a priori estimates for the solution.

Mathematical Modelling

Invited Talks

On the Pontryagin maximum principle

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Keywords: Pontryagin maximum principle, infinite-dimensional problems, control variations

A basic idea of the classical approach to obtain necessary optimality conditions in the calculus of variations and optimal control is to perturb with suitable variations the reference trajectory and then compare the unperturbed value of the cost functional with the perturbed one. Here we compare several classes of variations for the study of infinite-dimensional optimal control problems. Also, we present an abstract result on nonseparability of two closed sets and obtain as corollaries versions of the maximum principle under various assumptions. Finally, we give illustrative examples indicating some limits of applicability of the existing variational techniques.

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On differential inclusions with nonconvex right-hand side

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Keywords: differential inclusions with non-convex right-hand sides, colliding on a set, sweeping process, limiting normals

The talk is a survey of some results on the existence of local solutions of a differential inclusion with upper semicontinuous possibly nonconvex right-hand side. Some sufficient conditions will be presented. Moreau's sweeping process with the cone of limiting normals is a model example of a differential inclusion with upper semicontinuous possibly nonconvex right-hand side. Several results about existence of solution of a sweeping process involving o-minimal structures as assumptions will be discussed, as well as possible applications.

Controlled branching stochastic processes and their relatives

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Keywords: branching processes, immigration, migration, random control, discrete time

In 1974 Sevastyanov introduced a general class of branching stochastic processes for modeling the evolution of a population. A significant generalization of this

class pertaining to random control functions was proposed by Yanev (no relation to the speaker) in 1975. Controlled branching processes include the important sub-classes of processes with different regimes of immigration and emigration and regenerative processes among others.

The Bulgarian school of probability and statistics has been actively involved in the study of controlled branching processes since the beginning. Under the visionary leadership and mentorship of Nikolay Yanev (a.k.a. "The Captain"), a dynamic group of Bulgarian mathematicians has made significant contributions to both the theory and applications of these stochastic models (see [1]).

The aim of this talk is to present a brief overview of some results for controlled branching processes, especially those obtained by Bulgarian mathematicians and their collaborators. We will also discuss connections and applications to other branching models.

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Communications

Approximate solutions of fractional order boundary value problems by using Sinc-Galerkin method

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Keywords: Caputo derivative, quadrature rule, Riemann-Liouville derivative, sinc-Galerkin method, Mathematica

In this paper, we present sinc-Galerkin method for solving fractional order boundary value problems approximately. In order to illustrate the applicability and accuracy of the present method, the method is applied to some specific examples, and simulations of the approximate solutions are provided. The results are compared with the ones obtained by some methods in literature.

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Some probabilistic characteristics for queueing systems with two priority classes, semi-Markov orientation and strategy in the free state

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Keywords: distribution of the busy periods, distribution of queue length, probabilities of the system's states

It will be presented some performance results on the functioning of a class of queueing systems of the type M2—G2—1 with two priority classes and orientation time at the passing service process from one class of priority to another [1]. The requests of class 1 are endowed with absolute (preemptive) priority which arrived in the system while serving a request of class 2 or orientation to class 2, interrupt the service of this message how and the orientation towards the service of messages of class 2. In connection with interrupted message's fate how and interrupted orientation, there are cautious some schemes in functioning of models, which will be described. Among performance characteristics obtained we mention the following: the distribution of the busy periods and various auxiliary characteristics; the distribution of queue length both stationary as well as non-steady state; probabilities of the system's states and others. The research methods are based on Laplace-Stieltjes transforms and generators functions. The results are obtained in terms of Laplace-Stieltjes transforms, but in the convenient form for algorithmization, which allows the development and implementation, in modern programming languages, of numerical algorithms for solving characteristics [2]. Given that the distribution functions of service and orientation time are of general order, the mentioned algorithms can be used for modelling and analysing a wide spectrum of real problems [3].

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Regularized wavelet-Galerkin method for Fredholm integral equations of the first kind

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Keywords: Fredholm integral equation of first kind, ill-posed problem, Galerkin method, trigonometric wavelets

In this work, we use trigonometric wavelet bases to numerical solution of Fredholm integral equations of first kind in Holder space. We establish the convergence of the Tikhonov regularized solution in Holder space $C^{\alpha}[a,b]$. Some error estimates are also given under a suitable assumption on the exact solution. Many problems in applied mathematics and engineering can be formulated as Fredholm integral equations of the first kind:

$$Kf(x) = \int_a^b k(x,y)f(y)dy = g(x)$$

where the kernel $k(;)$ and the right-hand side g are smooth real-valued functions.

The determination of the solution f of this equation is an ill-posed problem in the sense of Hadamard; in the sense that the solution (if it exists) does not

depend continuously on the data. In this work, we suggest a numerical procedure based on the Galerkin projection method, where the solution is projected onto a subspace generated by trigonometric wavelet scaling functions.

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On the separation of optimal spherical configurations

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Keywords: optimal configurations, fullerenes, minimal energy, Riesz potentials, equilibrium measures

Spherical configurations with some optimal properties have wide-ranging application in science. In this talk we shall survey briefly the topic and focus on

minimal energy configurations and in particular on their separation properties. Our techniques naturally lead to investigation of optimal configurations in the presence of external field. Numerical computations will be presented as well.

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Generalized Padé approximants for plane condenser and distribution of points

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Keywords: Padé approximant, plane condenser, m_1-almost uniform convergence

Given a regular plane condenser (E, F), let α and β be triangular point tables, $\alpha \in E$, $\beta \in F$,

$$\omega_n^{\alpha}(z) := \prod_{k=1}^{n}(z - \alpha_{n,k}) \quad \text{and} \quad \omega_n^{\beta}(z) := \prod_{k=1}^{n}(z - \beta_{n,k}), \quad n = 1, 2, \ldots$$

We say that the points (α, β) are extremal with respect to the condenser (E, F), if

$$\lim_{n\to\infty}\left|\frac{\omega_n^{\alpha}(z)}{\omega_n^{\beta}(z)}\right|^{1/n} = C_0\exp\left(\frac{h(z)}{c}\right),$$

where $c = c(E, F)$ is the capacity for the condenser and $h(z)$ is the harmonic measure for the condenser, with C_0 being a positive constant.

In the present paper, a criterium relating to the extremal distribution of the tables α and β is proved. The results are provided in terms of generalized Padé approximant, associated with the point tables α and β.

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Optimal Bayesian classifier for model-based RNA-Seq classification

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Keywords: classification, RNA-Seq, model-based

Motivation: Sequencing datasets consist of a finite number of reads which map to specific regions of a reference genome. Most effort in modeling these datasets focuses on the detection of univariate differentially expressed genes. However, for classification, we must consider multiple genes and their interactions.

Results: Thus, we introduce a multivariate Poisson model (MP) and the associated optimal Bayesian classifier (OBC) for classifying samples using sequencing data. Lacking closed-form solutions, we employ a Monte Carlo Markov Chain (MCMC) approach to perform classification. We demonstrate superior or equivalent classification performance compared to typical classifiers for two synthetic datasets and over a range of classification problem difficulties. We also introduce the Bayesian minimum mean squared error (MMSE) conditional error estimator and demonstrate its computation over the feature space. In addition, we demonstrate superior or leading class performance over an RNA-Seq dataset containing two lung cancer tumor types from The Cancer Genome Atlas (TCGA).

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Growth behavior of sequences of rational functions

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Keywords: approximation by rational functions, Padé approximation

Given a regular compact set E in the complex plane $\mathbb{C}$, assume that the function f is holomorphic on E (analytic and single valued or a univalent branch

of an analytic function). Denote by $E_{\rho(f)}$ the largest domain, canonical with respect to the Green's function $G_E(z,\infty)$ with pole at infinity such that f admits a continuation as a meromorphic function from the compact set E into E_ρ, $\rho > 1$. We suppose that $\rho(f) < \infty$. Set Π_n for the collection of all polynomials of degree not exceeding n and introduce the class $R_{n,m} := \{r, r = P/Q, P \in \Pi_n, Q \in \Pi_m, Q \not\equiv 0\}$. We call a polynomial Q, $Q \not\equiv 0$ normalized with respect to $E_{\rho(f)}$, if

$$Q(z) := \prod(z - \alpha_j) \prod(1 - z/\beta_j),$$

where $\alpha_j \in E_{\rho(f)}$ and $\beta_j \notin E_{\rho(f)}$ are all zeros of Q. Let $\{m_n\}$ be an infinite sequence of positive integers with $m_n = o(n)$ as $n \to \infty$ and suppose that $\{r_{n,m_n}\}$ is a sequence of rational functions, $r_{n,m_n} \in R_{n,m_n}$ such that

$$\limsup_{n\to\infty} \|f - r_{n,m_n}\|_E^{1/n} \le 1/\rho(f); \tag{1}$$

here $\|\cdot\|_E$ stands for the Chebyshev norm on E and the denominators normalized with respect to $E_{\rho(f)}$.

We show first that under the above conditions the sequence $\{r_{n,m_n}\}$ converges in capacity to the function f inside the domain $E_{\rho(f)}$.

Set $r_{n,m_n} := p_{n,m_n}/q_{n,m_n}$, $n = 12, \cdots$ where p_{n,m_n} and q_{n,m_n} do not have common divisors.

Suppose now that the function f has a singularity of multivalued character on the boundary $\partial E_{\rho(f)}$. Then the sequence $\{r_{n,m_n}\}$ converges maximally to f inside $E_{\rho(f)}$. With other words, the inequality (1) is exact, i.e.

$$\limsup_{n\to\infty} \|f - r_{n,m_n}\|_E^{1/n} = 1/\rho(f)$$

and on each compact set K in $E_{\rho(f)}$

$$\limsup_{n\to\infty} \|q_{n,m_n}(f - r_{n,m_n})\|_K^{1/n} = e^{\|G_E(z,\infty)\|_K}/\rho(f).$$

An example of a function and a sequence of rational approximants will be provided such that (1) is fulfilled but the sequence is not maximally convergent.

The presented theorems may be applied to sequences of classical and of generalized Padé approximants, as well as to sequences of best rational approximants, the real case.

The talk is generalizing results in [1].

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Quasilinearization method and third order convergence for second order singular boundary value problems with solutions in weighted spaces

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Keywords: generalized quasilinearization, singular boundary value problem, weighted space, higher order of convergence

The method of generalized quasilinearization has been extended for second order singular boundary value problems with solutions in weighted spaces. Natural lower and upper solutions are considered. Two sequences have been obtained that converge uniformly and monotonically to the unique solutions of the second order singular boundary value problems with solutions in weighted spaces. The rate of convergence of these sequences is cubic when the nonlinearity of the iterates is quadratic.

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Multicriteria assessment scale of future cyberthreats identification

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Keywords: cyberthreats identification, multicriteria assessment, optimization.

The process of identifying future cyberthreats on the basis of practical implementation of "scenario method" is briefly described after the methodological approach from [1]. As it is relying on experts' beliefs, analysis and validation an ad-hoc defined multicriteria assessment matrix [2] is proposed. Further

on, the matrix data is optimized using different future beliefs criteria. The obtained results produce a user-oriented ad-hoc multicriteria assessment of future cyberthreats. Some practical examples concerning social networks and smart environments cyberthreats identification and assessment are also given.

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Explicit thanks for the smart environments cyber threats scenario context to: A Feasibility Study on Cyber Threats Identification and their Relationship with Users' Behavioural Dynamics in Future Smart Homes, Bulgarian Science Fund, Ministry of Education Youth and Science, 2012-2014, DFNI-T01/4, www.smarthomesbg.com.

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Sevastyanov branching processes with non-homogeneous Poisson immigration

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Keywords: branching processes, limit theorems

Sevastyanov age-dependent branching processes allowing an immigration component are considered in the case when the moments of immigration form a

non-homogeneous Poisson process with intensity $r(t)$. The asymptotic behavior of the expectation and of the probability for non-extinction is investigated in the critical case depending from the asymptotic rate of $r(t)$. Corresponding limit theorems are also proved using different types of normalization. Among them we obtained limiting distributions similar to the classical ones of Yaglom (1947) and Sevastyanov (1957) and discovered also new phenomena due to the non-homogeneity.

Inequalities for ultraspherical polynomials. Proof of a conjecture of I. Raşa

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Keywords: Bernstein polynomials, Legendre polynomials, ultraspherical polynomials

A recent conjecture by I. Raşa asserts that the sum of the squared Bernstein basis polynomials is a convex function in $[0,1]$. This conjecture turns out to be equivalent to a certain upper pointwise estimate for the logarithmic derivative of Legendre polynomials outside the interval $[-1,1]$. We prove both upper and lower pointwise estimates for the logarithmic derivatives of the ultraspherical polynomials. In particular, we validate Raşa's conjecture.

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Calibration of p-values for multiple testing problems in Genomics

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Keywords: conservative statistical tests, p-values, FDR, statistical power

Conservative statistical tests are often used in complex multiple testing settings in which computing the type I error maybe difficult. In such tests, the reported p-value for a hypothesis can understate the evidence against the null hypothesis and consequently statistical power may be lost. False Discovery Rate adjustments, used in multiple comparison settings, can worsen the unfavorable effect. Despite these effects, the problem seems to be somewhat overlooked within the biostatistics and bioinformatics communities, with many practitioners not even aware of the issue.

We present a computationally efficient and test-agnostic calibration technique that can substantially reduce the conservativeness of such tests. As a consequence, a lower sample size might be sufficient to reject the null hypothesis for true alternatives, and experimental costs can be lowered.

As an example, we apply the calibration technique to the results of DESeq, a popular method for detecting differentially expressed genes from high-throughput RNA sequencing data. The increase in power maybe particularly high in small sample size experiments, often used in preliminary experiments and funding applications. In some situations, after correction, statistical power can increase 3 fold without the need of additional experimental costs.

Crack formation in a flexible honeycomb membrane under tensile force: a molecular dynamics simulation study

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Keywords: computer modeling, molecular dynamics simulation, graphene-like membrane, external stretching force, crack formation

We consider the fracture of a free-standing two-dimensional (2D) elastic-brittle network to be used as protective coating subject to constant tensile stress applied on its rim. Using a molecular dynamics simulation with a Langevin thermostat, we investigate the scission and recombination of bonds, and the formation of cracks in the 2D graphene-like hexagonal sheet for different pulling force f and temperature T. We find that bond rupture occurs almost always at the sheet periphery, and the mean first breakage time $\langle\tau\rangle$ of bonds decays with membrane size as $\langle\tau\rangle \propto N^{-\beta}$, where $\beta \approx 0.50 \pm 0.03$ and N denotes the number of atoms in the membrane. The probability distribution of bond scission times t is given by a Poisson function $W(t) \propto t^{1/3} \exp(-t/\langle\tau\rangle)$. The mean failure time $\langle\tau_r\rangle$ necessary to rip-off the sheet declines with growing size N as a power law $\langle\tau_r\rangle \propto N^{-\phi(f)}$. We also find $\langle\tau_r\rangle \propto \exp(\Delta U_0/k_BT)$, where the nucleation barrier for crack formation $\Delta U_0 \propto f^{-2}$, in agreement with Griffith's theory. $\langle\tau_r\rangle$ displays an Arrhenian dependence of $\langle\tau_r\rangle$ on temperature T. Our results indicate a rapid increase in crack spreading velocity with growing external tension f.

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Contribution to an extremal problem of Markov: Largest forward and backward partial sums of polynomial coefficients

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Keywords: Chebyshev polynomial, extremal problem, partial sum, polynomial coefficients, V. A. Markov

We consider V. A. Markov's general extremal problem of 1892 for real univariate polynomials P_n of degree $\leq n$ with $P_n(x) = \sum_{k=0}^{n} a_k x^k$, see e.g. [1, p. 4], [4, p. 39], [5, p. 246]:

Markov's Extremal Problem. *To find the maximum of the coefficient functional* $L = |\sum_{k=0}^{n} a_k \beta_k|$ *subject to the condition* $||P_n||_\infty \leq 1$ *(uniform norm on the interval* $I = [-1; 1]$*), where the* β_k*,* $0 \leq k \leq n$*, are given real parameters.*

Markov himself [5, p. 248] solved this problem, inter alia, for the case $L = |a_k|$ (k-th coefficient, $0 \leq k \leq n$; the solution for $k = n$ is due to P. L. Chebyshev [2]).

G. Szegö gave notice to P. Erdös [3, p. 1176] that he had obtained a solution for the case $L = |a_{k-1} \pm a_k|$, if $k \equiv n \mod 2$ (pairs of consecutive coefficients). Recently we have found in [8, 9, 10] solutions for related cases. In all these instances the Chebyshev polynomial T_n (with $||T_n||_\infty = 1$) [13] plays a crucial role as an extremizer of the coefficient functional L in question. Here we consider, for $1 \leq k \leq n-1$ and $n \geq 4$, two further cases of Markov's extremal problem, which also find applications in numerical analysis [14]:

(1) $L = |a_0 + a_1 + a_2 + \cdots + a_k|$ (k-th forward partial sum of coefficients), and

(2) $L = |a_k + a_{k+1} + a_{k+2} + \cdots + a_n| = B(k, P_n)$ (k - th backward partial sum of coefficients).

The case (1) with $k \equiv n \mod 2$ we have already settled in [6], improving on partial results given in [11], [13, (2.38)], see also [7]. The solutions obtained here for the remaining three instances are believed to be new, and in particular our solution for the case (2) with $k \not\equiv n \mod 2$ (Theorem) covers the partial result given in [12], which is referred to in [13, (2.43)]:

Theorem. *Let* $||P_n||_\infty \leq 1$ *and* $k \not\equiv n \mod 2$*, then* $B(k, P_n) \leq B(k, T_n)$*, except for* $k = 1$ *if* $n = 4q$*, and except for* $k = 2$ *if* $n = 4q + 1$*, where* $q \geq 1$*.*

Our results for the case (1) with $k \not\equiv n \mod 2$ and for the case (2) with $k \equiv n \mod 2$ are different in flavour, but all three theorems reveal new extremal properties of the classical polynomial T_n.

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Exceedance-type tests

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Keywords: two-sample problem, exceedance and precedence statistics, location-shift alternative, stochastic ordering

Suppose X and Y are random variables with continuous univariate distributions F and G, respectively. For testing the hypothesis $H_0 : F(x) = G(x)$ against the alternative $H_A : F(x) \geq G(x)$, there are simple tests based available on precedences and exceedances. One can count the number of observations in the Y-sample above all observations in the X-sample, or the number of observations in the X-sample below all those in the Y-sample. As suggested by Tukey, one or both of these statistics might be used to test H_0 against H_A. The test based on the sum of these two quantities is mentioned as the earliest work of Šidák on nonparametric statistics.

The extreme sample values may get inflated by possible outliers, which may adversely affect these test statistics. For this reason, we may want to reduce their influence by defining thresholds above the smallest and below the largest observed values in the samples. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be two independent random samples from the distributions F and G, respectively and denote the ordered X's and Y's by $X_{(1)} < \cdots < X_{(m)}$, and $Y_{(1)} < \cdots < Y_{(n)}$, respectively. Thresholds based on $(r+1)$-th order statistic from the Y-sample and $(m-s)$-th order statistic from the X-sample define the exceedance and precedence statistics of the form

$$(1) \qquad \begin{array}{lllll} A_s = & \text{the number of} & Y\text{-observations} & \text{larger than} & X_{(m-s)}, \\ B_r = & \text{the number of} & X\text{-observations} & \text{smaller than} & Y_{(1+r)}, \end{array}$$

where $0 \leq s < m$ and $0 \leq r < n$.

We study a family of rank statistics for the two-sample problem in which the test statistic is a sum of A_s and B_r for appropriate choices of r and s. It includes the Šidák's test as a special case.

Stochastic fluid models with two priority classes

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Keywords: fluid flow models, static priority service discipline, multi-class priority networks, continuous time Markov chain

We present an analysis of a two-priority fluid model with input rates governed by an irreducible Continuous Time Markov Chain (CTMC). We derive exact analytic results on the Laplace Stieltjes transform (LST) of the steady-state distribution of the lower priority fluid which is treated as a fluid model with jumps. Then we show a possible approximation of the output processes in multi-class on-off Markov networks where the results of the two-priority model are used recursively. This study has been motivated by the analysis of multi-class fluid networks (MFN) which find various applications in the petroleum industry, systems of heavy traffic and models of nowadays Internet traffic.

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Necessary and sufficient conditions of optimality for a damped hyperbolic equation in one space dimension

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Keywords: maximum principle, optimality, hyperbolic equation, beam

The present study deals with the necessary optimality condition for a class of distributed parameter systems in which the system is modeled in one space dimension by a hyperbolic partial differential equation subject to the damping and mixed constraints on state and controls. Pontryagin maximum principle is derived to be a necessary condition for the controls of such systems to be optimal. With the aid of some convexity assumptions on the constraint functions, it is obtained that maximum principle is also sufficient condition for the optimality.

Mathematical Aspects of Computer Science

Invited Talks

Construction and classification of optical orthogonal codes and related to them combinatorial structures

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Keywords: optical orthogonal code, cyclic difference set, constant weight error-correcting code

Since the introduction of fundamental principles of optical code-division multiple access (OCDMA) using on-off pulses as signature sequences, the search for powerful code structures began. Among the most famous codes introduced to date are optical orthogonal codes (OOCs). They also have applications in mobile radio, frequency-hopping spread-spectrum communications, radar, sonar signal design, constructing protocol-sequence sets for the M-active-out-of T users collision channel without feedback, etc.

OOCs with specific parameters may also be viewed as constant weight error-correcting codes and cyclic difference families. Also, some balanced incomplete block designs satisfy the requirements of an OOC.

So far a number of families of optical orthogonal codes have been constructed by two types of methods – methods using mathematical structures (projective geometry, finite field theory, design theory) and search methods using computer

algorithms (greedy algorithms, outer-product matrix algorithms, evolutionary algorithms).

In this presentation we will present a computer assisted technique for construction and classification of optical orthogonal codes with different parameters. The constructed OOCs can be used for direct practical applications, in recursive constructions of OOCs of higher parameters and for construction of related to them combinatorial structures.

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Towards formal modelling of the neurobiological aspects of trust and trustworthiness

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Keywords: trust, artificial intelligence, game and decision theory, neuroscience, neuroeconomics

Trust pervades the digital and the "brick and mortar" worlds as an important facilitator that mitigates risk and enables cooperation. The development of trust is of special importance in socio-economic exchange where both humans and computers must decide whether to trust a new acquaintance or a business partner, to what extent and when. Current research on trust in artificial intelligence and game theory still overlooks the neurobiology of trust. Artificial intelligence models of trust as a cognitive stance, subjective belief, or a subjective probability all fail to recognize the powerful role biology plays in human behaviour, thereby unnecessary restricting the scope, the applicability and the predictive power of formal models. Human trust is deeply biologically grounded and affected by intricate feedback loops between brain and body, conscious and unconscious, neurotransmitters and hormones.

The talk presents an overview of current research in neuroscience, behavioural economics and biology, showing that human trust is quite different from the

prevailing intuitive understanding of trust as volitional, rational and conscious activity. The paper also provides links between formal models of trust in game and decision theory and the growing empirical data in neuroscience, hoping to contribute to bridging the gap between these two disciplines. Our research is motivated by the belief that the growing neurobiological data on brain functioning and neurochemistry can enrich research on trust in game and decision theory and help produce models that formally characterize the biological basis of human trust and trustworthiness.

The threshold implementation method and applications

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Keywords: side-channel attacks, masking, secret sharing

During the computation of a cryptographic algorithm the device leaks information. Side-channel analysis can exploit this information to reveal the secret key.

Differential power analysis (DPA) attacks as introduced by Kocher et al. [4] exploit unintentional information leakage of a device's internal processing through its power consumption. Over the years, many types of countermeasures against DPA attacks have been proposed. One family of countermeasures is called masking and consists of computing the algorithm on a randomized representation of the data. For this purpose, the data is split in several shares that are processed sequentially or in parallel.

The problem of most masking approaches is that they underestimate the amount of information that is leaked by hardware. Typically, the security proofs are based on an idealized hardware model, resulting in requirements on the hardware that are very expensive to meet in practice.

Threshold Implementation (TI) is a masking scheme based on secret sharing and multi-party computation [5]. It provides provable security against 1^{st} order

[1]This abstract is based on [1–3]

DPA even in the presence of glitches. The only requirement is that the shares leak independently, but this requirement holds for all masking schemes. The approach can be described in short as follows. An (n, n) secret sharing scheme is constructed to share the secret variable that has to be processed by the circuit. The circuit is divided into combinational blocks in such a way that no single combinational block needs all shares as input. Constructing a sharing for a nonlinear function with algebraic degree d requires at least $d + 1$ shares. TI sharings impose additional requirements and become more difficult to find with increasing d. Several examples of functions have been shown to possess a TI sharing with 3 shares, namely the quadratic Boolean functions of two and three variables, multiplication on the extension field $\mathrm{GF}(2^{2m})/GF(2^m)$ (e.g. multiplication in $GF(4)$), and the Noekeon S-box [5]. It has been shown that all 3×3, 4×4 and the DES 6×4 S-boxes have a TI sharing with 3, 4 or 5 shares [1].

To summarize: the main advantages of the TI approach are that it provides provable security against first-order DPA attacks, even in the presence of glitches, that it requires few assumptions on the hardware leakage behavior, and that it allows to construct realistic-size circuits without intervention and design iterations.

The TI approach has been applied by now to several widely used ciphers: PRESENT, AES, FIDES and KECCAK. In this presentation we will discuss those implementations and will revisit the TI method discussing some new insides and developments.

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Persuasion technology design: computer-supported behaviour change

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Keywords: persuasive technology, persuasive games, gamer types, user model, personalization

With the increasing pervasiveness of information technologies in all spheres of human activities, opportunities arise to use computers to change people's behaviour for good. Recently, we have witnessed the emergence of persuasive technologies that aim to make people live healthier, protect the environment, engage in social causes. Computer-Supported Behaviour Change is an emerging interdisciplinary area, cutting across psychology, anthropology, health sciences, political science and computer science. This talk will discuss our recent work in this area in the context of designing a casual game that encourages players in healthy eating habits. Our results show clearly that the currently dominant "one-size-fits-all" approach in the design of persuasive technologies is not optimal and shows the benefits of personalization towards certain features of the user, for example, the user's gamer type.

International Conference Mathematics Days in Sofia
July 7–10, 2014, Sofia, Bulgaria

Communications

Construction methods for self-dual codes

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Keywords: coding theory, self-dual codes, construction, classification, algorithms

The self-dual codes form one of the important classes of linear codes because of their rich algebraic structure and their close connections with other combinatorial configurations like block designs, lattices, graphs, etc.

There exist several methods to construct self-dual codes of length $n+2$ from self-dual codes of length n. In [1], the authors describe three such methods recalling them *the recursive construction, the building-up construction and the Harada-Munemasa construction*. The first one (recursive construction) gives all inequivalent $[n+2, n/2+1, d+2]$ codes starting from the self-dual $[n, n/2, d]$ codes. The other two constructions give all self-dual codes of length $n+2$ from the self-dual codes of length n. Combined with efficient algorithms for classification, these methods give the possibility to classify all binary self-dual codes of a given length (see for example [2]).

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Secure aggregation of distributed information in multi-agent systems

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Keywords: multi-agent systems, secure information exchange, safe and informative protocols, generalised Russian cards problems

We consider a generic problem of *Secure Aggregation of Distributed Information (SADI)*, where a group of agents have information distributed amongst them, i.e., included in their collective knowledge, while each agent has only partial knowledge of it. The agents act as a team that has to exchange and aggregate that information, either as common knowledge within the group or in the individual knowledge of at least one of them. The exchange is performed over insecure communication channels and is presumed intercepted by an adversary "eavesdropper". The task of the team is to achieve the aggregation of the distributed information, following a prearranged protocol, in such a way that the adversary does not learn important information.

More specifically, we model the SADI problem by assuming that the information that each agent has is encoded by a set of "cards" that she holds in her hands, where the cards are drawn from a publicly known deck[1] and every card is in the hands of exactly one agent of the team. The goal of the team is to exchange and spread across the whole team the information about how the cards are distributed amongst the agents. The agents can only communicate by making public announcements over insecure channels and the goal of the eavesdropper is to learn the location of at least one of the cards by intercepting and analyzing the announcements exchanged by the agents in the team. We further assume that in their exchange of announcements the agents follow a publicly known (hence, known by the eavesdropper, too) protocol.

[1]The drawing and distribution of these cards is considered secret and secure and we do not discuss the side issue of how exactly that is done, as we regard the card deal merely as a metaphor. In reality, we assume that each of the agents has obtained her initial information in some private way.

The problem described above is a variation of the well-known "Russian cards problem" [1], [2]. In the present work we, on the one hand, generalize it to any number of agents, but on the other hand we restrict it essentially by assuming that the eavesdropper has no cards in his hands.

In [3] we introduce the generic SADI problem, present a combinatorial construction of protocols that provides a direct solution of a class of SADI problems and develop a technique of iterated reduction of SADI problems to "simpler" ones which are eventually solvable directly. Eventually, we show that our methods provide a solution to all SADI problems of sufficiently large size.

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A case study in weakly monotonic time

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Keywords: weakly monotonic time, Hoare style proof system

Weakly monotonic time (WMT, [2]) is a model of time for hybrid processes such as the archetypal continuous plant-digital control. Hybrid CSP (HCSP) [1,4] is a process algebra which extends Hoare's CSP by real-time constructs, including differential equations to model continuous evolution. We propose two Hoare-style

proof systems for reasoning about hybrid systems which are modelled in HCSP. The first proof system is WMT-based. The second one is based on the *Duration Calculus* (DC, [3]) and features a real-time counterpart to WMT's discrete component. Both systems are complete wrt validity in their respective logical languages.

Acknowledgements. Dimitar P. Guelev's work on this topic was supported through Bulgarian NSF grants DID02/32/2009 and DID02/32/2009 extension, application pending. Chaochen Zhou, Shuling Wang and Zhan Naijun were supported through National Basic Research Program 2014CB340700, NSFC 91118007 and NSFC 6110006.

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Approaches to the parallelization of data mining algorithms with the aim of improving the accuracy

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Keywords: parallel data mining, bioinformatics

Ever since the rise of availability of parallel architectures, intense research in the field of Machine Learning and Data Mining has been focused on utilizing these compute resources by parallelizing known data mining algorithms. Due to the size and dimensionality of contemporary datasets, the focus has been on developing

efficient parallel data mining algorithms, which improve upon the performance of the existing serial algorithms.

Yet for many tasks, the main priority is the quality of result obtained by the algorithm. Often Data Mining algorithms employ a greedy heuristic, which relies on the local optimality property in order to make the search through an enormous space of potential solutions feasible. Due to this limited exploration, finding the optimal solution is not guaranteed. The focus of our work is to develop strategies for investing parallel compute resources to improve (the result obtained by) a greedy data mining heuristic. We propose a strategy *Widening* for using parallel resources to augment a greedy heuristic by investing parallel resources in simultaneous "searches" or the exploration of many paths in the solution space.

The goal of Widening to invest the available parallel resources in a way that maximizes the solution space exploration and the improvement of solution quality. A smart utilization of parallel resources in the search for the optimal solution relies on *diversity*, to prevent exploring similar or identical solutions in parallel and force the widened search to investigate more broadly the solution space. The goal to avoid undesired overhead, which arises from the communication between parallel workers lead us to look for *Communication-less Widening* approaches, in which the parallel workers do not communicate with each other when selecting diverse paths through the solution space. Achieving a diverse traversal of the solution space is not difficult when the parallel workers are allowed to communicate with each other, however achieving diversity in a communication-less setting, in which the parallel workers do not share information with each other, is far from trivial. We propose different strategies to Diversity-Driven Widening in a communication-less setting and demonstrate the improvement of accuracy using Rule Induction Algorithm and the Greedy Algorithm for the Set Cover Problem.

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The decontamination sweepwidth of planar shapes

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Keywords: computational geometry, algorithms, progressive search

Informally, *decontamination sweep* of a compact connected closed planar shape S is a continuous temporal process that starts with S being *contaminated*, *i.e.* every point of S being contaminated, and then sweeping P in a finite amount of time with finitely many continuous curves moving with finite speed until S becomes *decontaminated*. Any point $p \in S$ becomes decontaminated whenever a sweeping curve reaches it. However, if when the sweeping curve leaves p it is the case that p is connected to a contaminated point q by a continuous curve $c \in S$ then p becomes *recontaminated*, and has to be decontaminated again. *The sweepwidth of* S is the minimum, over all sweeps of S, of the maximum length of the sweeping curves at any given moment of the sweep. The computational problem is, given a planar shape S, to compute the minimum decontamination sweepwidth of S.

Alternatively, a decontamination sweep can be thought of as any $2D$ surface T in $S \times [0,1]$ that is a *separator* of $S \times [0,1]$ in the sense that every continuous curve in $S \times [0,1]$ that connects a point from $S \times 0$ to $S \times 1$ intersects T. The third, "vertical", axis clearly corresponds to time. The sweepwidth of a given sweep is the longest intersection of T with any plane perpendicular to the vertical axis; of course, under the assumption the said intersection is $1D$. We are looking for a separator that minimizes that quantity.

That idea stems from the *graph search games* originally introduced by Torrence Parsons in the 1970's [5]. For a detailed overview of different graph searches see [1]. The graph search that is closest—in the discrete world—to our decontamination sweep is *node search number*. It is well known graph searching is

$\mathcal{NP}$-hard [4] in general. An extremely important result with respect to graph searching is the proof that an optimal graph search can always be achieved without recontamination [2]. Only by using her result it is possible to claim $\mathcal{NP}$-completeness, rather than $\mathcal{NP}$-hardness, of the graph searching. Graph searching is $\mathcal{NP}$-complete even on planar graphs of maximum degree three [3], which are the direct discrete analogues of planar shapes.

The idea of a continuous analogue of discrete graph search is, to the best of our knowledge, novel. We provide a precise definition of decontamination sweep and decontamination sweep without recontamination. We discuss the ramifications that arise in the continuous case as opposed to the "classical" discrete search games and especially proving that an optimal sweep can always avoid recontamination.

We discuss the decontamination sweepwidth of polygons. On convex polygons the sweepwidth problem is trivial: numerically, the answer is *the thickness* of S, that is, the minimum over all directions D of the longest chord L that intersects S and is in direction D.

On concave polygons, however, the computational complexity is drastically worse. We prove that even for a relatively simple class of *orthogonal polygons* that we call *flag polygons*, the problem is $\mathcal{NP}$-hard.

Finally, we demonstrate on another class of orthogonal concave polygons called *comb polygons*, the sweepidth problem is solvable in linear time.

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Prioritization of confidence in the associative classifier PGN – strong and weak points

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Keywords: data mining, classifiers

Typically, the generation of association rules from a training set is guided by the support and confidence metrics. This approach, with a primary focus on support and confidence as the second criterion rejects high confidence rules if the support is too low. The PGN classifier [1] by its construction falls into the family of associative classifiers. But it turn the usual used priorities of support over confidence around and focuses on confidence first by retaining only 100% confidence rules and just in a second step looks on the support.

The eperimental results from the comparison with different classifiers showed that PGN is competitive with some of the more advanced classification methods such as neural networks, support vector machines and random forests, while outperforming techniques such as decision rules and trees. The experiments over different types of datasets showed the strong and weak points of this approach - multi-class datasets, uneven class-label distribution, noise datasets. In this talk we discuss these specifics.

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Mini-symposium

Advanced Analytical and Numerical Techniques for Applications

Organizer:

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The mini-symposium is dedicated to established and novel techniques for solving partial differential equations and their applications in science and engineering. These include the techniques of finite elements (weak and discontinuous Galerkin, mixed methods, and their hybrid variants), finite volumes, and efficient iterative methods for the resulting systems. The applications may include flows in physics, chemistry, and biology.

Invited Talks

Iterative solution of algebraic systems arising in viscoelastic models from Geophysics

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Keywords: viscoelasticity, finite elements, preconditioned iterative methods, inner-outer solvers, parallel performance

We consider efficient and accurate numerical methods for computer simulations of advanced Glacial Isostatic Adjustment (GIA) models, that describe the response of the solid Earth to redistribution of mass due to alternating glaciation and deglaciation periods [6]. The mathematical description of GIA processes employs a generalized visco-elastic nonlinear rheology that accounts for viscous flow as the main mechanism of stress relaxation. Models describing the response of the solid Earth to load are based on the concept of isostacy, where the weight of the load is compensated for by adjustment in, and buoyancy of, the viscous mantle. The space and time scales are huge and computer simulations become the only viable option.

We report results from numerical simulations of a simplified two-dimensional model, in which the Earth is assumed to be incompressible. This give raise to large scale linear systems with saddle point matrices. These linear systems are solved using Krylov subspace iteration methods and a block preconditioner, that requires inner-outer iterations. The efficiency of solving one system of the above type is crucial as it will be embedded in a time-evolution procedure, where systems with matrices with similar characteristics have to be solved repeatedly.

The simulations are performed using toolboxes from publicly available software packages – deal.ii [2], Trilinos [3], Paralution [4] and AGMG [5], using OpenMP-type parallelism on multicore CPU systems as well as on GPU. We present performance results in terms of numerical and computational efficiency, number of iterations and execution time, and compare the timing results against a sparse direct solver from a commercial finite element package [1].

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New boundary element methods for multiple scattering problems

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Keywords: Boundary element method, high-frequency, multiple scattering.

We introduce a class of hybrid boundary element methods for the solution of sound soft scattering problems. To facilitate the applicability of our algorithms throughout the entire frequency spectrum, we have enriched our Galerkin approximation spaces, through incorporation of oscillations in the incident field of radiation, into the algebraic and trigonometric polynomial approximation spaces. In connection with the single-scattering effects, the resulting methodologies have three distinctive properties. Indeed, from a theoretical point of view (1) *they can*

be tuned to demand only an $\mathcal{O}(k^{\varepsilon})$ *increase* (*for any* $\varepsilon > 0$) *in the number of degrees of freedom to maintain a fixed accuracy with increasing wavenumber* k, owing to the optimal adaptation of approximation spaces to asymptotic stretching (shrinking) of illuminated and deep shadow regions (shadow boundaries), and (2) *they are convergent for each fixed wavenumber* k, thanks to the additional approximation spaces in the deep shadow region. Perhaps more importantly, from a practical point of view (3) *they give rise to linear systems with significantly enhanced condition numbers* and this, in turn, allows for more accurate approximations to single-scattering effects if desired.

Extension of these methodologies to multiple scattering configurations, on the other hand, is facilitated by first resorting to microlocal analysis to extend the Melrose-Taylor theory for convex bodies to a finite collection of convex obstacles as this allows for a straightforward incorporation of the derived asymptotic behaviours into the (weighted) Galerkin approximation spaces.

Elliptic problems with jump coefficients: approximation and preconditioning

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Keywords: jump coefficients, weighted-norms, mixed finite elements, high contrast media, robust preconditioners

In this note we propose and analyze a preconditioner for the system arising in finite element approximation of second order elliptic problems describing processes, such as flow in highly heterogeneous media. Our approach uses the technique of multilevel methods and the recently proposed preconditioner based on additive Schur complement approximation. We prove that the preconditioner proposed here is robust with respect to the discretization parameters, and, most importantly with respect to the contrast of the media. This provides provably efficient numerical technique which has applications in oil reservoir simulations, and, in general, flow in porous media.

Central schemes for mean field games

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Keywords: mean field games, central schemes.

Mean Field Games (MFG) models have been recently introduced and analysed by Lasry and Lions. They describe a limiting behaviour of stochastic differential games as the number of players tends to infinity. Numerical methods for the approximation of such models have been developed by Achdou, Camilli, Capuzzo-Dolcetta, Gueant, and others. Efficient algorithms for such problems require special efforts and so far all methods introduced have been first order accurate. In this talk we will describe a new numerical method for time dependent MFG. The new discretization is based on central schemes which are widely used in hyperbolic conservation laws and it is second order accurate.

Communications

Numerical study of traveling wave solutions to 2D Boussinesq equation

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Keywords: Stationary Boussinesq equation, initial approximations

The aim of this report is to compare two different initial approximations for the Simple-Iteration Method applied on the two-dimensional nonlinear stationary Boussinesq Paradigm Equation (BPE). The first type of initial approximation uses the best fit formulae given in [1]. The other is obtained by the numerical solution of the ground state equation corresponding to BPE. Numerical tests with finite difference schemes (of 2nd, 4th, and 6th order of approximation) show that both initial approximations converge to the one and the same solution of the nonlinear BPE.

Acknowledgements. This work is partially supported by the Bulgarian Science Fund under grant DDVU 02/71.

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Finite time blow up of the solutions to Klein-Gordon and Boussinesq type equations with arbitrary positive initial energy

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Keywords: nonlinear Klein-Gordon equation, finite time blow up, Nehari functional

We consider the nonexistence of global solutions to the nonlinear Klein-Gordon equation of the form

$$u_{tt} - \Delta u + mu = a|u|^{p-1}u, \quad x \in \mathbb{R}^n, \quad t \in \mathbb{R}^1,$$

where m and a are positive constants, $1 < p < \infty$ for $n = 1, 2$ and $1 < p < (n+2)/(n-2)$ for $n \geq 3$. We give sufficient conditions on the initial data with arbitrary high positive initial energy such that the solution blows up in a finite time or at infinity. The performed numerical experiments support the theoretical results.

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International Conference Mathematics Days in Sofia
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Preconditioning of finite element methods for flows in heterogeneous media

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Keywords: heterogeneous media, robust preconditioners, discontinuous Galerkin

We shall discus preconditioners for systems arising in finite element approximation of second order elliptic problems, describing processes in highly heterogeneous media, modeled by Darcy (Fourier) laws or Brinkman equations. The approximations are based on the discontinuous Galerkin finite element method that employs Raviart-Thomas or Brezzi-Douglas-Marini finite elements. Our approach uses the technique of multigrid methods and the preconditioner of Arnold, Falk, and Winther [1], developed for second order problems in mixed form and further extended to Stokes and Brinkman systems in [2].

The main result is design and numerical justification of a preconditioner that is optimal with respect to the mesh-size and robust with respect to the contrast of the heterogeneous media. The contrast is defined as a ratio, which may range up to 10^8, between the maximum and the minimum values of the permeability/conductivity coefficient. We confirm the robustness of such preconditioners on a number of numerical tests representing examples of porous media, including 3-D computations for SPE10 benchmark.

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Minisymposium

Algebraic Methods in Quantum Field Theory

Organizer:

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email: dobrev@inrne.bas.bg

The mini-symposium is designed to serve the community of mathematicians, mathematical physicists and theoretical physicists, working on mathematical structures that may be applied to Quantum Field Theory and/or building models for physical systems based on algebraic methods. The algebraic methods are meant in their widest sense, i.e., representation theory, algebraic geometry, infinite-dimensional Lie algebras and groups, superalgebras and supergroups, quantum groups, noncommutative geometry, number theory, symmetries of linear and nonlinear PDE, special functions.

Invited Talks

Towards a complete classification of multipartite entanglement

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Keywords: entanglement, quantum information, SLOCC, invariant polynomials, schur-weyl duality

Multi-particle entanglement is an essential resource for a variety of quantum information processing tasks. Yet, despite an enormous amount of literature dedicated to its study, our current understanding of it is still in its infancy. In this talk I will introduce a systematic classification of multiparticle entanglement in terms of equivalence classes of states under stochastic local operations and classical communication (SLOCC). I will show that such an SLOCC equivalency class of states is characterized by ratios of homogenous polynomials that are invariant under local action of the special linear group. I will then introduce a complete construction of the set of all such SL-invariant polynomials (SLIPs). The construction is based on Schur-Weyl duality and applies to any number of qudits in all (finite) dimensions. In addition, I will introduce an elegant formula for the dimension of the homogenous SLIPs space of a fixed degree as a function of the number of qudits. The expressions for the SLIPs involve in general many terms, but for the case of qubits can be written in a much simpler form.

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A new venue of spontaneous supersymmetry breaking in supergravity

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Keywords: non-Riemannian volume form, modified supergravity theories, dynamical generation of cosmological constant, new mechanism of dynamical breakdown of supersymmetry

The principal result to be presented is a qualitatively new mechanism for dynamical spontaneous breakdown of supersymmetry. Specifically, we consider a modified formulation of standard minimal $N = 1$ supergravity. The modification is based on an idea worked out in detail in previous papers by some of us, where we proposed a new formulation of (non-supersymmetric) gravity theories employing an alternative volume form (volume element, or generally-covariant integration measure) in the pertinent Lagrangian action, defined in terms of auxiliary (pure-gauge) fields instead of the standard Riemannian volume form. Invariance under supersymmetry of the new modified $N = 1$ supergravity action is preserved due to the addition of an appropriate compensating antisymmetric tensor gauge field. We find as a result of the above modification, that a non-zero cosmological constant is naturally produced as an integration constant when solving some of the relevant (supersymmetric) equations of motion. It is this dynamically generated cosmological constant which signifies a dynamical spontaneous breakdown of supersymmetry in the supergravity context. Even more importantly, one can appropriately choose the value of the above integration constant so as to achieve simultaneously a small effective observable cosmological constant and a large (dynamically generated) mass of the gravitino as required by modern realistic cosmological scenarios for the slowly accelerated universe today.

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Renormalization theory as a common playground for physicists and mathematicians

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Keywords: quantum field theory, position space renormalization, residue

The conceptual meaning of ultraviolet renormalization of a local quantum field theory is revealed in the x-space picture. It takes a particularly transparent form in a dilation invariant (massless) theory [1]. A causal factorization condition, which uses an adiabatic procedure requiring all vertices to be treated as external, serves as a basis for an inductive procedure in the order of a Feynman graph. Integration over internal vertices does not respect the causal ordering and needs a special treatment. The fact that whenever such integration is infrared convergent it does not alter the degree of ultraviolet divergence and preserves the order of the highest logarithm opens the way to its inclusion in the position space approach [2]. The residue of the pole subtracted from an analytically regularized primitively divergent amplitude is independent of the renormalization ambiguity and is a *period* in the sense of [3]. We review recent results of Brown and Schnetz [4] in which the periods of a sequence of primitive n-loop φ^4 *zig-zag graphs* are calculated to be rational multiples of $\zeta(2n-3)$. These (single odd) zeta values are known to be primitive elements in the algebra of motivic multiple zeta values and appear as generators of the Lie algebra of an associated Galois group. The zig-zag graphs are conjectured to play a similar role in the φ^4 theory.

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Rigidity of quantum tori and the Andruskiewitsch-Dumas conjecture

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Keywords: automorphisms of algebras, quantum groups, quantum tori

The full automorphism groups of infinite dimensional algebras are often difficult to describe. Precise answers are only known for very particular algebras. In some cases Joseph, Alev, Shestakov, and Umirbaev proved the existence of wild automorphisms. On the other hand, Andruskiewitsch and Dumas conjectured that the positive parts of all quantized universal enveloping algebras of simple Lie algebras have small automorphism groups which can be described explicitly.

We will outline a proof of the latter conjecture in full generality. The key step in this proof is a rigidity theorem for an important class of "bifinite" automorphisms of completions of quantum tori. It has a broad range of applications. It allows one to control the full automorphism groups of large classes of associative algebras arising in the areas of quantum groups and quantum cluster algebras.

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Communications

A generalization of Calabi-Yau fourfolds arising from M-theory compactifications

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Using a reconstruction theorem, we prove that the supersymmetry conditions for a certain class of flux backgrounds are equivalent with a tractable subsystem of relations on differential forms which encodes the full set of contraints arising fom Fierz identities and from the differential and algebraic conditions on the internal part of the supersymmetry generators. The result makes use of the formulation of such problems through Kähler-Atiyah bundles, which we developed in previous work. Applying this to the most general $N = 2$ flux compactifications of 11-dimensional supergravity on 8-manifolds, we extract the conditions constraining such backgrounds and give an overview of the resulting geometry, which generalizes that of Calabi-Yau fourfolds.

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Berezin-Toeplitz quantization: some open questions

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Keywords: Kähler manifold, line bundle, Toeplitz operator, semiclassical limit, second quantization, hyperkähler manifold

Berezin-Toeplitz quantization is a well-known technique [3]. The word *quantization* usually means a way to pass from classical mechanics to quantum mechanics. In Berezin-Toeplitz setting the classical side is an integral Kähler manifold (M, ω), and the quantum side is obtained by choosing a holomorphic hermitian line bundle $\mathcal{L}$ with curvature of the hermitian connection $-2\pi i\omega$. Let's assume that M is compact. Let k be a positive integer. Let f be a complex-valued smooth function on M. A Berezin-Toeplitz operator $T_f^{(k)}$ is an endomorphism of the space of holomorphic sections of $\mathcal{L}^{\otimes k}$, defined as multiplication by f followed by the orthogonal projection from the space of L^2 sections of $\mathcal{L}^{\otimes k}$ onto the subspace of holomorphic sections. Berezin-Toeplitz quantization is the map $f \mapsto T_f^{(k)}$ (with a fixed k, interpreted as $1/\hbar$). It is known that this map has good semiclassical properties [2].

Berezin-Toeplitz operators have been mostly studied for smooth symbols f. I will mention recent work [1] where the case of non-C^∞ symbols is discussed to a great extent.

I will discuss open questions related to the case of several particles or second quantization.

I may also touch the subject of quantization on hyperkähler manifolds, when three Kähler forms are present, there are three Berezin-Toeplitz quantizations and three quantum-mechanical vector spaces, and a natural open question is how to treat second quantization in this setting.

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Fundamental group of a monstrous (?) discriminant complement

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Keywords: braids, Coxeter groups, hyperbolic orbifolds, lattices, monster

We shall study the fundamental group of a curious 13 dimensional complex hyperbolic orbifold X. Daniel Allcock has conjectured (see [1]) that the fundamental group of X maps onto the semi-direct product of $M \times M$ with $\mathbb{Z}/2\mathbb{Z}$, where M is the monster simple group. We shall explain the rationale behind this conjecture, evidences for it, and recent progress towards its proof. We shall explain how this orbifold might be related to the monster manifold, sought by Hirzebruch, to give a geometric explanation for monstrous moonshine. The Leech lattice will play a key role in our construction.

Acknowledgements. Part of the talk will be based on joint work with Daniel Allcock.

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Higher-order singletons and partially massless fields

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Keywords: higher-dimensional conformal field theory, fusion rules, higher symmetries, AdS/CFT

Using ambient space we develop a fully gauge and $\mathfrak{o}(d,2)$ covariant approach to boundary values of AdS_{d+1} gauge fields. It is applied to the study of (partially) massless fields in the bulk and (higher-order) conformal scalars, i.e. singletons. The corresponding generalization of the Flato-Fronsdal theorem [1] on the decomposition of the tensor product of two singletons is obtained and agrees with the known structure of symmetries for the (higher-order) wave operators [2]. All these facts support a generalization of the higher-spin holographic duality [3].

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Quantization of principal $U(1)$-connections on globally hyperbolic spacetimes

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Keywords: locally covariant quantum field theory, quantum field theory on curved spacetimes, gauge theory on principal bundles

I will address an approach to the functorial quantization of pure electromagnetism as a Yang-Mills theory of connections on a principal $U(1)$-bundle over a globally hyperbolic Lorentzian manifold [1, 2]. As a first step to each principal bundle we assign a suitable presymplectic Abelian group of gauge-invariant on-shell functionals (observables), whose pairing with the space of gauge classes of on-shell connections is non-degenerate [1, 3]. That done, extending to presymplectic Abelian groups the usual canonical commutation relations for symplectic vector spaces, we obtain a C^*-algebra associated to each presymplectic Abelian group of observables. The resulting functor fulfils the requirements of a generally covariant quantum field theory, except for injectivity of the induced morphisms,

which is violated due to the failure of a similar property at a cohomological level. Actually, it turns out that no quotient of this functor allows one to recover injectivity. Yet, fixing any principal bundle, it is still possible to realize a quantum field theory in the sense of Haag and Kastler by performing a suitable quotient.

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Discrete derived categories

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Keywords: discrete derived categories, Auslander–Reiten quivers, t-structures, Bridgeland stability conditions

Discrete derived categories, as defined by Vossieck, form a class of triangulated categories which are sufficiently simple to make explicit computation possible, but also non-trivial enough to manifest interesting behaviour. In this talk, I will describe what they are, and explain some recent work with D. Pauksztello and D. Ploog, where we study the autoequivalences and bounded t-structures on these categories. I will then talk about spaces of Bridgeland stability conditions, and on-going work to fully understand these spaces for discrete derived categories.

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Principal subspaces for affine Lie algebra of type $B_2^{(1)}$

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Keywords: affine Lie algebra, vertex operator algebra, principal subspace, combinatorial basis

We consider principal subspaces $W_{L(k\Lambda_0)}$ and $W_{N(k\Lambda_0)}$ of standard module $L(k\Lambda_0)$ and generalized Verma module $N(k\Lambda_0)$ at level $k \geq 1$ for affine Lie algebra of type $B_2^{(1)}$. By using the theory of vertex operator algebras, we find combinatorial bases of principal subspaces in terms of quasi-particles. From quasi-particle bases, we obtain character formulas for $W_{L(k\Lambda_0)}$ and $W_{N(k\Lambda_0)}$.

Topology of moduli spaces of free group representations in reductive groups

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Keywords: character varieties, reductive groups, representation varieties

Let G be a reductive algebraic group and Γ be a finitely generated group. Moduli spaces of representations of Γ into G, the so-called G-character varieties

of Γ, play important roles in hyperbolic geometry, the theory of bundles and connections, knot theory and quantum field theories.

Let K be a maximal compact subgroup of G, and let F_r be a rank r free group. We show that the space of closed orbits in $\mathrm{Hom}(F_r, G)/G$ admits a strong deformation retraction to the orbit space $\mathrm{Hom}(F_r, K)/K$. In particular, all such spaces have the same homotopy type. We compute the Poincaré polynomials of these spaces for some low rank groups G. We also compare, for real G, the real moduli spaces to the real points of the corresponding complex moduli spaces, and describe the geometry of many examples.

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KP hierarchy for a cyclic quiver

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Keywords: integrable hierarchies, Cherednik algebra, quiver varieties, Calogero–Moser system.

We introduce a generalisation of the KP hierarchy, which is intimately related to the cyclic quiver with m vertices; the case $m = 1$ corresponds to the usual KP hierarchy. Generalising the result of [3], we show that our hierarchy admits special solutions parameterised by suitable quiver varieties. Using the approach of [2], we identify the dynamics of the singularities for these solutions with the classical Calogero–Moser system for the complex reflection groups $G(m, 1, n)$. The link to the bispectral operators from the work [1] will be indicated. This is joint work with A. Silantyev.

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Higher-dimensional Heegaard Floer homology

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Keywords: Heegaard Floer homology, higher dimension, contact geometry

In a work in progress with Ko Honda, we extend the definition of the hat version of Heegaard Floer homology to contact manifolds of arbitrary odd dimension using higher-dimensional open book decompositions and the theory of Weinstein domains. This also suggests a reformulation and an extension of Symplectic Khovanov homology to links in arbitrary 3-manifolds.

Invariant differential operators for non-compact Lie algebras parabolically related to conformal Lie algebras

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Keywords: invariant differential operators, non-compact Lie algebras, parabolic relation

In the present talk we review the recent progress of our project of systematic construction of invariant differential operators for non-compact semisimple Lie groups. Our starting points is the class of algebras, which we call 'conformal Lie algebras', which have very similar properties to the conformal algebras of Minkowski space-time, though our aim is to go beyond this class in a natural way. For this we introduce the new notion of *parabolic relation* between two non-compact semisimple Lie algebras $\mathcal{G}$ and $\mathcal{G}'$ that have the same complexification and possess maximal parabolic subalgebras with the same complexification.

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Fine gradings on Heisenberg algebras and related structures

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Keywords: Heisenberg algebra, graded Lie algebra, fine grading

In the last years there has been an increasing interest in the study of the group gradings on Lie theoretic structures. Gradings reveal the structure properties and appear naturally in a variety of contexts, for instance, superalgebras, loop algebras, Lie color algebras, automorphisms of finite order, symmetric spaces and generalized symmetric spaces, contractions and deformations of Lie algebras, and so on. In the complex case, fine gradings on simple Lie algebras are a natural generalization of the root decomposition relative to a Cartan subalgebra, which has had great relevance in many fields of Mathematics. In Physics, fine gradings provide maximal sets of quantum observables with additive quantum numbers. A recent exhaustive survey on the subject is the monograph [1], which collects the results about the classification of gradings on simple finite-dimensional Lie algebras over algebraically closed fields.

On the contrary, there is not much work done in the field of gradings on nilpotent or solvable Lie algebras. Some of the few references on this topic are [2] and [3]. Our work [4] gives one step further in this direction. Some of its contributions are the descriptions of the fine (group) gradings on the Heisenberg (nilpotent) algebras H_n, on the Heisenberg superalgebras $H_{n,m}$ and on the twisted Heisenberg (solvable) algebras H_n^λ, as well as of their Weyl groups (symmetry groups of the gradings). The choice of this kind of structures is motivated by the important role played by the Heisenberg group in several branches of mathematics, such as representation theory, Fourier analysis, partial differential equations, number theory, several complex variables and quantum mechanics.

During the talk we will exhibit some of the results of [4] jointly with some of the possible applications and continuations of the work. For instance, we wonder if the essentially unique representation given by the Stone-von Neumann theorem is also unique in the graded setting.

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Villamayor-Zelinsky sequence for braided finite tensor categories

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Keywords: finite tensor category, bimodule category, Brauer-Picard group, fusion category, braided monoidal category

The classical Crossed Product Theorem states that the relative Brauer group with respect to a Galois field extension is isomorphic to the second Galois cohomology group. In 1977 Villamayor and Zelinsky introduced a cohomology for an extension of commutative rings and constructed an infinite exact sequence involving the respective cohomology groups. These groups are evaluated at three types of coefficients and the three types of cohomology groups appear periodically in the sequence. If the ring extension is faithfully flat, the relative Brauer group embeds into the middle term on the second level of the sequence. This sequence generalizes the Crossed Product Theorem to the case of commutative rings.

In 2005 Caenepeel and Femić introduced the Brauer group of Azumaya corings and proved that it is isomorphic to the mentioned middle term cohomology group. This resolves the deficiency in the cohomological interpretation of the Brauer group of a commutative ring. In 2008 the same authors constructed a version of the infinite exact sequence for a commutative bialgebroid and we interpreted the middle terms on the first three levels of the sequence. If $R \to S$ is a commutative ring extension, then $S \otimes_R S$ is a commutative bialgebroid and the new sequence generalizes the previous one.

In the present work we introduce a cohomology for a braided finite tensor category and construct an infinite exact sequence of the respective cohomology groups. We investigate how much of the previous results can be recovered in this setting. This is a work in progress.

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Two-dimensional scattering and unitarity methods

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Keywords: two-dimensional models, AdS/CFT correspondence, on-shell methods

We consider several massive two-dimensional models, including the world-sheet $\mathrm{AdS}_5 \times S^5$ superstring, and apply the standard unitarity-cut method to compute $2 \to 2$ scattering S-matrices at one loop from tree level amplitudes. Evidence is found for the cut-constructibility of supersymmetric integrable models, while for models without supersymmetry (but integrable) the missing rational terms can be interpreted as a shift in the coupling.

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Enumeration of vacua in string theory via algebraic combinatorics

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Keywords: string landscape, enumeration, conjugacy classes, Lie groups, symmetry breaking patterns

How to count vacua is one of the most interesting questions that can arise in the study of the string landscape. Resolutions to this question will enhance our ability to classify the crowded string landscape and pinpoint the desired vacuum that represents the gauge group of the world, namely, the standard model. In this talk, we introduce the progress that we made on this question by translating it into an equivalent problem in algebraic combinatorics that has not been posed before in the literature. We do so in the context that involves the moduli space of M-theory compactifications on singular manifolds with G2 holonomy; it takes into consideration dualities between gauge theories with different gauge groups but equal numbers of U(1) factors. We show that counting these dual vacua is equivalent to enumerating conjugacy classes of elements of finite order inside Lie groups. We pose and solve these enumeration problems. We are also able to point out that symmetry breaking patterns by Wilson lines and Higgs fields are different in certain cases, unlike the conventional expectation that they are the same.

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Role of Lie algebra for confinement in non-Abelian gauge field scheme

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Keywords: classical field, soliton, wilson loop, path integral, step function

The nonlinearity of non-Abelian gauge fields with Lie matrices has led to classical analytic and quantum numerical studies. Considering the status of the solutions in literature to be more understood [1], we derived a new solution of a classical vector potential composed of localized and unlocalized functions [1]. For the confining classical configuration shifted from the zero field, the initial energy balance condition at the scale-invariant time, which is a function of the scale-invariant length measured from a pair creation spacetime point, determines a classical continuum field parameter. The localized soliton-like function contributes to the linear confinement potential of the Wilson loop due to the trace of the polynomials of the Lie matrix, with no contribution from the unlocalized function. The string tension derived in our study is compatible with the expected value. The quantum field is expressed in terms of step functions and their derivatives, with a finite degree of freedom in order to construct the gauge-invariant and spacetime covariant continuum theory [1,2]. The quantized field renders the Coulomb potential independent of the gauge group, and produces a local mass. Our solution has an explicit form of functions, revealing the detailed confinement mechanism. The step function has a zero value at upper and lower bounds of the support. The advanced basis set of step functions can contain step functions that are inserted between the original step functions, taking unity at the upper and lower bounds of the support, without influencing the results. This mechanism works for the pure Yang-Mills case, when a pair creation occurs.

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Some bialgebras related to quantum field theory from a simplicial viewpoint

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Keywords: bialgebras, operads, decomposition spaces

In this talk, we will look at some general constructions of bialgebras, with many relevant examples for quantum field theory among them, from operadic and categorical insights that originate in the combinatorics of simplicial sets.

We begin with a general construction of a bialgebra or Hopf algebra structure from a cooperad with a suitably compatible multiplication. This has several "classical" applications: for different choices of cooperad we can recover the bialgebras of trees of Connes and Kreimer, Baues's bialgebra structure on the cobar construction of a 1-reduced simplicial set, and Goncharov's bialgebra structure on motivic multiple zeta values. This is joint work in progress with Ralph Kaufmann and Andrew Tonks.

From another direction, we turn then to the consequences of our work with Joachim Kock and Andrew Tonks, where we established that the Faa di Bruno formula for the Green function in the bialgebra of operadic trees may be obtained as an equivalence of groupoids, on taking cardinality. [1].

Building from there, in our current work in progress we introduce the notion of "decomposition space" as a broad generalisation of the notion of poset in which to construct combinatorial incidence bialgebras. We show there how the Connes-Kreimer Hopf algebra arises from a decomposition space of combinatorial forests, and derived Hall algebras from the Waldhausen S-dot construction of a stable infinity-category.

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Quadratic algebras, Yang-Baxter equation, and Artin-Schelter regularity

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Keywords: Yang-Baxter equation, Artin-Schelter regularity, PBW-algebras

We study two classes of n-generated quadratic algebras over a field K. The first is the class of all n-generated PBW algebras with polynomial growth and finite global dimension. We show that a PBW algebra A has polynomial growth and finite global dimension *iff* its Hilbert series is $H_A(z) = 1/(1-z)^n$. Furthermore the class contains a unique (up to isomorphism) monomial algebra. The second is the class of n-generated quantum binomial algebras A, where the defining relations R are nondegenerate square-free binomials $xy - c_{xy}zt$, with nonzero coefficients c_{xy}. Our main result shows that the following conditions are equivalent: (i) A is an Artin-Schelter regular PBW algebra. (ii) A is a Yang-Baxter algebra, that is the set of quadratic relations R defines canonically a solution of the Yang-Baxter equation. (iii) A is a binomial skew polynomial ring. (iv) The Koszul dual $A!$ is a quantum Grassmann algebra.

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Geometric quantization of Poisson manifolds

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Keywords: quantization, groupoids, C*-algebras

Although all formal deformation quantizations of Poisson manifolds were famously constructed and classified by Kontsevich 17 years ago, strict deformation quantizations (using C*-algebras) remain less well understood.

There do exist explicit constructions for strict deformation quantization of some types of Poisson manifold. For a symplectic manifold, this is constructed by geometric quantization. For the dual bundle of a Lie algebroid, it is constructed using the convolution C*-algebra of a Lie groupoid.

More generally, a C*-algebra can be constructed from a Poisson manifold by using a symplectic groupoid with a prequantization and polarization [1]. This construction generalizes the C*-algebras in the cases mentioned above and in a few other examples of quantization.

This construction can be applied to a "Heisenberg-Poisson manifold" to give a family of C*-algebras (corresponding to different values of $\hbar$) [2]. In cases where the values of $\hbar$ should be restricted by integrality conditions, this is automatically taken into account by the construction.

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A compactness theorem for the Seiberg-Witten equations with multiple spinors

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Keywords: Seiberg-Witten equations, compactness of the moduli space, harmonic spinors, higher dimensional gauge theory

Motivated by higher dimensional gauge theory, we consider the compactness problem for the Seiberg–Witten equations with n spinors in dimension three. We show that a sequence of solutions of the Seiberg–Witten equations has a subsequence converging to a Fueter section, which is a non-linear version of a harmonic spinor.

This is a joint project with Th. Walpuski.

Diophantine subsets of representations of $sl(2, k)$

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Keywords: special linear Lie algebra, finite-dimensional representations, free abelian category, von Neumann algebra, Diophantine subsets

A global theory of finite-dimensional representations of the Lie algebra $L = sl(2, k)$, k a field of characteristic 0, is presented. This is accomplished by considering the free abelian category $Ab(U(L))$ of the universal enveloping algebra $U(L)$ and localizing at the Serre subcategory of those objects that vanish on the affine k-plane $k[x, y]$. This localization is itself the free abelian category over a von Neumann regular k-algebra $U'(L)$, whose space of simple representations is the closure, in the Jacobson topology on the space of primitive ideals, of the homogeneous components of the affine k-plane. This space of simple torsion representations of $U'(L)$ is totally disconnected. Our main result is that it is isomorphic to the Stone space of a mysterious Boolean algebra of Diophantine subsets of the natural numbers.

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Positive representations and quantum higher Teichmüller theory

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Keywords: positive representations, split real quantum groups, modular double, GNS-representation, higher Teichmüller theory, quantum dilogarithm

We review the notion of positive representations of split real quantum groups $\mathcal{U}_{q\tilde{q}}(\mathfrak{g}_{\mathbb{R}})$ introduced in a joint work with Igor Frenkel [1], and describe the tensor product decomposition when restricted to the Borel part. This generalized the essential step of the construction of the quantum Teichmüller theory from the representation of the quantum plane studied in [2], and provide a candidate for the quantum higher Teichmüller theory.

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A Hopf algebra with the dimensions of the Bell numbers

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Keywords: Bell numbers, Solomon algebra, Hopf algebras

In their work [1], A. I. Solomon, G. E. Duchamp, P. Blasiak, A. Horzela and K. Penson defined a co-commutative Hopf algebra BELL, described as a vector space spanned by some family of graphs with an additional bialgebra structure. The *labelled* version of these graphs is in bijection with non-ordered partitions of the set $\{1, \ldots, n\}$, for all natural number n. We want to describe a Hopf algebra spanned by all non-ordered partitions, and show that it can be endowed with an additional structure of module over the Solomon algebra. We shall give a general way to described bialgebra structures, not necessarily cocommutative, on sets of partitions of the natural numbers, and try to explain how to reconstruct them from the subspaces of their primitive elements.

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Toric quiver moduli

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Keywords: quiver representations, toric varieties, combinatorial ring theory

When the dimension vector of a quiver takes value 1 on each vertex, the moduli spaces obtained from the usual geometrical invariant theory constructions are toric varieties. This class has been the topic of several recent studies (see for example [1], [2], [4]). In this talk we will discuss our current research (see [3]) in which we showed that, up to isomorphism, there are only finitely many such toric moduli in any given dimension, and outlined a process for their classification. Furthermore, by applying a result of T. Yamaguchi, M. Ogawa and A. Takemura from 2013 (see [5]) we proved that in the case when the moduli spaces are also projective varieties their homogenous ideals can be generated by the elements of degree at most 3.

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Birdtrack construction of SU(N) projection operators

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Keywords: representation theory, SU(N), diagrammatic methods, QCD

I outline a general recipe for constructing orthogonal bases for the calculation of color structures appearing in QCD for any number of partons and arbitrary N_c. The bases are constructed using hermitian gluon projectors onto irreducible subspaces invariant under SU(N_c). Thus, each basis vector is associated with an irreducible representation of SU(N_c). The resulting multiplet bases are not only orthogonal, but also minimal for finite N_c. As a consequence, for calculations involving many colored particles, the number of basis vectors is reduced significantly compared to standard approaches employing overcomplete bases.

This contribution is based on joint work with Malin Sjödahl (Lund).

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Geometric Supersymmetry on Cahen-Wallach spaces

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Keywords: geometric supersymmetry, Cahen-Wallach space, homogeneous structure, supergravity

We will present a classification result on manifolds that can be used as supergravity backgrounds. The manifolds taken into account are of Cahen-Wallach type, see [1]. There are many known examples of such spaces that occur in the physics and the mathematical physics literature, see [2–6, 10], for example.

We give a precise definition for a superalgebra structure on such spaces that is defined by geometric data. We will argue how far this structure is unique. The moduli space of such superalgebra structures on Cahen-Wallach spaces is non compact in general. Nevertheless, there exist points in the moduli space for which the superalgebra turns into a Lie superalgebra, the so called geometric supersymmetry. These points are in general singular in the moduli space.

Imposing some natural conditions, we will give a full list of low-dimensional Cahen-Wallach spaces admitting geometric supersymmetry. We will also provide some partial results in more general situations that show how our tools can be used in general.

In particular, we will present a 2-parameter family of eleven-dimensional indecomposable Cahen-Wallach spaces that admit geometric supersymmetry. The compactification of the moduli space admits three points for which the underlying Cahen-Wallach space is decomposable. One of them can be interpreted as an oxidation from a six-dimensional Cahen-Wallach space with a geometric $N = 4$ supersymmetry, [9].

Part of the results we present can be found in [7]. Some of the necessary tools are already published in [8].

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Systems of differential operators and generalized Verma modules

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Keywords: quasi-invariant differential operators, intertwining differential operators, generalizedl flag manifolds, generalized Verma modules, standard maps

Intertwining differential operators between homogeneous vector bundles over generalized flag manifolds, equivalently, homomorphisms between generalized Verma modules, have received a lot of attention in representation theory, parabolic geometry, and mathematical physics. In contrast that intertwining differential operators between line bundles are intensively investigated, much less is known about such differential operators between vector bundles that are not necessarily line bundles. In this talk we will introduce a construction method for such differential operators from a line bundle to a vector bundle.

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Quantizations of cluster algebras

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Keywords: cluster algebra, quantum group, Lie theory, canonical bases

In this lecture we wish to give an introduction to Sergey Fomin and Andrei Zelevinsky's theory of cluster algebras [2] and study their quantizations. Although Fomin-Zelevsinky's initial motivation comes from Lie theory, the definition of a cluster algebra is very elementary. In fact, a cluster algebra is a commutative algebra and we obtain generators and relations by a combinatorial mutation process. To establish a connection to Lie theory, and Lusztig's canonical bases in particular, Berenstein and Zelevinksy [1] have introduced quantum cluster algebras as non-commutative deformations of ordinary cluster algebras.

A given cluster algebra does not necessarily admit a quantization, and if a quantization exists, then it is not necessarily unique. At the end of the talk we present some joint work with Florian Gellert to decide whether a cluster algebra admits a quantization. Moreover, we give an explicit and natural basis for the space of all possible quantizations.

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Algebras, deformations and the Standard Model

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Keywords: deformations of algebras, non-commutative phase spaces, Dirac derivations, time-space, standard model cosmology and the Big Bang

In this talk I shall continue the study of the geometry of the moduli-space of pairs of points in 3 dimensions. I show that this space, $\underline{\tilde{H}}$, is the base space of a canonical family of associative k -algebras in dimension 4, furnishing a possible mathematical model for a Big Bang-scenario in cosmology.

The study of the corresponding family of derivations leads to a natural way of introducing an action of the gauge Lie algebras of the Standard Model, in $\underline{\tilde{H}}$. Introducing the notion of quotient spaces in non-commutative algebraic geometry, we obtain a geometry that seems to fit well with the set-up of the Standard Model. These subjects are all treated within the set-up of [7].

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Weyl transformations for quantum and semiclassical cosmological billiards

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Keywords: hyperbolic spaces, quantum gravity, symmetries of the spacetime, mathematical aspects of early cosmology, classical and quantum chaos, discrete subgroups of Lie groups

The wavefunction for the Early Universe for the General Cosmological Solution in discussed in the asymptotic limit towards the cosmological singularity under the BKL (Belinski, Khalatnikov and Lifshitz) paradigm, for which the evolution of the metric tensor is mapped to a chaotic system (Cosmological Billiard) in the suitable hyperbolic target space.

Close to the cosmological singularity, the gravitational field is described at the quantum level, at the semiclassical transition and at a classicalized scale.
The wavefunction of the universe is defined by the causal structure of the spacetime, as encoded in the topology of the target space where it is defined, and shaped according to the statistical properties of the considered cosmological model.

The energy spectrum associated to the eigenvalue problem for the Laplace-Beltrami operator corresponds with the WDW (Wheeler-DeWitt) equation, such that quantum-gravitational effects are investigated as modifications to the energy levels and as to the geometry of the spacetime: the reciprocity between the symmetries of the cosmological setting and the gravitational field are analyzed, for which the implementation of Weyl symmetries on the corresponding scenario remains non-trivial.

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Conserved charges in metric-affine gravity: the paradox of non-metricity

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Keywords: conserved charges, superpotentials, non-metricity, soldering form

Superpotentials offer a direct means of calculating conserved charges associated with the asymptotic symmetries of space-time. Yet, they have been plagued with inconsistencies, related to their non-uniqueness and non-covariance. These inconsistencies have been resolved by providing a variational differential equation that uniquely determines the variation of the superpotential. A hallmark solution of this equation is the Katz-Bičák-Lynden-Bell superpotential, obtained from the first-order Lovelock Lagrangian. In this lecture, I introduce the formalism of superpoten-tials and review the covariant phase space approach of Julia and Silva. I re-derive the variational equations for two first-order formulations of metric-affine gravity: the general linear and the Palatini formulations. A comparison of both formulations re-veals a discrepancy in their corresponding superpotentials, attributable to the com-bined roles of the soldering form and the non-metricity tensor. Different approaches are shown to remedy this situation. The contributing role of non-metricity to con-served charges is elaborated with numerous examples, showing that the choice of dynamic fields affects the superpotential for equivalent on-shell dynamics.

Acknowledgements. The author acknowledges fruitful discussions with J. Katz and the support of D. Porath and the Institute of Chemistry.

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Factorisation algebras and the Koszul duality

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Keywords: factorization algebra, topological chiral homology, topological quantum field theory, Koszul duality, little disks algebra

A factorisation algebra is an algebraic structure closely related to quantum field theory. Originally, a version of this on an algebraic curve was introduced by Beilinson and Drinfeld as a rigourous formulation of the notion of a chiral algebra in confomal field theory. I introduce factorisation algebras with the ideas of their connection to quantum field theories. Then I concentrate on "locally constant" such algebras, corresponding to topological field theories, and discuss the "Koszul" duality for them, generalising the classical Verdier (and Poincaré) duality, and its implication on a toplogical field theory.

Physical applications of noncommutative localization

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Keywords: noncommutative localization, representation theory, q-algebras, deformation theory

Localizations of algebras provide a powerful mathematical technique for relating certain aspects of differing physical structures. For example, using certain localizations of the Poincare group, much of the representation theory of the Poincare group can be related to the representation theory of certain deformations of it, namely SO(2,3) and SO(1,4). Here we describe these results and also give other physically interesting applications, such as similar results for the Lorentz and homogeneous Galilean groups. We also give results on generalizations to supersymmetry and q deformations, for example for $U_q(sl(2))$ and $U_q(osp(1,2))$. Our method leads to new representations of $U_q(osp(1,2))$.

On the Yang-Baxter equation

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Keywords: Yang-Baxter equation, quantum groups, Hopf algebras

The Yang-Baxter equation first appeared in theoretical physics, in a paper by the C. N. Yang, and in statistical mechanics, in R. J. Baxter's work. Later, it turned out that this equation plays a crucial role in: quantum groups, knot theory, braided categories, analysis of integrable systems, quantum mechanics, non-commutative descent theory, quantum computing, non-commutative geometry, etc. The full classification of its solutions is an open problem. Solutions

for the Yang-Baxter Equations from algebra structures, coalgebra structures, Lie (super)algebras and Boolean algebras are presented. Various aspects of the Yang-Baxter equation, related algebraic structures, and some applications are discussed (see [1, 2]).

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Topological quantum spin models from conformal field theory

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Keywords: conformal field theory, quantum many-body physics, topology

Conformal field theory has turned out to be an interesting tool for investigating certain aspects of quantum many-body physics. Here, we use conformal field theory to construct models of quantum many-body systems on lattices with interesting physical and mathematical properties. The quantum states of the models are taken to be correlators of conformal fields, and we propose to use null vectors to derive Hamiltonians, which provide the interactions needed to realize the states physically. In particular, we use the approach to derive families of semi-analytical models with topological or critical properties. Semi-analytical models

are valuable guides to understand many-body systems because the complexity of the systems limits the possibilities for doing numerical computations.

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Vertex algebras and renormalization

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Keywords: vertex algebras, renormalization theory, quantum filed theory, operads

A construction will be presented that provides a correspondence between models of vertex algebras and renormalization groups in perturbative massless Quantum Field Theories. It is based on a common symmetric operad that appears in both areas. This opens various perspectives, in particular, for constructing nontrivial models of quantum fields in higher dimensions.

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The de Rham cohomology of quantum projective space

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Keywords: noncommutative geometry, quantum groups

In the newly emerging field of noncommutative complex geometry, the de Rham complexes of the irreducible quantum flag manifolds (as introduced by Heckenberger and Kolb [1]) are examples of central importance. Given the dimension drop problem for the cyclic cohomology of quantum groups, it is natural to ask if these complexes have classical cohomological dimension. In this talk we show that for the special case of the quantum projective spaces this is indeed the case. The dimensions are calculated using a direct q-deformation of the classical Hodge theoretic approach.

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Quantization holography and the universal coefficient theorem

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Keywords: quantum field theory, universal coefficient theorem, quantization of gravity, dualities

I present a method of performing geometric quantization using cohomology groups extended via coefficient groups of different types. This is possible according to the Universal Coefficient Theorem (UTC). I also show that by using this method new features of quantum field theory not visible in the previous treatments emerge. The main observation is that the ideas leading to the holographic principle can be interpreted in the context of the universal coefficient theorem from a totally different perspective. Implications on subjects like "Wheeler bags of gold" in the context of holography are briefly discussed.

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Grothendieck's dessins d'enfants in quantum contextuality

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Keywords: quantum contextuality, Grothendieck's dessins d'enfants, permutation groups, finite geometries, geometric hyperplanes

In quantum mechanics, due to the non-commutativity of observables, the reality of the particle prior to the measurement has to be questioned. This reality relies on the arrangement set up and on compatible unperformed/counterfactual measurements: this is called quantum contextuality (QC). Proofs of QC, the Kochen-Specker theorems [1], rely on remarkable point/line incidence geometries $\mathcal{G}$'s of the projective (or polar type), as the Tits generalized polygons [2].

In this work, one finds that such finite geometries arise from (are stabilized by) some Grothendieck's dessins d'enfants [3]. A dessin d'enfant is a bipartite graph embedded in a Riemann surface defined over the field $\bar{\mathbb{Q}}$ of algebraic numbers (i.e. an algebraic curve). Following Grothendieck's "Esquisse d'un programme", the (two-generator) permutation group P of a $\mathcal{D}$, of prescribed characteristics, is readily derived from the cosets of the relevant subgroup of the free group on two generators. Then, the two-point stabilizers of P ensure the derivation of the corresponding $\mathcal{G}$'s. Restricting to $\mathcal{G}$'s with three points on a line, we also investigate the organization of their geometric hyperplanes h_i, whose (set theoretical) 'addition law' $h_i \oplus h_j$ is the complement of their symmetric difference.

Acknowledgements. Part of this work is currently performed jointly with the authors of [2].

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Quantum plactic monoid

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Keywords: noncommutative symmetric functions, combinatorics, quantum groups

Jean-Yves Thibon and Daniel Krob [1] introduced quantum Schur functions as noncommutative characters of comodules of the quantized function algebra of GL(n). The latter algebra is conjectured to be isomorphic to a combinatorial algebra with cubic relations, the so called quantum plactic algebra. We give some evidences in support of this conjecture.

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Supersymmetry and neutral bions: hints about deconfinement?

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I will discuss the conjecture that the thermal deconfinement transition in pure Yang-Mills theory is continuously connected to a quantum phase transition in softly-broken $N = 1$ supersymmetric Yang-Mills theory on $R^3 \times S^1$. The latter is driven by a competition between the contributions of various exotic "topological" molecules, classified via the affine Dynkin diagram of the gauge group. The transition occurs in a calculable weak-coupling regime, thus a great deal can be learned about its properties. I shall present evidence in favor of the continuity conjecture for theories based on all simple compact Lie groups and discuss possible directions for future study and speculations.

Boson-fermion correspondence in the language of symmetric functions

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Keywords: boson-fermion correspondence, generalized symmetric functions, Jacobi-Trudi identity

Boson-fermion correspondence describes equivalence of two representations of the Heisenberg algebra and uses the fact that algebra of symmetric functions can be identified with a polynomial algebra. We will show that the main components of the famous correspondence can be interpreted as equally famous identities in symmetric functions – such as Cauchy identity, Jacobi-Trudi identity, relations between generating functions of homogeneous, elementary and power

sum functions. This combinatorial interpretation allows to construct analogues of boson-fermion correspondence in analogues of symmetric functions that appear naturally in representation theory and combinatorics. The examples include characters of finite-dimensional irreducible representations of classical Lie groups, shifted and double Schur functions, particular evaluations of Jacobi polynomials of type BC_N.

Symmetries and supersymmetries of Dirac operators with skew-symmetric torsion

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Keywords: symmetry groups, Dirac-type operators with torsion, generalized Killing-Yano tensors

We propose to study the symmetries of the Dirac operator with skew-symmetric torsion [1] following the appraoch in [2, 3] for regular Dirac and Dirac-type operators. We will study the properties of covariantly constant generalized Killing-Yano tensors for HKT[4] and also non-Kähler manifolds. We also investigate the superalgebras generated by the modified Dirac operators with torsion.

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Classification of quantum groups and Lie bialgebra structures on $sl(n, \mathbb{F})$

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Keywords: quantum group, Lie bialgebra, classical double, r-matrix, admissible triple

Given an arbitrary field $\mathbb{F}$ of characteristic 0, we study Lie bialgebra structures on $sl(n, \mathbb{F})$, based on the description of the corresponding classical double. For any Lie bialgebra structure δ, the classical double $D(sl(n, \mathbb{F}), \delta)$ is isomorphic to $sl(n, \mathbb{F}) \otimes_{\mathbb{F}} A$, where A is either $\mathbb{F}[\varepsilon]$, with $\varepsilon^2 = 0$, or $\mathbb{F} \oplus \mathbb{F}$ or a quadratic field extension of $\mathbb{F}$. In the first case, the classification leads to quasi-Frobenius Lie subalgebras of $sl(n, \mathbb{F})$. In the second and third cases, a Belavin–Drinfeld cohomology can be introduced which enables one to classify Lie bialgebras on $sl(n, \mathbb{F})$, up to gauge equivalence. The Belavin–Drinfeld untwisted and twisted cohomology sets associated to an r-matrix are computed. For $\mathbb{F} = \mathbb{C}((\hbar))$, we obtain the classification of quantum groups whose quasi-classical limit is $sl(n, \mathbb{C})$.

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Spontaneous supersymmetry breaking and instanton sum in 2D type IIA superstring theory

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Keywords: matrix models, superstring theory, noncritical strings, supersymmetry, nonperturbative effects

We consider a double-well supersymmetric matrix model and its interpretation as a nonperturbative definition of two-dimensional type IIA superstring theory. The interpretation is confirmed by direct comparison of symmetries and amplitudes in both sides of the matrix model and the string theory [1, 2].

We show that instanton contributions in the matrix model survive in the double scaling limit and induce spontaneous supersymmetry breaking [3]. It implies that the target-space supersymmetry is spontaneously broken due to nonperturbative effects in the string theory. Finally, we obtain the full nonperturbative free energy in terms of the Tracy-Widom distribution in random matrix theory [4].

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Characterizations of Ricci flat metrics and Lagrangian submanifolds in terms of the variational problem

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Keywords: Einstein metrics, Ricci flat metrics, Lagrangian submanifolds, slant immersions, harmonic maps

In this talk, we introduce a new variational problem on the space of (0, 2)-tensors with non-zero determinant. In particular, we calculate the 1st and 2nd variational formulaes for the new variational problem.

We show that Einstein metrics and slant immersions are critical points of the newly defined functional. Moreover, Ricci flat metrics and Lagrangian immersions can be considered as stable critical points of it.

This is a collaborative research with Seiichi Udagawa.

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Functional renormalization group approach for composite–particle excitations

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Keywords: functional renormalization group, vertex IR regulator, superfluidity

Functional renormalization group (FRG) [1] is a pragmatic realization of Wilson's original ideas of renormalization group [2], which provides a nonperturbative formulation of quantum field theories. After reviewing its formal and mathematical properties, I would like to talk about the problem of composite–particle excitations from the viewpoint of FRG [3–5]. We will find usefulness of the "vertex IR regulator" in the appearance of bound states [4,5].

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Quantum mechanics on large circles: an adelic approach

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Keywords: quantum mechanics on rational numbers, adeles

Harmonic analysis on Q (rational numbers) and its Pontryagin dual group A_Q/Q is studied. The relevant Schwartz-Bruhat space of functions is defined. The

corresponding Heisenberg-Weyl group is studied. Other time-frequency topics in this context (e.g., Wigner and Ambiguity functions) are also discussed. The work can be viewed as harmonic analysis on "large circles". It reviews and extends further the work in [1, 2]

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Maurer-Cartan elements and cyclic operads

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Keywords: operads, BV algebras, deformation theory, moduli spaces

Getzler introduced the notion of a 'gravity algebra' to model the algebraic structure arising on the equivariant cohomology of a topological conformal field theory. The corresponding algebraic structure in the non-equivariant setting is a Batalin-Vilkovisky (BV) algebra. In this talk we will give a construction of a pair of differential graded Lie algebras whose Maurer-Cartan solutions determine such a gravity-BV pair on the twisted cohomology. Moreover the failure of the gravity relations to hold on the cochain level can be given by explicit homotopies. The input for this construction is the notion of a cyclic operad, which we shall review.

Minisymposium

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A generalization of the extended Jacobi polynomials in two variables

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Keywords: extended Jacobi polynomials, Jacobi polynomials, recurrence relation, generating function, hypergeometric function

The main object of this paper is to construct a two-variable analogue of extended Jacobi polynomials and to give some properties of these polynomials. We obtain various differential formulas for two-variable extended Jacobi polynomials and give recurrence relations involving them. We derive various families of bilinear and bilateral generating functions. Furthermore, some special cases of the results are presented in this study. We mainly use the following references: [1–4].

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Summability process on the Baskakov-type approximation theory

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Keywords: Korovkin theory, Baskakov-type approximation, summability process

The classical Korovkin theory was generalized by Baskakov [2] by means of a wider class of operators than positive linear operators. In [1], some statistical variants of Baskakov's results were obtained. The main purpose of this study is to investigate the effects of summability process introduced by Bell [2] on the Baskakov-type approximation. We also show that our results improve the ones in [2] and not comparable with the ones in [1].

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Voronovskaja type approximation theorem for q-Stancu-Szász-Beta type operators

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Keywords: q-Stancu-Szász-Schurer-Beta type operators, Voronovskaja's theorem

In this paper, we study on q-analoque of Stancu-Szász-Beta type operators. We give a Voronovskaja type theorem for these operators. We mainly use the following papers: [1–3].

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Summability methods in the weighted approximation to derivatives of functions

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Keywords: weighted approximation, weighted spaces, summability process

In [3], Èfendiev constructed an approximation by linear operators to derivatives of functions defined on weighted spaces. Their statistical approximation properties were examined by Anastassiou and Duman [1]. In this study, we mainly use the summability process introduced by Bell [2] in order to get a weighted approximation by linear operators to derivatives of functions. Furthermore, we prove that our approximation method is more applicable than Èfendiev's result.

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On modified Kantorovich-type operators in polynomial weighted spaces

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Keywords: Kantorovich-type operators, modulus of continuity, weighted spaces

In this paper, we introduce a modification of Kantorovich-type operators in polynomial weighted spaces of functions of one variable and we study some approximation properties of these operators. We give some estimations by means of the weighted modulus continuity. Our main motivation for this study is the following papers: [1–3].

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On the Poisson-Charlier polynomials

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Keywords: Poisson-Charlier polynomials, recurrence relation, generating function, hypergeometric function, Lauricella functions

In this paper we study on the Poisson-Charlier polynomials. Some of their recurrence relations are obtained. Various families of bilinear and bilateral generating functions for these polynomials are derived. Furthermore, some special cases of the results are presented. We mainly use the following references: [1, 2, 4, 4, 5].

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Optimal quadratic Lagrange interpolation: extremal node systems with least Lebesgue constant via symbolic computation

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Keywords: extremal nodes, optimal lagrange interpolation, minimal Lebesgue constant, quadratic polynomial, symbolic computation

We consider optimal Lagrange interpolation with polynomials of degree at most 2 on the unit interval $[-1, 1]$. In a largely unknown paper, Schurer [3] has analytically described the infinitely many zero-symmetric (see also Tureckii [4]) and zero-asymmetric extremal node systems $-1 \leq x_1 < x_2 < x_3 \leq 1$ which all lead to the minimal Lebesgue constant 1.25 that had already been determined by Bernstein [1]. As Schurer's proof is not given in full detail, we formally verify it by providing two new and sound proofs of his theorem with the aid of symbolic computation using quantifier elimination. Additionally, we provide an alternative, but equivalent, parameterized description of the extremal node systems for quadratic Lagrange interpolation which seems to be novel. It is our purpose to bring the computer assisted solution of the first nontrivial case of optimal Lagrange interpolation to wider attention and to stimulate research of the higher-degree cases (see [2] for the cubic case). This talk will be presented by the first author within the mini-symposium on Approximation Theory and Special Functions.

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Some recursion formulas for Horn hypergeometric functions

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Keywords: Horn hypergeometric functions, recursion formulas.

Recently, Opps, Saad and Srivastava [1] have given the recursion formulas of Appell's function F_2 by the contiguous relation of the Gauss hypergeometric series ${}_2F_1$. Then, Wang [2] has presented various recursion formulas for F_1, F_2, F_3 and F_4 Appell's hypergeometric functions. The aim of this paper is to present various recursion formulas for Horn hypergeometric functions by the contiguous relations of hypergeometric series.

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Recent developments on convergence methods on time scales

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Keywords: time scale, statistical convergence, lacunary statistical convergence

Using the theory of time scales (see [1]), it is possible to unify discrete and continous analysis. The main object of this study is to apply this theory to convergence methods from the summability theory, such as, statistical convergence [2, 4, 5], lacunary statistical convergence [3, 6].

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International Conference Mathematics Days in Sofia
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Some results on convergence properties of singular integrals

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Keywords: pointwise approximation, singular integral, point of continuity

In this talk, we give some theorems about pointwise approximation to the functions belong to the Lebesgue space $L_p(\mathbb{R}^2)$ by a family of convolution type singular integral operators and the order of convergency. Also some computational results are presented.

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Weighted approximation by the q-Szász-Schurer-Beta type operators

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Keywords: q-Szász-Schurer-Beta type operators, weighted approximation.

In this talk, we study weighted approximation properties of a sequence of q-Szász-Schurer-Beta type linear positive operators. We give a weighted approximation theorem and obtain rates of convergence of these operators for continuous functions of polynomial growth on the interval $[0, \infty)$. This work is motivated by the following papers: [1–3].

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Minisymposium

Geometry Days in Sofia

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- **Johann Davidov**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria
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- **Oleg Mushkarov**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria
- **Velichka Milousheva**, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria – secretary
  email: `vmil@math.bas.bg`

The aim of the event is to provide a coherent forum for scientific interaction of participants working in various facets of modern differential geometry.

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Stacky compactifications of minimal resolutions of singularities of type A_k and gauge theory

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Keywords: ALE spaces, stacks, framed sheaves, instantons, gauge theories

We use stacky compactifications of minimal resolutions of singularities of type A_k (ALE spaces) and a theory of framed sheaves on projective stacks to give rigorous definitions of partition functions for supersymmetric gauge theories on ALE spaces. We show that in dimension 2 the moduli functor for framed sheaves on projective stacks is representable, and is represent by a scheme, for which it is possible to compute the obstruction to smoothness.

The resulting partition functions show nice factorization properties with respect to the toric structure of the ALE spaces and it is possible to prove Nekrasov's conjecture for them; in the vanishing limit of the equivariant parameter they define suitable pre-potentials.

We use the notion of root stack to be able to incorporate instantons on the ALE spaces that have nontrivial holonomy at infinity.

Joint work with F. Sala and M. Pedrini and R. Szabo

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Higher codimension CR structures, Levi-Kähler reduction and toric geometry

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Keywords: CR structure, toric geometry, Kähler metric

CR structures in codimension one play an increasingly important role in differential geometry, deeply intertwined with Kähler geometry. In this talk, based on joint work in progress with V. Apostolov, P. Gauduchon and E. Legendre, I discuss the relation between CR structures in higher codimension and Kähler geometry, through a process called "Levi–Kähler reduction". I focus in particular on the toric case, where Levi–Kähler reduction provides a new way to construct distinguished metrics on toric varieties. When the Delzant polytope is a product of simplices, explicit quotients of products of spheres are obtained, generalizing Bryant's Bochner-flat metrics on weighted projective spaces.

Orientability in real Gromov-Witten theory

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Keywords: moduli spaces, orientability, real Gromov-Witten theory

The orientability problem in real Gromov-Witten theory is one of the fundamental hurdles to enumerating real curves. In this talk I will describe topological conditions on the target manifold which ensure that the uncompactified moduli spaces of real maps are orientable for all genera of and for all types of involutions on the domain. In the case of a fixed-point free involution on the target the result yields real Gromov-Witten invariants of arbitrary genus. This is a joint work with A. Zinger.

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Approximate Hitchin-Kobayashi correspondence for Higgs G-bundles

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Keywords: principal (Higgs) bundles, semistability, approximate Hermitian-Yang-Mills structures, Kähler manifolds

We generalize the Hitchin-Kobayashi correspondence between semistability and the existence of approximate Hermitian-Yang-Mills structures to the case of principal Higgs bundles. We prove that a principal Higgs bundle on a compact Kähler manifold, with structure group a connected linear algebraic reductive group G, is semistable if and only if it admits an approximate Hermitian-Yang-Mills structure.

Joint work with U. Bruzzo.

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Labelled convex quadrilaterals and toric Kähler-Levi structures on the CR structure of $S^3 \times S^3$

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Keywords: toric Kähler structures, CR-structures with symmetries, generalized contact forms

I will explain a correspondance between labelled convex quadrilaterals and the set of T^4 invariant connection 1-forms on $S^3 \times S^3$ whose differential define a positive definite 2-form on the natural CR structure of $S^3 \times S^3$. These structures define a natural toric Kähler metric on any compact symplectic toric orbifold with first Betti number 2, thus generalizing the well-known construction of Bochner-flat metrics on weighted projective planes, due to Webster and R. Bryant. I will discuss generalisations of this correspondance in higher dimension. This is a part of joint work with V. Apostolov, D. M. Calderbank, P. Gauduchon.

The J-flow and stability

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Keywords: moment map, the J-flow, CSCK metrics, the K-stability

The J-flow is a parabolic flow introduced by Donaldson [1]. In this talk, we present a new algebro-geometric stability condition which is equivalent conjecturally to the existence of solutions of the critical equation of the J-flow. We present also examples due to Fang-Lai [2] and explain how this is related to the stability condition.

Acknowledgements. The first named author is grateful to Gábor Székelyhidi and the rest of Department of Mathematics at University of Notre Dame for their hospitality. The second named author thanks Jeff Diller for several useful conversations on the topic in the appendix. The first named author was supported by a FQRNT grant during his visit to University of Notre Dame. The second named author is supported in part by NSF grant DMS-1306298.

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Existence of holomorphic functions on nilpotent Lie groups

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Keywords: almost complex manifolds,holomorphic functions, nilpotent Lie groups

We classify real 4- and 6-dimensional nilpotent Lie algebras for which every left-invariant almost complex structure on the corresponding Lie group has non-constant local holomorphic functions. We also obtain a simple algebraic characterization of 2-step nilpotent Lie algebras with the same property.

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Comparison results in CR geometry under positive "Ricci" bound

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Keywords: Lichnerowicz-Obata theorem, pseudohermitian manifold, sub-Laplacian, eigenvalues

The CR version of Obata type result for the first eigenvalue of the sub-Laplacian in the setting of a compact strictly pseudoconvex pseudohermitian manifold which satisfies a Lichnerowicz type condition will be presented together with some other comparison results.

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Degenerate twistor spaces for hyperkähler manifolds

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Keywords: hyperkähler manifolds, twistor spaces

Let M be a hyperkähler manifold, and η a closed, positive (1,1)-form which is degenerate everywhere on M. We associate to η a family of complex structures on M, called a degenerate twistor family, and parametrized by a complex line. When η is a pullback of a Kähler form under a Lagrangian fibration L, all the fibers of degenerate twistor family also admit a Lagrangian fibration, with the fibers isomorphic to that of L.

Mini-symposium

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Invited Talks

Nonclassical convolutions and their uses

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Keywords: convolution, operational calculus, boundary value problem, Duhamel principle

The following generalization of the classical Duhamel convolution

$$(f \overset{t}{*} g)(t) = \chi_\tau \left\{ \int_\tau^t f(t+\tau-\sigma) g(\sigma) d\sigma \right\} \tag{1}$$

with arbitrary linear functional χ, found independently by the author (1974) and by L. Berg (1976), has similar algebraic properties as these of (1). It can be used to build a generalization of Mikusinski's operational calculus, intended for nonlocal Cauchy problems.

As a next step, we proposed (1976) the operation

$$\begin{aligned}(f \overset{x}{*} g)(x) \quad = \quad & \Phi_\xi \left\{ \int_x^\zeta f(\zeta + x - \eta) g(\eta) d\eta \right. \\ & \left. - \int_{-x}^\zeta f(|\zeta - x - \eta|) g(|\eta|) \operatorname{sgn}(\eta(\zeta - x - \eta)) d\eta \right\},\end{aligned} \tag{2}$$

which happened to be useful for solving of nonlocal BVPs connected with the square of differentiation [1].

A detailed study of operations (1) and (2) and revealing their differential and functional properties is made in N. Bozhinov's book [2]. Recently, it became clear that both convolutions and their closest extensions are useful for practical applications too. Their multidimensional extension allow to develop multivariate operational calculi and to apply them to local and nonlocal BVPs of mathematical physics.

The generalizations of the classical Duhamel principle give explicit solutions of BVPs which solutions had been known till now only in series form.

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On associativity of the convolution of ultradistributions

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Keywords: convolution of ultradistributions, associativity of convolution of ultradistributions

New results on the existence and associativity of the convolution in various spaces of ultradistributions are proved.

Acknowledgements. This work was partly supported by the Centre for Innovation and Transfer of Natural Sciences and Engineering Knowledge, Poland.

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Nonstandard quadratures of Gauss-Lobatto type and applications in the fractional calculus

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Keywords: Gauss-Lobatto quadratures, numerical methods, fractional derivatives, CAS Mathematica; 65D30, 33C45, 41A55, 65D32

In a joint paper with S. Esmaeili [2], a family of nonstandard Gauss-Lobatto quadratures for numerical calculating integrals of the form $\int_{-1}^{1} f'(x)(1-x)^{\alpha}\,\mathrm{d}x$, $\alpha > -1$, has been derived and applied to approximation of fractional derivatives of Riemann-Liouville and Caputo type.

In this lecture we start with a general weight function $w : (-1,1) \to \mathbb{R}$ for which all its moments $\mu_\nu = \int_{-1}^{1} x^\nu w(x)\,dx$, $\nu = 0, 1, \ldots$, exist and are finite, and we consider nonstandard (algebraic) quadrature formulas of Gaussian or Gauss-Lobatto type of the form

$$I(f) = \int_{-1}^{1} (Lf)(x)w(x)\,\mathrm{d}x = A_0 f(-1) + \sum_{k=1}^{n} A_k f(x_k) + A_{n+1} f(1) + R_n(f),$$

where L is a linear operator acting between certain functional spaces, and R_n is the remainder term, which is equal zero for all algebraic polynomials of degree at most $2n+1$. A special attention is devoted to an important case when $(Lf)(x) = \alpha f(x) + (1+x)f'(x)$, $\alpha > 0$. Under some conditions on the moment sequence $\{\mu_\nu\}_{\nu\geq 0}$ we prove that such kind of quadratures exist for each $n \in \mathbb{N}$. The nodes x_k are real, mutually different and lie in $(-1,1)$. The weights A_k can be expressed in terms of the corresponding Christoffel numbers of an equivalent Gauss-Christoffel quadrature formula (cf. [3]). We also analyze some special weight functions, including weights of Jacobi type, and give some applications of such quadrature rules in the fractional calculus.

A software implementation of these quadratures was done by the recent MATHEMATICA package `OrthogonalPolynomials` (cf. [1] and [4]), which is downloadable from the Web Site: `http://www.mi.sanu.ac.rs/~gvm/`. Several numerical examples are presented and they show the effectiveness of the proposed approach.

Acknowledgements. The author was supported in part by the Serbian Ministry of Education, Science and Technological Development (No. #OI 174015).

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Stronger Rolle's Theorem for Complex Polynomials

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A domain Θ_n is called *Rolle's domain* if for every complex polynomial $p(z)$ of degree $n \geq 2$ and $p(-i) = p(i)$ there exists at least one $\zeta \in \Theta_n$, such that $p'(\zeta) = 0$.

A Theorem X is called Rolle's theorem for complex polynomials if it states that a given domain Θ_n^X is a Rolle's domain.

A Rolle's Theorem X is *stronger* than the Rolle's Theorem Y, if $\Theta_n^X \subset \Theta_n^Y$ and $\Theta_n^X \neq \Theta_n^Y$.

A Rolle's Theorem X is *sharp*, if from $\Theta_n^Y \subset \Theta_n^X$ follows that $\Theta_n^Y = \Theta_n^X$.

There are several Rolle's theorems for complex polynomials. The most famous one is, see [1, p. 126]:

Theorem 1 (Grace-Heawood). *The disk*

$$\Theta_n^{GH} = D\left[0; \cot\frac{\pi}{n}\right] = \left\{z : \ |z| \leq \cot\frac{\pi}{n}\right\} \tag{1}$$

is a Rolle's domain.

Another complex Rolle's theorem, see [1, Theorem 4.3.4, p. 128], is the following:

Theorem 2. *The double disk* $\Theta_n^F = DD[c;r] = D[-c;r] \bigcup D[c;r]$, *where*

$$c = \cot\frac{\pi}{n-1}, \quad r = \sin^{-1}\frac{\pi}{n-1}; \quad n \geq 3,$$

is a Rolle's domain.

Neither one of the above two theorems is stronger than the other.

The main goal of this lecture is to prove the following:

Theorem 3. *The double disk* $\Theta_n^{SS} = DD[c;r]$, *where*

$$c = \cot\frac{2\pi}{n}, \quad r = \sin^{-1}\frac{2\pi}{n}; \quad n \geq 3,$$

is a Rolle's domain.

It is easy to see that Theorem 3 is stronger than Theorem 1 and Theorem 2. In Figure 1, the Rolle's domains of Theorems 1, 2 and 3, for $n = 20$, are presented.

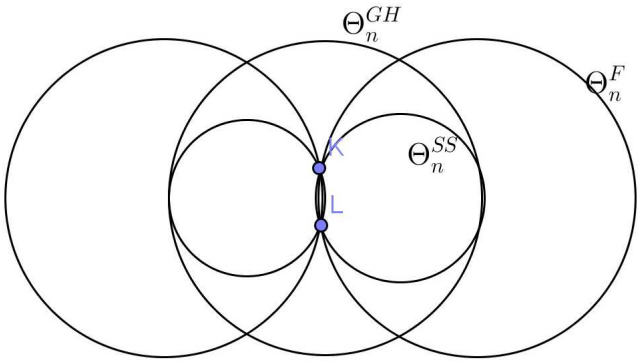

Fig. 1, K=i, L=-i.

The prove of Theorem 3 is based on the notion *locus holder*, on an analogue of the Grace-Walsh-Szegő coincidence theorem, called Argument coincidence theorem and on the Sector theorem, see [2].

To make the lecture selfcontent, we present the needed facts for the notion locus holder, see [3] and the formulation of the Argument coincidence theorem. We also emphasize on the Sector theorem, which is an analogue of the Gauss-Lucas theorem for the algebraic polynomials with real end non negative coefficients.

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Communications

Application of the operational calculus approach of Dimovski for solving the backward heat problem

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Keywords: operational calculus, non-classical convolution, Duhamel principle, ill-posed problem, quasireversibility

The final value problem for the heat equation is known to be ill-posed. To deal with this, in the method of quasireversibility, the equation or the final value condition is perturbed to form an approximate well-posed problem, depending on a small parameter ε. In this work, several known quasireversibility techniques for the backward heat problem are considered and the obtained new problems are treated using the operational calculus approach developed by Dimovski [1]. For every approximate problem, applying an appropriate bivariate operational calculus, a Duhamel-type representation of the solution is obtained. It is in the form of a convolution product of a special solution of the problem and the given final value function. The idea for such Duhamel-type representations for ill-posed problems is originally proposed in [2]. Here it is further developed for different regularizations of the backward heat problem and its application for calculating the numerical solution is illustrated on some test problems.

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Simulation of viscoelastic flows with fractional derivative models: an approach via the operational calculus of Dimovski

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Keywords: operational calculus, non-classical convolution, Riemann-Liouville fractional derivative, generalized Oldroyd-B fluid, finite difference scheme

The boundary value problem for the velocity distribution of a viscoelastic flow with generalized fractional Oldroyd-B constitutive model is studied. The model contains two Riemann-Liouville fractional derivatives in time of different orders. Based on the eigenfunction expansion, the unique existence of the solution is established and some regularity results and qualitative properties are obtained. Further, applying the operational calculus approach proposed by Dimovski [1], a Duhamel-type representation of the solution with respect to the space variables is found. This is a compact representation, containing a non-classical convolution product of a special solution and the given initial function. It is appropriate for numerical computation of the solution. To illustrate this, a finite difference scheme is also constructed and the solutions of some test problems are calculated numerically in different ways: using the finite difference approximation, using the Duhamel-type representation, or combining both of them. Numerical results for one- and two-dimensional examples are presented and the different techniques are compared in terms of efficiency, accuracy, and CPU time.

Acknowledgements. This research is in frames of the working program of the Project "Mathematical Modelling ..." under bilateral agreement (2012–2014) between Bulgarian and Serbian Academies of Sciences.

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Existence of solutions to boundary value problem for impulsive fractional equations

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Keywords: fractional differential equations, impulsive conditions, weak solution, classical solution, three critical point theorem

In this paper we study the existence and the multiplicity of solutions for an impulsive boundary value problem for fractional differential equations. The notions of classical and weak solutions are introduced. Then, the existence of at least one and three solutions are proved. An example is given.

For related studies, see references below.

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Commutant of Sturm-Liouville operator in an invariant subspace

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Keywords: commutant, invariant subspace, Sturm-Liouville operator, convolution

We characterize the continuous linear operators $M : C \to C$, with $C = C[0,\infty)$, commuting with the Sturm-Liouville operator $D = \dfrac{d^2}{dx^2} - q(x)$ in the invariant subspace $C_{h,\Phi} = \{f \in C, f'(0) - hf(0) = 0, \Phi\{f\} = 0\}$, where Φ is an arbitrary nonzero continuous linear functional. Additionally, we assume $M : C^k \to C^k$, $k = 1, 2$.

Using a convolution $f * g$ found by the authors (see [1]) in 1976, we found explicitly the commutant as consisting of all operators of the form

$$Mf(x) = \mu f(x) + m * f,$$

where $\mu = \text{const}$ and $m \in C$.

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Explicit solutions of BVPs for multidimensional heat equation

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Keywords: convolution, operational calculus, nonlocal boundary value problem, Duhamel principle

We consider a general nonlocal BVP (boundary value problem) of the form:

$$\begin{aligned}
&u_t = u_{x_1x_1} + \cdots + u_{x_nx_n} + F(x_1, \ldots, x_n, t),\\
&u(x_1, \ldots, x_n, 0) = 0,\\
&u(x_1, \ldots, x_{j-1}, 0, x_{j+1}, \ldots, x_n, t) = 0,\\
&\Phi_{j,\xi}\left\{u(x_1, \ldots, x_{j-1}, \xi, x_{j+1}, \ldots, x_n, t)\right\} = 0, \quad j = 1, 2, \ldots, n,
\end{aligned}$$

with given linear functionals $\Phi_1, \ldots, \Phi_n$.

It is shown that using a multidimensional operational calculus, the problem could be reduced to n one-dimensional BVPs of the form

$$\begin{aligned}
&v_t = v_{x_kx_k}, \quad v(x_k, 0) = x_k,\\
&v(0, t) = 0, \quad \Phi_{j,\xi}\{v(\xi, t)\} = 0, \quad k = 1, \ldots, n,
\end{aligned}$$

with corresponding solutions $\Omega_k(x_k, t)$.

Then a general solution is obtained as an extension of the Duhamel principle for the space variables in the form

$$u(x_1, \ldots, x_n, t) = \frac{\partial^{2n}}{\partial x_1^2 \ldots \partial x_n^2}\left[(\Omega_1 \ldots \Omega_n) * F\right],$$

where $*$ is a multidimensional convolution.

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On the convolutors in the D_{Lp}-type spaces associated with a singular second order differential operator

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Keywords: D_{Lp}-type spaces, convolution product, differential operator

We consider the D_{Lp}-type spaces associated with a singular second order differential operator $\triangle_A$. Some results are established. Next, using the convolution associated with $\triangle_A$, we study the convolutors and the surjective convolution operators acting on spaces of distributions of L_p^A-growth.

For more details, see References.

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From the hyper-Bessel operators of Dimovski to the generalized fractional calculus

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Keywords: hyper-Bessel operators and functions, integrals and derivatives of fractional order, special functions and integral transforms related to fractional calculus

In several works since 1966, Dimovski [1] introduced and studied a very wide class of singular differential operators with variable coefficients of arbitrary integer order $m \geq 1$, known nowadays as *hyper-Bessel differential operators* and appearing in alternative forms in many problems of mathematical physics:

$$B = x^{\alpha_0} D x^{\alpha_1} D x^{\alpha_2} \cdots D x^{\alpha_m} = x^{-\beta} \prod_{i=1}^{m} (xD + \beta\gamma_i)$$

$$= x^{-\beta} \left(x^m D^m + a_1 x^{m-1} D^{m-1} + \cdots + a_{m-1} x D + a_m \right), \quad \beta > 0, \quad m \geq 2, \tag{1}$$

with $D := d/dx$, and real parameters α_k, γ_i, a_j. The best known example, giving rise to their name, is the second order differential operator of Bessel $B_\nu = x^{-2}(xD+\nu)(xD-\nu) = x^{\nu-1} D x^{2\nu+1} D x^{\nu} = D^2 + x^{-1}D - x^{-2}\nu^2$, related to

the Bessel function $y(x) = J_\nu(x)$ as a solution of the equation $B_\nu y(x) = -y(x)$. Another simple representative of higher order is the operator of m-fold differentiation $D^m = (d/dx)^m$.

Dimovski constructed a very general operational calculus for the operators (1) following the algebraic approach of Mikusinski, and giving rise to the new notion of *convolutional calculus*, [2]. Besides, he discovered that an integral transform introduced by another Bulgarian mathematician in 1958 – the Obrechkoff transform – can serve as a transform approach to the same operational calculus for (1). Many well-known mathematicians (to mention Ditkin, Prudnikov, Meller, Botashev, Krätzel, Rodriguez, etc) have studied later very particular cases of the hyper-Bessel operators (1) and rediscovered operational calculi and Laplace-Meijer or Hankel type integral transforms related to them, years after Dimovski's most general works.

The present author has started her studies on Dimovski's hyper-Bessel operators and on the Obrechkoff transform since 1975, and finally benefitted of them in developing a theory of the *generalized fractional calculus* (GFC) [3] and in introducing new classes of integral transforms [4,5] and special functions [6]. The GFC deals with generalized integrals and derivatives of *fractional multi-orders* $(\delta_1, \delta_2, \dots, \delta_m)$ as analogues of the Riemann-Liouville (R-L) and Erdélyi-Kober integrals (E-K) $I^\delta, I_\beta^{\gamma,\delta}$ and derivatives $D^\delta, D_\beta^{\gamma,\delta}$ of arbitrary order $\delta > 0$ in the classical fractional calculus. Having the structure of commuting compositions of E-K operators $I_{(\beta_k)_1^m,m}^{(\gamma_k)_1^m,(\delta_k)_1^m} = \prod_1^m I_{\beta_k}^{\gamma_k,\delta_k}$, the generalized fractional integrals of our GFC are represented by means of integral operators involving special functions:

$$(2)\quad I_{(\beta_k),m}^{(\gamma_k),(\delta_k)} f(x) = \int_0^1 \Phi(\sigma) f(x\sigma) d\sigma, \quad \text{where } \Phi(\sigma)$$

$$= H_{m,m}^{m,0}\left[\sigma \middle| \begin{matrix} (\gamma_i + \delta_i + 1 - \frac{1}{\beta_i}, \frac{1}{\beta_i})_1^m \\ (\gamma_i + 1 - \frac{1}{\beta_i}, \frac{1}{\beta_i})_1^m \end{matrix}\right]$$

is the Fox H-function, a generalized hypergeometric function of very general nature. The corresponding R-L and Caputo-type generalized fractional derivatives $D_{(\beta_k),m}^{(\gamma_k),(\delta_k)}$ are defined by means of suitable differ-integral expressions, see [3,7].

The other operators of fractional calculus and many generalized integrations and differentiations used in applied analysis are shown to be special cases of (2). But the worthy fact to emphasize is that *the hint to introduce the GFC came from the hyper-Bessel operators* (1) of integer order m that appear also to be generalized "fractional derivatives" of multiorder $(1, 1, \dots, 1)$: $B = x^{-\beta} D_{(\beta,\dots,\beta),m}^{(\gamma_1,\dots,\gamma_m),(1,\dots,1)}$.

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Fractional order linear autonomous system with distributed delay

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Keywords: fractional order differential equations, Riemann-Liouville fractional derivative, Caputo derivative, distributed delay

Asymptotic properties of the solutions of fractional order linear autonomous system with distributed delay are studied.

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Evolution equations for the Stefan problem

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Keywords: Stefan problem, boundary value problem, abstract parabolic evolution equations

The Stefan problem is a particular kind of a free boundary value problem which models phase transition phenomena, for example melting of ice and freezing of water.

We study a quasi-steady variant and propose in our model a boundary condition with surface tension and kinetic undercooling that reflects the relaxation dynamics. In our approach to the problem we use the theory of abstract quasilinear parabolic evolution equations. The obtained results are in Sobolev spaces.

Mean value theorems for analytic functions

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Keywords: mean value theorems, real and complex analytic functions, Rolle's theorem in the complex domain

The problem of extending Rolle's theorem to the complex domain, as well as other related questions, have been of enduring interest (see for example [2]). In this talk, we will examine some interesting and little-known mean value theorems concerning real and complex analytic functions, focusing on the complex case. A sharper Evard-Jafari theorem (see [1]) will be proved. The remarkable contributions of the Bulgarian school of Mathematics to this field will be emphasized throughout the presentation, and the paper will be dedicated to the 118th anniversary of the birth of Academician Nikola Obrechkoff.

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On various existence conditions for the convolution of Beurling ultradistributions

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Keywords: ultradistribution of Beurling type, convolution of ultradistributions of Beurling type

Theorems on the existence of the convolution as well as on the convergence of convolutions in some spaces of ultradistributions of Beurling type are proved under certain general conditions.

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Extending the Stieltjes Transform

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Keywords: abelian theorem, Boehmians, generalized function, iterated Laplace transform, Stieltjes transform

Several authors have extended the classical Stieltjes transform onto spaces of generalized functions. Many have investigated the Stieltjes transform on the space $J'(r)$, which consists of distributions of the form $T = D^n f$ (for some $n \in \mathbb{N}$), where f is a locally integrable function supported on the interval $[0, \infty)$ and satisfies a growth condition at infinity.

The space of generalized functions known as Boehmians, whose construction is algebraic, has been used to extend integral transforms such as Fourier, Laplace, Hilbert, and Hankel. Roopkumar [2] has extended the Stieltjes transform onto a space of Boehmians. However, the transform is a Boehmian, not a function.

In this note, by using iteration of the Laplace transform, we extend the Stieltjes transform onto a subspace of Boehmians which contains a proper subspace that can be identified with $J'(r)$. In this case, the transform is an analytic function in the half-plane $\operatorname{Re} z > 0$. This allows, in a natural way, to establish an abelian theorem of the final type.

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A family of hyper-Bessel functions and convergent series in them

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Keywords: hyper-Bessel function, hyper-Bessel differential operator, series in hyper-Bessel functions, convergence of series in complex plane

In 1953, Delerue introduced generalizations $J^{(m)}_{\nu_1,\ldots,\nu_m}(z)$ of the Bessel function of the first type $J_\nu(z)$ with vector indices $\nu = (\nu_1, \nu_2, \ldots, \nu_m)$. Later these functions were studied and explored also by other authors, for example Marichev, Kljuchantcev, Dimovski, Dimovski and Kiryakova (for details see e.g. [1]– [4]), etc. The Delerue hyper-Bessel functions are closely related to the hyper-Bessel differential operators of arbitrary order $m > 1$, introduced by Dimovski [1], see [3].

In this work we consider an enumerated family of hyper-Bessel functions and study the convergence of series in such kind of functions. The obtained results are analogues to the ones in the classical theory of widely used power series.

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A note on a subclass of close-to-convex functions

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Keywords: univalent functions, close-to-convex functions

Let S denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

which are analytic and univalent in the open unit disk $E = \{z : |z| < 1\}$.

Let C denote the class of convex functions [1]:

$$f(z) \in C \text{ if and only if for } z \in E, \ \operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > 0.$$

Let S^* denote the class of starlike functions [2]:

$$f(z) \in S^* \text{ if and only if for } z \in E, \ \operatorname{Re}\frac{zf'(z)}{f(z)} > 0.$$

A function $f(z)$ analytic in E is said to be close-to-convex in E, if there exists a function $g(z) \in S^*$ such that for $z \in E$

$$\operatorname{Re}\frac{zf'(z)}{g(z)} > 0.$$

The class of such functions is denoted by K, [3].

The classes S, K, S^* and C are related by the proper inclusions

$$C \subset S^* \subset K \subset S.$$

Now we will consider a class $\widetilde{K}$ defined as follows:

Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be analytic in E. Then $f(z) \in \widetilde{K}$ if and only if there exists a function $g(z) \in C$ such that for $z \in E$

$$\operatorname{Re}\frac{zf'(z)}{g(z)} > 0.$$

Since $C \subset S^*$, it follows that $\widetilde{K} \subset K$ and so, functions in $\widetilde{K}$ are univalent.

Theorem 1. *Let $f(z) \in \widetilde{K}$. Then for $z = re^{i\theta} \in E$:*

$$\frac{1-r}{(1+r)^2} \le |f'(z)| \le \frac{1+r}{(1-r)^2},$$

$$-\ln(1+r) + \frac{2r}{1+r} \le |f(z)| \le \ln(1-r) + \frac{2r}{1-r}.$$

Each inequality is sharp for $f_0(z)$ defined by

$$f_0(z) = \overline{x}\log(1-zx) + \frac{2z}{1-xz} \quad \textit{with} \quad |x| = 1.$$

Theorem 2. *Let $f(z) \in \widetilde{K}$, with $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, then for $z \in E$:*

$$|a_n| \le 1 - \frac{1}{n}$$

for $n \ge 2$. Equality is attained for $f_0(z)$.

Theorem 3. *Let $f(z) \in \widetilde{K}$ and $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Then,*

$$\left|a_3 - \mu a_2^2\right| \le \begin{cases} \dfrac{5}{3} - \dfrac{9}{4}\mu, & \textit{if} \quad \mu \le \dfrac{2}{9} \\ \dfrac{2}{3} + \dfrac{1}{9}\mu, & \textit{if} \quad \dfrac{2}{9} \le \mu \le \dfrac{2}{3} \\ \dfrac{5}{6}, & \textit{if} \quad \dfrac{2}{3} \le \mu \le 1. \end{cases}$$

For each μ there is a function in $\widetilde{K}$ such that equality holds.

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Somos-4 property of Hankel determinants derived from number sequences in an elliptic integral form

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Keywords: special numbers, determinants, polynomials, recurrence relations

In continuation of our paper [1], we will consider the special number sequences in the next integral form

$$g_n^{(p)} = \frac{1}{2\pi}\int_a^b x^{n-\delta_{1,p}}(b-x)^{\nu-1}(x-a)^{\mu-1}(b_1-x)^{\nu_1-1}(x-a_1)^{\mu_1-1}\,dx,$$

where $a_1 < b_1 < 0 < a < b$; $p \in \{0,1\}$; $\mu, \nu, \mu_1, \nu_1 > 0$; $n \in \mathbb{N}$ and their *Hankel transform* $H = \{h_n\}$ given by $h_n = |g_{i+j-2}|_{i,j=1}^n$.

We are interested in the special cases of $\{g_n\}$ which satisfy the *generalized convolution property*

$$g_n = \sum_{k=1}^{r} \alpha_k g_{n-k} + \beta \sum_{k=0}^{n-r} g_k g_{n-r-k},$$

and the Hankel determinants have the *generalized Somos-4 property*

$$h_n h_{n-4} = r\ h_{n-1}h_{n-3} + s\ h_{n-2}^2 \quad (n = 4, 5, \ldots) \quad (r, s \in \mathbb{N}).$$

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Representation of holomorphic functions by Schlömilch's series

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Keywords: Schlömilch's series, Bessel functions, representations of holomorphic functions, Erdélyi-Kober operators

A necessary and sufficient condition is given for a holomorphic function to be represented by a series of the kind

$$\sum_{n=0}^{\infty} a_n J_0(nz), \quad z,\ a_n \in \mathbb{C},$$

where J_0 is the Bessel function of first kind with zero index.

To derive the result, we use an Erdélyi-Kober operator of fractional order known as Uspensky transform, and the Poisson integral representation for $J_0(z)$ via $\cos(z)$.

Some of these results have been published in [1].

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An operational calculus approach for periodic and anti-periodic solutions in the environment of a computer algebra system

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Dedicated to the 80th anniversary of Professor Ivan Dimovski, Corresponding member of the Bulgarian Academy of Sciences

Keywords: convolution, operational calculus, Mikusiński calculus, linear ordinary differential equation, periodic solution, boundary value problem

An operational calculus approach for obtaining periodic and anti–periodic solutions of Linear Ordinary Differential Equations (LODE) with constant coefficients in the environment of a Computer Algebra System (CAS) is considered.

A Miikusiński's type operational calculus, based on the non–classical convolution of Dimovski

$$(f * g)(t) = \Phi_\tau \left\{ \int_\tau^t f(t+\tau-\sigma)g(\sigma)d\sigma \right\},$$

where Φ is a linear functional in $C(\mathbb{R})$, is used (see [1]). In the framework of this operational calculus an extension of the classical Heaviside algorithm is proposed. It is intended for solving nonlocal boundary value problems for LODEs with constant coefficients. The problems of obtaining periodic and anti–periodic solutions of LODEs with constant coefficients are reduced to such problems.

The algorithms based on the considered approach are implemented in the environment of the CAS *Mathematica*. Illustrative examples are presented.

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On various polynomials of Mittag–Leffler type

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Keywords: hypergeometric function, generating function, polynomial sequence, recurrence, orthogonality

The Mittag–Leffler polynomials $\{g_n(y)\}$ can be represented by the following hypergeometric function or by its generating function, as follows:

$$g_n(y) = 2y\ {}_2F_1\Big(\begin{matrix}1-n,\ 1-y\\ 2\end{matrix}\ \Big|\ 2\Big): \quad \Big(\frac{1+x}{1-x}\Big)^y = \sum_{n=0}^{\infty} g_n(y)x^n \quad (|x|<1).$$

They were introduced by Mittag-Leffler in a study on the integral representations. Their main properties were found by H. Bateman [1]. They can be considered as

a special case of the Meixner polynomials $M_n(x;\beta,c)$ for $\beta = 2$ and $c = -1$ (in spite of the fact that the Meixner polynomials require the constraint $0 < c < 1$) or the Pidduck polynomials by the expression $P_n(y) = ((e^D + 1)/2)g_n(y)$, where we use series for the exponential function and D is differentiation.

A few new papers considering these polynomials, appeared recently, see [2].

Based on the generalized integer powers of real numbers

$$z^{(0,h)} = z^{[0,h]} = 1, \quad z^{(n,h)} = \prod_{k=0}^{n-1}(z - kh), \quad z^{[n,h]} = \prod_{k=0}^{n-1}(z + kh) \quad (n \in \mathbb{N}),$$

and on the deformed exponential function

$$e_h(x,y) = (1 + hx)^{y/h} \qquad (x \in \mathbb{C} \setminus \{-1/h\},\ y \in \mathbb{R}),$$

we introduce *the deformed Mittag–Leffler polynomials* as the coefficients in the expansion

$$G_h(x,y) = e_h(x,y)\ e_{-h}(x,y) = \sum_{n=0}^{\infty} g_n^{(h)}(y)x^n\ .$$

We investigate their recurrence relations, hypergeometric representation and orthogonality. Since they have all zeros on the imaginary axis, we consider the associated real polynomials.

Acknowledgements. This research is supported by Ministry of Science and Technology of Serbia, under Projects No. 174011 and 44006. It is also in frames of the Project "Mathematical Modelling ..." under bilateral agreement (2012–2014) between Bulgarian and Serbian Academies of Sciences.

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On the asymptotic behavior of generalized functions

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Keywords: Stieltjes transform, asymptotic behavior, distributions

In this paper we present some new versions of the distributional Stieltjes transform, and apply it to the asymptotic behaviour and asymptotic expansion.

These integral transforms allow corresponding Abelian and Tauberian type results.

Acknowledgements. This paper is on the working program of bilateral project between Bulgarian and Serbian Academies of Sciences, "Mathematical modeling by means of integral transform methods, partial differential equations, special and generalized functions" and is also partly supported by Environment Protection, within the Project No. 144016.

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Some operational solutions of higher order fuzzy differential equations

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Keywords: fuzzy calculus, operational calculus, fuzzy differential equations, Mikusiński calculus and operators

In this paper fuzzy differential equations of higher order with fuzzy coefficients are studied within the frames of the Mikusiński calculus. Some preliminaries on the subject can be seen in the references.

The exact and the approximate solutions of the considered problem are constructed and their characters are analyzed.

Acknowledgements. This paper is on the working program of bilateral project between Bulgarian and Serbian Academies of Sciences, "Mathematical modeling by means of integral transform methods, partial differential equations, special and generalized functions" and is also partly supported by Environment Protection, within the Project No. 144016.

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Minisymposium

Variational Analysis

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The purpose of the mini-symposium "Variational Analysis" is to bring together researchers in applied analysis related to optimization and control. Most of the participants are active researchers that are in the beginning of their careers and are connected in some way to the developments of the field in Bulgaria.

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Metric regularity and convergence of iterative schemes

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Keywords: metric regularity, set-valued perturbations, Clarke's theorem, Robinson's theorem, non-smooth Newton's method

Given Banach spaces X and Y, a single-valued (possibly non-smooth) mapping $f : X \to Y$ and a multivalued mapping $F : X \rightrightarrows Y$, we investigate the properties of the solution mapping corresponding to a generalized equation:

$$\text{Find} \quad x \in X \quad \text{such that} \quad 0 \in f(x) + F(x). \tag{1}$$

This model has been used to describe in a unified way various problems such as equations, inequalities, variational inequalities, and in particular, optimality conditions. In the first part, we present a result concerning the stability of metric regularity under (set-valued) perturbations as well as an infinite-dimensional generalization of the Izmailov's theorem [6] which is an extension both of the Clarke's theorem [4] and finite-dimensional version of the Robinson's theorem [8]. In the latter part, we study the convergence properties of the following iterative process for solving (1): *Choose a sequence of set-valued mappings $A_k : X \times X \rightrightarrows Y$ approximating the function f and a starting point $x_0 \in X$, and generate a sequence (x_k) in X iteratively by taking x_{k+1} to be a solution to the auxiliary generalized equation*

$$0 \in A_k(x_{k+1}, x_k) + F(x_{k+1}) \quad \textit{for each} \quad k \in \{0, 1, 2, \dots\}. \tag{2}$$

The results from the first part are applied in the study of (super-)linear convergence of (2). Especially, several particular cases are discussed in detail. The presentation is based on the forthcoming paper with Samir Adly and Huynh Van Ngai as well as on the one with Asen L. Dontchev.

Acknowledgements. This work was partially supported by the European Regional Development Fund (ERDF), project NTIS – "New Technologies for the Information Society", European Centre of Excellence, CZ.1.05/1.1.00/02.0090.

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On the solution set of evolution inclusions

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Keywords: m-dissipative evolution inclusions, lemma of Filippov–Pliss

In a Banach space X we study the evolution inclusion of the form $x'(t) \in Ax(t) + F(t, x(t))$, where A is an m-dissipative operator and F is an upper hemicontinuous multifunction with nonempty convex and weakly compact values. If X^* is uniformly convex, F is almost upper hemicontinuous and one-sided Perron

with sublinear growth, then we prove a variant of the well known Filippov–Pliś theorem. Afterward, sufficient conditions for near viability and (strong) invariance of a set $K \subseteq \overline{D(A)}$ are established.

The result is then applied to prove the connectedness of the solution set of evolution inclusions without compactness and afterward the existence of attractor of autonomous evolution inclusion when the perturbations are one-sided Lipschitz with negative constant. We derive also $\varepsilon - \delta$ lower semicontinuity of the solution set and, consequently, the propagation of continuity of the minimum time function associated with the null controllability problem.

When X^* is not necessarily uniformly convex and F is almost lower semicontinuous with nonempty closed and bounded values we assume that A generates compact semigroup. If F is one-sided Perron with sublinear growth, then, we establish the relation between the solutions of the considered differential inclusion and the solutions of the relaxed one, i.e., $x'(t) \in Ax(t) + \overline{\text{co}}F(t, x(t))$.

Acknowledgements. This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS–UEFISCDI, project number PN-II-ID-PCE-2011-3-0154.

Monotonicity of set-valued maps and existence of solutions of differential inclusions

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Keywords: monotone maps, set-valued map, differential inclusions, existence

We survey some older and newer notions of monotonicity of set-valued mappings, recall some old and present some new results on existence of solutions of differential inclusions with convex and non-convex right-hand sides. We discuss some basic approaches to these proofs – compactness theorems, existence of special selections or discrete approximations.

Foundations of asymptotic control theory for a system of linear oscillators

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Keywords: minimum-time problem, reachable set, DiPerna–Lions theory

We present asymptotical control theory for a system of an arbitrary number of linear oscillators under common bounded control. We suggest a method for a design of a feedback control for the system. We prove by using the DiPerna-Lions theory of singular ODE that the suggested control law correctly defines a motion of the system. The obtained control is asymptotically optimal: the ratio of motion time to zero with this control to the minimum one is close to 1, if the initial energy of the system is large. Some of the results are based on a new lemma about observability of perturbed autonomous linear systems.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research grant 14-08-00606.

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Perturbation theory of observable linear systems and asymptotic control theory

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Keywords: minimum-time problem, reachable set, observability

The present contribution is motivated by asymptotic control theory for a system of linear oscillators: the problem is to design common bounded scalar control for damping all oscillators in asymptotically minimal time [1]. Motion of the system is described in terms of a canonical system similar to that of the Pontryagin maximum principle. We consider evolution equation for adjoint variables as a perturbed observable linear system. Due to the perturbation, the unobservable part of the state trajectory cannot be recovered exactly. We estimate the recovering error via the L_1-norm of perturbation. This allows us to prove that the control makes the system approach the equilibrium with a strictly positive speed.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research grant 14-08-00606.

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A relaxation result for state constrained inclusions in infinite dimension

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Keywords: state constrained differential inclusions, infinite dimensional state space

We consider the semilinear differential inclusion $x' \in Ax + F(t,x)$, under state constraints of the form $x(t) \in K$. Here A is the infinitesimal generator of a strongly continuous semigroup on an infinite dimensional separable Banach space X, $F : [0,1] \times X \to X$ is a set-valued map and K is a closed subset of X. We provide sufficient conditions for a relaxation theorem and analyze some examples involving partial differential equations.

Metric regularity in nonlinear semidefinite programming

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Keywords: metric regularity, strong regularity, Kojima function, coherent orientation, nonlinear semidefinite programming

The one-to-one relation between the points fulfilling the KKT conditions of an optimization problem and the zeros of the corresponding Kojima function is well-known. In the present paper we study the interplay between metric regularity and strong regularity of this a priori nonsmooth function in the context of semidefinite

programming. We shall identify a class of locally Lipschitz functions which turn out to have coherently oriented B-subdifferentials if metric regularity is assumed. This class is general enough to contain the Kojima function corresponding to the nonlinear semidefinite programming problem. Using a characterization of strong regularity for semismooth functions in terms of B-subdifferentials we arrive at an equivalence between metric regularity and strong regularity provided that an assumption involving the topological degree is fulfilled.

Acknowledgements. This research was supported by the EPSRC grant EP/D502535/1 and by the VEGA projects 1/0406/09 and 1/1005/12.

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Metric subregularity at an order q and the solving of inclusions

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Keywords: metric regularity at order q, calmness at order q, covering property

We consider some metric regularity properties at order q for set-valued mappings and we establish several characterizations of these concepts in terms of Hölder-like properties of the inverses of the mappings considered. In addition, we show that even if these properties are weaker than the classical notions of regularity for set-valued maps, they allow us to solve variational inclusions under mild assumptions.

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An inverse mapping theorem for H-differentiable set-valued maps

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Keywords: generalized differentiability, positively homogeneous maps, inner norm, outer norm, Fisher convergence

Taking advantage of recent developments in the theory of generalized differentiation, we present an inverse mapping theorem for set-valued maps and prove its stability under small linear perturbations. Using variational convergences of set-valued mappings, we present as well an approximate version of our inverse theorem.

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A Newton iteration for differentiable set-valued maps

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Keywords: set-valued maps, Newton iteration

We employ recent developments of generalized differentiation concepts for set-valued mappings and present a Newton-like iteration for solving generalized equations of the form $f(x) + F(x) \ni 0$ where f is a single-valued function while F stands for a set-valued map, both of them being smooth mappings acting between two general Banach spaces X and Y. The Newton iteration we propose is constructed on the basis of a linearization of both f and F; we prove that, under suitable assumptions on the derivatives of f and F, it converges Q-linearly to a solution to the generalized equation in question. When we strengthen our assumptions, we obtain the Q-quadratic convergence of the method.

Second-order Karush–Kuhn–Tucker optimality conditions for problems with continuously differentiable data

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Keywords: nonlinear programming, nonsmooth optimization, second-order Karush–Kuhn-Tucker necessary optimality conditions, second-order constraint qualifications

Some necessary and sufficient optimality conditions for inequality constrained scalar problems with continuously differentiable data were obtained in the papers [1, 4].

In the present work, we continue these investigations. We obtain some new second-order necessary conditions of Karush–Kuhn–Tucker (in short, KKT) type for a local minimum.

Consider the nonlinear programming problem with inequality constraints

Minimize $f(x)$ subject to $g_i(x) \leqq 0,\ i = 1, 2, \ldots, m,$ (P) where f and g_i, $i = 1, 2, \ldots, m$ are scalar functions, defined on some open set X, $X \subset \mathbb{R}^n$.

We introduce a new second-order constraint qualification (in short, CQ) of Zangwill type (see [5]).

We obtain second-order KKT necessary optimality conditions for the problem (P) in terms of this CQ. Our second-order CQ fits to problems with C^1 data. In our knowledge, it is an open question to apply second-order CQ in such problems.

The Guignard CQ (see [3]) is the most general one among the known first-order CQs. The first-order KKT necessary conditions in terms of the Guignard CQ are particular case of our results.

Additionally, we introduce a second-order Slater CQ. It extends the Slater CQ (see [3]). This CQ arises from the notion of a second-order pseudoconvex function, which was introduced in the paper [2]. We derive more optimality conditions applying the second-order Slater CQ.

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Generic solvability of some optimisation problems involving integral functionals

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Keywords: integral functional, solvability

We develop new perturbed optimisation method suited for integral functional. We apply it to the following two problems.

Let f be convex and lower semicontinuous function on Banach space X such that $f(0) = 0$ is a strong minimum of f and let $a \in X$ be such that $f(a) > 0$. The first problem is:

$$\int_0^\infty \|v(t)\|^2 + f(u(t))\,dt \to \min,$$

where $v \in L_2([0,\infty), X)$ and $u(t) = \int_0^t v(s)\,ds$.

For the second problem f is positive and lower semicontinuous. Note that it is not required to be convex. The problem is:

$$\int_0^1 f(u(t))\,dt \to \min,$$

where $u : [0,1] \to X$ is 1-Lipschitz and $u(0) = 0$.

For each of the problems, we show that there is arbitrary small convex perturbation of f such that the corresponding problem has solution.

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A solvability theorem for relaxed one-sided Lipschitz algebraic inclusions and some consequences

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Keywords: solvability theorem, relaxed one-sided Lipschitz multivalued mappings, differential and algebraic inclusions, numerical methods

In this talk, a recently published solvability theorem for relaxed one-sided Lipschitz algebraic inclusions will be proved and examined. Its applications include an implicit function theorem and a zero-finding method for algebraic inclusions, implicit numerical schemes for ordinary differential inclusions as well as schemes that only propagate the boundaries of reachable sets, and an existence and stability result for elliptic partial differential inclusions. I will briefly sketch these applications and the role of the solvability theorem in these contexts.

Newton diagrams and sums of squares

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Keywords: semidefinite programming, Newton diagrams

A polynomial optimization problem is a problem to find a minimum of a polynomial function over the set defined by polynomial equations and inequalities. There is an algorithm which obtains the global minimum, by solving a sequence of semidefinite programming.

The key of the algorithm is the property that a given polynomial can be written as a sum of square polynomials (SOS-representability). On the other hand, SOS-representability has strong connections with optimality conditions. We study rolls of optimality conditions using Newton diagrams in SOS-representability.

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Loci of complex polynomials

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Keywords: zeros and critical points of polynomials, apolarity, locus, polar derivative, Grace's theorem, Grace-Walsh-Szegő coincidence theorem, complex Rolle's theorem, Laguerre's theorem

For every complex polynomial $p(z)$, closed point sets are defined, called loci of $p(z)$. A closed set in the extended complex plane is a locus of $p(z)$ if it contains a zero of any of its apolar polynomials and is the smallest such set with respect to inclusion. Using the notion locus, several classical theorems in the geometry of polynomials can be refined. We establish several fundamental properties of the loci and investigate their intriguing connections with the higher-order polar derivatives of $p(z)$. This is a joint work with Blagovest Sendov.

Acknowledgements. The first author was partially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada

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On metric regularity of Voronoi cells

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Keywords: Voronoi cells

Let T be a given subset of $\mathbf{R}^n$, whose elements are called sites, and let $s \in T$. The Voronoi cell of s with respect to T consists of all points closer to s than to any other site. In this talk, we consider the stability of the Voronoi cells from a different perspective, as those given in the papers of Goberna, Reem, etc. We try to investigate the property of metric regularity, which is a central concept in variational analysis. In the special case, where the Voronoi sites is a discrete set, we have found all the couples of sites and points in $\mathbf{R}^n$, where the solution mapping, associated with the Voronoi cells is metrically regular.

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