A Multiple Access System for a Disjunctive Vector Channel

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Outline

1. Task Statement
2. Signal-code construction
3. Probability of denial
4. Relative group rate
Our task is to propose a signal-code construction using A-channel and to study the properties of multiple access system built on the basis of this construction.
Let us denote the number of active users by $S$, $S \geq 2$.

**Input** at time $t$

Vectors $\mathbf{x}_i^{(t)} \in \{0, 1\}^q$, $|\mathbf{x}_i^{(t)}| = 1, \quad i = 1, \ldots, S$.

**Output** at time $t$

$y^{(t)} = \bigvee_{i=1 \ldots S} \mathbf{x}_i^{(t)}$

The channel is noiseless.
Transmission

Each user encodes the information transmitted by \( q \)-ary \((n, k, d)\)-code \( C \) (all users use the same code). Consider the process of sending the message by \( i \)-th user.

Let us denote the codeword to be transmitted by \( c_i \), each symbol \( c_i \) is associated with a binary vector of length \( q \) and weight 1, the unit is in a position corresponding to the element of \( GF(q) \) to be transmitted. We denote the matrix constructed in this way by \( C_i \).

Transmission is performed symbol by symbol. Before sending a binary vector a random permutation of its elements is performed. The permutations used are selected independently and with equal probability.
Transmission

Example

Let $q = 3$, $C = \{(0,0,0,0),(1,1,1,1),(2,2,2,2)\}$, $c_i = (1,1,1,1)$. Let the mapping $\left( GF(q) \rightarrow \{0,1\}^q \right)$ be defined in such a way: $0 \rightarrow (100)^T$, $1 \rightarrow (010)^T$, $2 \rightarrow (001)^T$, then

\[
C_i = \begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
The base station sequentially receives messages from all users. Let us consider the process of receiving a message from the $i$-th user. At receiving of each column the reverse permutation is performed. Thus, we obtain the matrix

$$
Y_i = C_i \lor \left( \bigvee_{m=1 \ldots S, m \neq i} X_m \right),
$$

where $C_i$ is a matrix corresponding to $c_i$ and matrixes $X_m, m = 1 \ldots S, m \neq i$ are the results of another users activity.
For all \( c_t \in C \)

1. construct a matrix \( C_t \) corresponding to \( c_t \).
2. if the condition \( C_t \land Y_i = C_t \) follows add \( c_t \) to a list of possible codewords.
3. go to next \( c_t \).

In case of only one word in the list output the word, else output a denial of decoding.
Probability of denial

**Theorem**

The estimate follows

\[ p_\ast \leq \sum_{W=d}^{n} \left[ A(W) \left( 1 - \left( 1 - \frac{1}{q} \right)^{S-1} \right)^W \right] < \]

\[ < q^k \left( 1 - \left( 1 - \frac{1}{q} \right)^{S-1} \right)^d, \]

where \( A(W) \) is a number of codewords of weight \( W \) in a code \( C \).
Corollary

Let $q, k, S$ and $p_r$ be fixed, than if the condition

$$d \geq \frac{k - \log_q p_r}{\beta},$$

follows, where $\beta = -\log_q \left(1 - \left(1 - \frac{1}{q}\right)^{S-1}\right)$, than $p_* < p_r$
Definitions

The rate for one user

\[ R_i(q, S, k, c) = \frac{k}{n(q, S, k, c)} \log_2 q. \]

Group rate

\[ R_\Sigma(q, S, k, c) = S \frac{k}{n(q, S, k, c)} \log_2 q. \]
Dependencies of group rate on number of active users

\[ q = 2^{11}, \ p_r = 10^{-10} \]

\[ q = 2^{13}, \ p_r = 10^{-10} \]
Relative number of users

\[ \gamma = S/q. \]

Relative asymptotic group rate \((p_r = 2^{-cn}, c > 0)\)

\[ \rho(\gamma, k, c) = \lim_{q \to \infty} \frac{R_\Sigma(q, \gamma q, k, c)}{q}. \]
An asymptotic estimate of group rate

**Theorem**

If \( \gamma < -\ln (1 - 2^{-c}) \) than the following inequality follows

\[
\rho (\gamma, k, c) \geq \rho (\gamma, c) = \gamma \left( \log_2 \left( \frac{1}{1 - e^{-\gamma}} \right) - c \right).
\]

Let us introduce a notion

\[
\rho^* (c) = \max_{\gamma} \left[ \rho (\gamma, c) \right].
\]
The dependency of $\rho^*(c)$ on $c$

Note that $\rho^*(\varepsilon) \geq (1 - \varepsilon) \ln 2 = (1 - \varepsilon)0.693 \ldots$
Main results:

1. A novel signal-code construction has been proposed. The construction does not need block synchronization.

2. A lower bound on a group rate in the multiple access system built on the basis of this construction is derived. The bound coincides with an upper bound in case of $c = \varepsilon$. 

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Thank you for the attention!