Quadrature Formulae Based on Interpolation by Parabolic Splines

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The standard way for construction of quadrature formulae is based on polynomial interpolation. A good reason for such an approach is the Weierstrass theorem about density of algebraic polynomials in the space $C[a, b]$, with $[a, b]$ being the integration interval (supposed to be finite and closed). However, there are lot of suggestions in the literature that in many situations spline functions provide better tool for approximation than polynomials, especially when the approximated functions are of low smoothness.

In the early nineties of the last century the first named author initiated study of quadrature formulae based on spline interpolation. In 1993 he formulated a conjecture about the asymptotical optimality of the Gauss-type quadratures associated with the spaces of polynomial splines with equidistant knots in the Sobolov classes of functions. This conjecture was proved (in a joint paper with P. Köhler) in 1995, and since then several papers devoted to Gaussian quadratures associated with spaces of low degree splines appeared.

A basic difficulty in the construction of Gaussian quadrature formulae associated with a given linear space of spline functions is to determine the mutual location of the quadrature nodes and the spline knots. Moreover, the highest "spline degree of precision" is achieved at the expense of irregular nodes distribution, and hence lack of possibility for building sequences of quadratures with nested nodes.

Here we present quadrature formulae, based on interpolation with parabolic splines with equidistant knots on a regular mesh of interpolation nodes. Explicit formulae are found for the weights, which are shown to be all positive. The numerical experiments indicate that our quadratures are competitive to the classical quadrature formulae based on polynomial interpolation.