Discontinuous Galerkin Finite Element Method for Convection-Diffusion Problems and Compressible Flow

M. Feistauer

In this paper we shall be concerned with several aspects of the numerical solution of nonlinear, nonstationary, convection-diffusion problems by the discontinuous Galerkin finite element method (DGFEM) and applications to compressible flow. The DGFEM is based on a piecewise polynomial approximation of the sought solution without any requirement on the continuity on interfaces between neighbouring elements. It is particularly convenient for the solution of conservation laws with discontinuous solutions or singularly perturbed convection-diffusion problems with dominating convection, when solutions contain steep gradients.

In the first part we shall be concerned with theoretical analysis of error estimates of various versions of the DG discretization applied to a scalar initial-boundary value problem. We shall discuss the error estimates for space semidiscretization and some types of full space-time discretization.

In the second part, some applications of the DGFEM to the simulation of compressible flow, i.e. the solution of the compressible Euler and Navier-Stokes equations, will be presented. Our goal is to develop sufficiently accurate, efficient and robust numerical schemes allowing the solution of compressible flow for a wide range of Reynolds and Mach numbers, applicable to flow simulation in time dependent domains and to fluid-structure interaction. The efficiency and accuracy of the method will be demonstrated by computational results.