Generalized Expo-rational B-splines and Unconditionally Stable Explicit Finite Element Methods for Initial-Value Problems for ODEs

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The exposition will begin with a very concise overview of relevant recent progress in the theory of expo-rational B-splines (ERBS) and their generalizations (GERBS) including, among others, the so-called Beta-function B-splines (BFBS). The generalized Vandermonde matrix for Hermite interpolation using these new B-spline bases (which have recently been designed also for Hermite interpolation on scattered point sets in domains of any dimension) is always block-diagonal, in Jordan normal form. Thus, these B-splines provide an excellent isogeometric representation, being simultaneously a convenient and efficient tool in Computer Aided Geometric Design and a similarly convenient and efficient tool for solving initial-value and boundary-value problems for domains with difficult geometry in dimensions 2, 3, 4, and higher, and for high-order PDEs. In the present communication, which will be first on this topic, we shall start only with the simplest type of 1-variate problems: Cauchy problems for linear ODEs of any order with variable coefficients and right-hand side. As we shall show, already in this simple case the new type of B-splines makes an impact. The approximate solution is an Hermite interpolant based on GERBS over a possibly non-uniform knot-vector, and the numerical solution has the following remarkable properties: (a) The issue of stability of the numerical solution is completely eliminated. The numerical solution is always stable for any knot-vector, provided that there is an a priori estimate for the *exact* solution. (b) The ERBS-based Hermite interpolant has transfinite order of accuracy. (c) For the initial-value problems considered, the stiffness matrix is upper-triangular and band-limited, i.e., the method is explicit. (d) Modification and refining of a mesh lead to a very easy recomputation of the solution. Thus, multigrid methods with such approach are easy in implementation, cheap in computations, and very fast in convergence. (e) The method works without any modifications also when the ODE degenerates (has variable order).

If the presentation time and the publication space permits, we may briefly consider the question about preservation of order constraints by the GERBS-based approximate solution in the cases of positivity, monotonicity, convexity, k-monotonicity of the exact solution, as well as the question about one-sided approximation in these cases.