On the Numerical Solution of a Chemotaxis System in Haematology

G. Bencheva

Many haematological diseases, including various types of leukaemia, are caused by abnormal production of particular blood cells. Their treatment consists mainly of two steps. The first step is chemotherapy and a whole body irradiation to eradicate the patient's haematopoietic system. The second step is the transplantation of haematopoietic stem cells (HSCs) obtained from the mobilized peripheral blood of a donor. After transplantation, HSCs find their way to the stem cell niche in the bone marrow. It has been shown that HSCs migrate *in vitro* and *in vivo* following the gradient of a chemotactic factor SDF-1 produced by stroma cells. Upon homing HSCs have to multiplicate rapidly to regenerate the blood system. Adequate computer models for these steps would help medical doctors to shorten the period in which the patient is missing their effective immune system.

Our attention at this stage is focused on the numerical solution of the mathematical model for the chemotactic movement of HSCs, proposed by A. Kettemann, M. Neuss-Radu in 2008. It consists of a nonlinear system of chemotaxis equations coupled with an ordinary differential equation on the boundary of the domain in the presence of nonlinear boundary conditions. The unknowns of the system are the concentrations of HSCs, of SDF-1, and of the stem cells bound to the stroma cells. Various classical numerical methods applied directly to a general chemotaxis system and in particular to HSCs migration model may lead to numerical instabilities and loss of the positivity property of the solution. A finite-volume method, based on a second-order positivity preserving central-upwind scheme is proposed by A. Chertock and A. Kurganov in 2008 for a class of chemotaxis and haptotaxis models with homogeneous Neumann conditions. Their approach is applied in the current paper for the numerical solution of the HSCs migration model. Special attention is focused here on the approximation of the ODE and of the boundary conditions. Theoretical analysis of the stability, consistency, convergence and positivity properties of the modified scheme is made. Results of numerical tests illustrating the theoretical estimates are also presented.