

Generalized Expo-rational B-splines and Isogeometric Analysis

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At the Seventh International Conference on Mathematical Methods for Curves and Surfaces in Tønsberg, Norway, in June 2008, at the lecture of Thomas J.R. Hughes, on which I was also present, Tom Hughes informed the geometric modelling community of the world of his vision of a united approach to geometric modelling in Computer Aided Geometric Design (CAGD) and finite element analysis (FEA) in the modelling and simulation via boundary-value problems for PDEs. The main common tool which Tom Hughes proposed was the current industrial standard in CAGD: Non-Uniform Rational B-splines (NURBS), and the NURBS-based methods proposed by him gave the start of *Isogeometric Analysis*.

On the next day of the afore-mentioned conference, I gave for the first time a communication on the topic of Generalized Expo-rational B-splines (GERBS), with Tom Hughes, Larry Schumaker, Tom Lyche and other well-known spline specialists in the audience. What seemed to impress the audience most, was the possibility to easily construct GERBS-based smooth convex partitions of unity on triangulations, where each GERBS had the support of the usual piecewise linear/affine B-spline (i.e., the star-1 neighbourhood of 'its' vertex in the triangulation) while at the same time GERBS was smooth, and multiplication of each GERBS with a coefficient which was not constant, but a Taylor polynomial 'around the vertex of the GERBS' immediately implied Hermite interpolation at this vertex of all derivatives present in the Taylor polynomial. The conversion to 'Bezier form' was also done effortlessly by simply changing the monomial basis in the Taylor polynomial around each vertex with respective *tensor-product* Bernstein basis; moreover, this conversion was done independently for every vertex in the triangulation, i.e., the procedure was readily parallelized.

During and after the conference there was a lot of interest in the multivariate constructions based on GERBS, but I requested (at least) one year more to work on the development of the theory before starting to publish relevant results. This is why the first results on this topic began to appear in the late 2009 and in 2010.

The purpose of the present communication is to inform the Numerical Analysis community about the great potential GERBS have in CAGD and FEA, and to convince the audience that the Non-Uniform Rational Generalized Expo-Rational B-splines (NURGERBS) vastly outperform NURBS as a universal tool of Isogeometric Analysis.