Optimal Order Multilevel Solvers in a Projection Scheme for Navier-Stokes Equations

P. Boyanova, S. Margenov

The numerical solution of the incompressible Navier-Stokes equations has been the focus of the computational fluid dynamics community for over six decades. However, the pursuit of constructing optimal schemes, in terms of computational cost and accuracy, is still not over.
We consider the implementation of a projection scheme which is based on nonconforming Crouzeix-Raviart finite element approximation of the velocities and piece-wise constant approximation of the pressure. The most significant advantage of this approximation is that the divergence of the velocity field is zero inside each element, i.e. the approximation is locally conservative.
We show that some recently developed Algebraic MultiLevel Iteration (AMLI) preconditioners can be successfully applied to get a composite time-stepping solution method which has a total computational complexity of optimal order. ¿From computational point of view, the prediction step consists of two decoupled scalar parabolic problems. At the projection step a coupled mixed FEM problem is to be solved. Eliminating the velocities we reduce this problem to a system with a weighted graph-Laplacian for the pressure unknowns.
The presented numerical tests aim to show both the numerical stability and the computational efficiency of the algorithm.