On the Mean Square Worst-Case Error of the Quasi-Monte Carlo Integration in Weighted Sobolev Spaces

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In our talk, we will consider problems of the multivariate quasi-Monte Carlo integration in weighted Sobolev spaces, which are reproducing kernel Hilbert spaces. We consider a concrete weighted Sobolev space $H_{Sob,s,\gamma,\mathcal{B}_\mathbf{4}}$, containing functions which partial derivatives up to order two have to be square integrable. This space has a reproducing kernel, based on using the Bernoulli polynomials up to fourth degree.
We will approximate the integrals through quasi-Monte Carlo algorithm with equal quadrature weights. We use a randomization of deterministic sample point nets, called (s,b)-digital shift, and consider the notion of mean square worst-case error of the integration in reproducing kernel Hilbert spaces.
As a tool of our investigation we use the Walsh functional system in base $b \geq 2$.
We obtain an exact formula for the mean square worst-case error of the integration in the space $H_{Sob,s,\gamma,\mathcal{B}_\mathbf{4}}$. This formula is an expresion in the terms of the Walsh functions in base b and the Fourier-Walsh coefficients of the reproducing kernel, which generates the space $H_{Sob,s,\gamma,\mathcal{B}_\mathbf{4}}$.
The formula for the mean square worst-case error is applied to two concrete choices of digital $b-$adic nets. First, we use an arbitrary $(t,m,s)-$net in base b and obtain the order of the mean square worst-case error. Second, we use the uniform lattice point net in base b and obtain the order of the mean square worst-case error. The obtained orders are compared.