Analysis of the Constant in the Strengthened Cauchy-Bunyakowski-Schwarz Inequality for Quadratic Finite Elements

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This article considers the second order scalar elliptic boundary value problem. By discretisation using conforming quadratic finite elements, the problem can be reduced to finding a solution of a linear system of algebraic equations. Preconditioners based on various multilevel extensions of two-level finite element methods (FEM) lead to iterative methods which often have an optimal order of computational complexity with respect to the number of degrees of freedom of such a system. The key role in the derivation of optimal convergence rate estimates is played by the constant γ in the strengthened Cauchy-Bunyakowski-Schwarz (CBS) inequality, associated with the angle between the two subspaces of the splitting. More precisely, the upper bound for $\gamma \in [0, 1)$ contributes significantly to the construction of various multilevel extensions of the related two-level methods.

We study the behavior of the CBS constant for the Differences and Aggregates (DA) and First Reduce (FR) splitting of the unknowns of the system which would ultimately lead to the construction of an efficient multilevel preconditioner. The presented CBS constant estimates utilise the hierarchical basis $p - method$ for $p = 2$ where piecewise linear and piecewise quadratic basis functions are used at the vertex and mid-edge points respectively. The obtained estimates are compared with earlier results. The numerical tests are run on the software package Mathematica.