Convergence of Finite Difference Schemes for a Multidimensional Boussinesq Equation

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We consider the Cauchy problem for the nonlinear Boussinesq equation

$$\frac{\partial^2 u}{\partial t^2} = \Delta u + \beta_1 \Delta \frac{\partial^2 u}{\partial t^2} - \beta_2 \Delta^2 u + \alpha \Delta f(u), \quad x \in \mathbb{R}^d, \ t > 0,$$
$$u(x,0) = u_0(x), \quad \frac{\partial u}{\partial t}(x,0) = u_1(x),$$

where α, β_1, β_2 are positive constants and the solution u additionally satisfies the asymptotic boundary conditions $u(x,t) \to 0$, $\Delta u(x,t) \to 0$ as $|x| \to \infty$. Typically, the nonlinear term is $f(u) = u^2$.

Depending on the way the nonlinear term $f(u)$ is approximated, we develop two families of finite difference schemes.

We obtain error estimates for these numerical methods in Sobolev space.

The extensive numerical experiments for the one-dimensional problem show good precision and full agreement between the theoretical results and practical evaluation for single soliton and the interaction between two solitons.