Stochastic Algorithms in Linear Algebra - beyond the Markov Chains and Neumann - Ulam Scheme

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In this talk I will present some new ideas around the stochastic approach for large systems of equations, in particular, solving very large systems of linear equations, randomized singular value decomposition (SVD) based methods for solving integral equations and ‘ ill-posed and inverse problems. In Monte Carlo methods, one has to cope often with very large dimensions, in problems like the integration, solution of integral equations, PDEs,simulation of random fields, etc. It is customary to think that the Monte Carlo methods based on the Markov chains and Neumann - Ulam scheme are able to resolve problems for very high dimensions, however it is true only under the following restricted conditions: (1) the variance of the MC estimator is small, (2) the desired accuracy is not high, (3) the complexity of construction of the random estimator is a slow function of the dimension. The condition (3) can be often fulfilled, however the conditions (1) and (2) are of the main concern, because the convergence rate of MC methods is slow, scaling as $\sigma/N^{1/2}$ where σ is the standard deviation, and N is the sample size. In my talk I will present absolutely other approach, beyond the Markov chains and Neumann - Ulam scheme, which is much closer to deterministic methods but which uses some remarkable probabilistic properties of large matrices. So I will discuss in details the SVD based randomized low rank approximation methods which is capable to handle systems of huge dimension. Another important class of stochastic methods I am going to talk about is the randomized version of projection methods. Applications to different practical problems in random field simulation, transport in porous media, crystal structure analysis by x-ray diffraction, nucleation and coagulation of large ensembles of interacting particles.